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IMPLEMENTING FRIES MODEL FOR THE FIXED LIFETIME INVENTORY SYSTEM

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Abstract

Many methods have being used for finding optimal ordering policy for the fixed lifetime inventory system. The dynamic programming approach is one of such methods. Hitherto, the claim is that the approach is not applicable to systems with lifetime greater than two periods. In this work, we used a total cost function in the literature to derive the equations for the ordered quantity in each period, for a products with lifetime greater than two periods. MATHEMATICA 8 was used to solve the equations.

***Keywords*:** inventory, dynamic programming, fixed lifetime, useful lifetime, ordering policy.

1. Introduction

The dynamic programming approach is one of the methods used in obtaining optimal solution for multi-stage inventory problems (other methods were discussed in[1] and [4]). Fixed lifetime inventory model which include the assumptions that orders are placed every period can be formulated as a dynamic programming problem. This is exemplified by the works in [2] and [3] . To use the method, the associated cost function is formulated as a recursive equation. The dynamic programming approach for the fixed lifetime inventory problem was extensively discussed in [3] and concluded that "Because the dimension of the state variable is proportional to the lifetime of the stock in periods, computing an optimal policy is feasible only for relatively short lifetimes. One quickly faces the "curse of dimensionality" that plagues many dynamic programming formulations. We re-examine the dynamic programming approach for the fixed lifetime inventory system using the model in [3] as a case study. Ordering policies were obtained for each period and the resulting equations were solved using MATHEMATICA 8. Numerical examples was carried out for items with lifetime between 3 and 21periods.

Fries Model.

The dynamic programming approach is applicable whenever we order m times in m periods. For example, if $m = 4,\ m \textit{ is lifetime of product}$, we order four times in four periods. The number of useful periods remaining on the items brought forward after their first period in inventory is $n = 3$. New orders are received every period and items are issued from inventory following FIFO (oldest units first). Table 1 gives the model outlook for $m = 4$.

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Table1: model outlook for $m = 4$

Period	1	2	3	4	5	6	7	8	9	10	11	12
	y_1	x_{12}	x_{13}	x_{14}								
		y_2	x_{22}	x_{23}	x_{24}							
			y_3	x_{32}	x_{33}	x_{34}						
				y_4	x_{42}	x_{43}	x_{44}					
					y_5	x_{52}	x_{53}	x_{54}				
						y_6	x_{62}	x_{63}	x_{64}			
							y_7	x_{72}	x_{73}	x_{74}		
								y_8	x_{82}	x_{83}	x_{84}	
									y_9	x_{92}	x_{93}	x_{94}
										y_{10}	$x_{10,2}$	$x_{10,3}$
											y_{11}	$x_{11,2}$
												y_{12}

In Table1, y_1 arrives in period 1. At the start of period 2, items from the first order reduces to x_{12} and y_2 arrives. At the start of period 3, items from the first and second orders reduces to x_{13} *and* x_{22} and y_3 arrives. At the start of period 4, items from the first, second and third orders reduces to x_{14}, x_{23} *and* x_{32} and y_4 arrives. The items from the first order will not go beyond period 4 since the lifetime of the product is 4. Any item from the first order not used to meet demand at the end of period 4 outdate and must be discarded. The process continues for the remaining orders. This was the dynamic process considered in [3]. The total cost function obtained is ;

$$f_n(w) = \min_{y \geq 0}\{-cW_1 + V(w_{m-1}) + G_{n-1}(w, y)\},\ w \geq 0,\ (n = 1,2,3,...) \qquad (1)$$

where

$$G_{n-1}(w, y) = cy + L(y) + \alpha \int_0^\infty f_{n-1}(z)\ \phi(t)\, dt$$

$$L(y) = h\int_0^y (y-t) f(t)dt + p\int_y^\infty (t-y) f(t)dt$$

$$V(x) = v\int_0^x (x-t) f(t)dt$$

$c = ordering\ \cos t$

$h = holding\ \cos t$

$p = shortage\ \cos t$

$v = outdate\ \cos t$

$\alpha = discounting\ \ factor,\ 0 < \alpha < 1$

$m = lifetime$

$y = order\ quantity$

$w_j = w_1 + w_2 + .. + .w_{m-1}$ *on hand inventory*

$w = (w_1, w_2, ...w_{m-1}) \geq 0$

The model in [3] has a parameter $\overline{x}$, that determine whether to order or not to order. $\overline{x}$ is defined as the positive solution to the equations

$$L^1(x) + c = 0 \tag{2}$$

or

$$L^1(x) + c - \alpha c F(x) = 0 \tag{3}$$

If $x < \overline{x},$ where $x = x_1 + x_2 + ... + x_{m-1}$, we order y. Where y is the solution to the equation

$$G^1{}_{n-1}(w, y) = cy + L(y) + \alpha \int_0^\infty f_{n-1}(z)\phi(t)dt \tag{4}$$

If $x \geq \overline{x}$ we do not order.

The time horizon in [3] is divided into three time eras, namely:
$n = 1 \;,\quad 1 < n < m \quad and \quad n \geq m$

The decision to order or not to order in each era depends on whether or not the total inventory on hand is less than the critical number . For each time era in equation (1), we derive an equation for the critical number and the ordered quantity.

$$L(y) = h\int_0^y (y-t) f(t)dt + p\int_y^\infty (t-y) f(t)dt$$

$$V(x) = v\int_0^x (x-t) f(t)dt$$

$$f_n(w) = \min_{y \geq 0}\{-cW_1 + V(w_{m-1}) + G_{n-1}(w, y)\}$$

$$G_{n-1}(w, y) = cy + L(y) + \alpha\int_0^\infty f_{n-1}(z)\phi(t)dt$$

$$f_0(w) = 0$$

$$f_n(w) = -cW_1 + V(w_{m-1}) + cy + L(y) + \alpha\int_0^\infty f_{n-1}(z)\phi(t)dt\,;\; n = 1,2,3,...m-1$$

$$f_1(w) = -cW_1 + V(w_{m-1}) + cy + L(y)$$

$$f_2(w) = -cW_1 + V(w_{m-1}) + cy + L(y) + \alpha\int_0^\infty f_1(z)\phi(t)dt$$

$$f_3(w) = -cW_1 + V(w_{m-1}) + cy + L(y) + \alpha\int_0^\infty f_2(z)\phi(t)dt$$

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$$f_{m-1}(w) = -cW_1 + V(w_{m-1}) + cy + L(y) + \alpha\int_0^\infty f_{m-2}(z)\phi(t)dt$$

Case 1: $n = 1$

If $W_1 < x^*$, order up to x^*, otherwise, do not order; where x^* is the unique solution to the equation

$L^1(x^*) + c = 0$ *and the demand density is* $f(t) = \lambda e^{-\lambda t}$
where

$$L(x^*) = h\lambda\int_0^{x^*}(x^* - t)e^{-\lambda t}dt + p\lambda\int_x^{\infty}(t - x)e^{-\lambda t}dt$$

$$L(x^*) = h\lambda\{\int_0^x xe^{-\lambda t}dt - \int_0^x te^{-\lambda t}dt\} + p\lambda\{\int_x^{\infty} te^{-\lambda t}dt - \int_x^{\infty} xe^{-\lambda t}dt\}$$

$$= h\lambda\{\frac{-xe^{-\lambda t}}{\lambda}\Bigg|_{t=0}^{t=x} + (\frac{te^{-\lambda t}}{\lambda} + \frac{e^{-\lambda t}}{\lambda^2})\Bigg|_{t=0}^{t=x}\} + p\lambda\{(\frac{-te^{-\lambda t}}{\lambda} - \frac{e^{-\lambda t}}{\lambda^2})\Bigg|_{t=x}^{t=\infty} + \frac{ye^{-\lambda t}}{\lambda}\Bigg|_{t=x}^{t=\infty}\}$$

$$= h\lambda\{\frac{-xe^{-x\lambda}}{\lambda} + \frac{x}{\lambda} + \frac{xe^{-x\lambda}}{\lambda} + \frac{e^{-x\lambda}}{\lambda^2} - \frac{1}{\lambda^2}\} + p\lambda\{\frac{xe^{-x\lambda}}{\lambda} + \frac{e^{-x\lambda}}{\lambda^2} - \frac{xe^{-x\lambda}}{\lambda}\}$$

$$= h\lambda\{\frac{-xe^{-x\lambda}}{\lambda} + \frac{x}{\lambda} + \frac{xe^{-x\lambda}}{\lambda} + \frac{e^{-x\lambda}}{\lambda^2} - \frac{1}{\lambda^2}\} + p\lambda\{\frac{xe^{-x\lambda}}{\lambda} + \frac{e^{-x\lambda}}{\lambda^2} - \frac{xe^{-x\lambda}}{\lambda}\}$$

$$= hx + \frac{he^{-x\lambda}}{\lambda} - \frac{h}{\lambda} + \frac{pe^{-x\lambda}}{\lambda}$$

$$= e^{-x\lambda}\{\frac{h}{\lambda} + \frac{p}{\lambda}\} + hx - \frac{h}{\lambda}$$

so that

$$\frac{dL}{dx^*} + c = 0, \text{ yields}$$

$$-e^{-x\lambda}\{h + p\} + h + c = 0$$

$$e^{-x\lambda}(h + p) = h + c$$

$$e^{-x\lambda} = \frac{h + c}{h + p}$$

$$x^* = \frac{Log[\frac{h + c}{h + p}]}{-\lambda} \qquad (5)$$

This shows we can only proceed if $c < p$.

To obtain W_1 , we differentiate $f_1(w)$ with respect to y and evaluate at $y = W_1$ Thereafter we compare the value of x from equation (5) with W_1 obtained. If W_1 obtained is less than x then we order up to x that is order $y_1 = x - W_1$ otherwise we do not order. Now

$$f_1(w) = -cW_1 + V(w_{m-1}) + cy + L(y) + \alpha\int_0^{\infty} f_0(w)\phi(t)dt$$

but $f_0(w) = 0$, *so we have*

$$f_1(w) = -cW_1 + V(w_{m-1}) + cy + L(y)$$

$$\frac{\partial f_1}{\partial y}\Bigg|_{y=W_1} = L^1(W_1) + c$$

W_1 is the solution to the equation

$$L^1(W_1)+c=0$$
$$-e^{-W_1\lambda}(\frac{h}{\lambda}+\frac{p}{\lambda})+h+c=0$$
$$e^{-W_1\lambda}(h+p)=h+c$$
$$e^{-W_1\lambda}=\frac{h+c}{h+p}$$
$$W_1=\frac{Log[\frac{h+c}{h+p}]}{-\lambda} \qquad (5.1)$$

Case2: $1<n<m$

If $w\in A_n,(where\ A_n$,is the ordering region) order up to $y_n^*(w)>W_1$; otherwise, do not order; where $y_n^*(w)$ solves $G_{n-1}^1\{w,y_n^*(w)\}=0\ and\ y_n^*(w)\le x$, and x is the solution to the equation

$$L^1(x)+c-\alpha cF(x)=0$$
$$where$$
$$F(x)=1-e^{-\lambda t}$$
$$L(x)=h\lambda\int_0^x(x-t)e^{-\lambda t}dt+p\lambda\int_x^\infty(t-y)e^{-\lambda t}dt$$
$$L(x)=e^{-x\lambda}\{\frac{h}{\lambda}+\frac{p}{\lambda}\}+hx-\frac{h}{\lambda}$$
$$\frac{dL}{dx}=-e^{-x\lambda}(h+p)+h$$
$$so\ that$$
$$\frac{dL}{dx}+c-\alpha cF(x)=0 \qquad ,yields$$
$$-e^{-x\lambda}(h+p)+h+c-\alpha c(1-e^{-x\lambda})=0$$
$$-e^{-x\lambda}(h+p)+h+c-\alpha c+\alpha ce^{-x\lambda}=0$$
$$e^{-x\lambda}(h+p-\alpha c)=h+c-\alpha c$$
$$e^{-x\lambda}=\frac{h+c-\alpha c}{h+p-\alpha c}$$
$$x^*=\frac{Log[\frac{h+c-\alpha c}{h+p-\alpha c}]}{-\lambda} \qquad (6)$$

For each $n=2,3,...m-1$, we compute $y_n(w)$ from $G_{n-1}^1\{w,y_n(w)\}=0$ and We compare $y_n(w)\ \ with\ \ x$ obtained from (6). Table 2 and Table 3 gives the required equations for computing $y_n(w)$.

Table 2: $G_{n-1}(w,y),\ \ for\ \ some\ \ values\ \ of\ \ n.$

n	$G_{n-1}(w,y)$
2	$G_1(w,y)=cy+L(y)+\alpha\int_0^\infty f_1(w)\phi(t)dt$
3	$G_2(w,y)=cy+L(y)+\alpha\int_0^\infty f_2(w)\phi(t)dt$
4	$G_3(w,y)=cy+L(y)+\alpha\int_0^\infty f_3(w)\phi(t)dt$
5	$G_4(w,y)=cy+L(y)+\alpha\int_0^\infty f_4(w)\phi(t)dt$

Table 3: $G_{n-1}(w,y), \quad for \quad some \quad values \quad of \quad n.$

n	$G_{n-1}(w,y)$
2	$G_1(w,y) = cy + L(y) + \alpha\lambda\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y))e^{-\lambda t}dt$
3	$G_2(w,y) = cy + L(y) +$ $\alpha\lambda\{\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) + [\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y))e^{-\lambda t}dt)])e^{-\lambda t}dt\}$
4	$G_3(w,y) = cy + L(y) +$ $\lambda\alpha\{\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) + [\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) +$ $[\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y))e^{-\lambda t}dt)])e^{-\lambda t}dt])e^{-\lambda t}dt\}$
5	$G_4(w,y) = cy + L(y) +$ $\alpha\lambda\{\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) + [\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) +$ $[\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) + [\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y))e^{-\lambda t}dt])e^{-\lambda t}dt])e^{-\lambda t}dt)e^{-\lambda t}dt\}$

y_2 *is obtained from* $G_1^1(w,y) = 0$

Now

$$G_1(w,y) = cy + L(y) + \alpha\lambda\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y))e^{-\lambda t}dt$$

$$G_1(w,y) = cy + e^{-\lambda y}(\frac{h}{\lambda} + \frac{p}{\lambda}) + hy - \frac{h}{\lambda} - cW_1\alpha + cy\alpha + vx\alpha - vx\alpha e^{-x\lambda} - \frac{v\alpha}{\lambda} + \frac{v\alpha e^{-x\lambda}}{\lambda} + vx\alpha e^{-x\lambda}$$
$$+ \frac{\alpha e^{-\lambda y}(h+p)}{\lambda} + h\alpha y - \frac{h\alpha}{\lambda^2}$$

so that

$$\frac{\partial G_1}{\partial y} = c\alpha - \alpha e^{-\lambda y}(h+p) + h\alpha + c - e^{-\lambda y}(h+p) + h = 0$$

$$e^{-\lambda y}(\alpha(h+p) + (h+p)) = c + h + c\alpha + h\alpha$$

$$e^{-\lambda y} = \frac{c + h + c\alpha + h\alpha}{\alpha(h+p) + h + p}$$

$$y_2 = \frac{Log[\frac{c + h + c\alpha + h\alpha}{\alpha(h+p) + h + p}]}{-\lambda}$$

$$y_2 = \frac{Log[\frac{c+h)(1+\alpha)}{(h+p)(1+\alpha)}]}{-\lambda}$$

y_3 *is obtained from* $G_2^1(w,y)=0$

Now

$$G_2(w,y)=cy+L(y)+$$
$$\alpha\lambda\{\int_0^\infty(-cW_1+V(w_{m-1})+cy+L(y)+[\alpha\lambda(\int_0^\infty(-cW_1+V(w_{m-1})+cy+L(y))e^{-\lambda t}dt)])e^{-\lambda t}dt\}$$

$$G_2(w,y)=cy+\frac{e^{-\lambda y}}{\lambda}(h+p)+hy-\frac{h}{\lambda}-cW_1\alpha+cy\alpha+vx\alpha-vx\alpha e^{-x\lambda}-\frac{v\alpha}{\lambda}+\frac{v\alpha e^{-x\lambda}}{\lambda}+\frac{\alpha e^{-\lambda y}(h+p)}{\lambda}$$
$$+hy\alpha-\frac{h\alpha}{\lambda}-cW_1\alpha^2+cy\alpha^2+vx\alpha^2+vx\alpha^2e^{-x\lambda}-\frac{v\alpha^2}{\lambda}+\frac{v\alpha^2e^{-x\lambda}}{\lambda}+v\alpha^2xe^{-x\lambda}+\frac{\alpha^2e^{-y\lambda}(h+p)}{\lambda}$$
$$+h\alpha^2y+\frac{h\alpha}{\lambda^2}$$

so that

$$\frac{\partial G_2}{\partial y}=c-e^{-\lambda y}(h+p)+h+c\alpha-\alpha e^{-\lambda y}(h+p)+h\alpha+c\alpha^2-\alpha^2e^{-\lambda y}(h+p)+h\alpha^2=0$$

$$e^{-\lambda y}(h+p+\alpha(h+p)+\alpha^2(h+p))=c+h+c\alpha+h\alpha+c\alpha^2+h\alpha^2$$

$$y_3=\frac{Log[\frac{c+h+c\alpha+h\alpha+c\alpha^2+h\alpha^2}{h+p+\alpha(h+p)+\alpha^2(h+p)}}{-\lambda}$$

$$y_3=\frac{Log[\frac{c(1+\alpha+\alpha^2)+h(1+\alpha+\alpha^2)}{h+p(1+\alpha+\alpha^2)}]}{-\lambda}$$

$$y_3=\frac{Log[\frac{(c+h)(1+\alpha+\alpha^2)}{(h+p)(1+\alpha+\alpha^2)}]}{-\lambda}$$

y_4 *is obtained from* $G_3^1(w,y)=0$

$$G_3(w,y)=cy+L(y)+$$
$$\lambda\alpha\{\int_0^\infty(-cW_1+V(w_{m-1})+cy+L(y)+[\alpha\lambda(\int_0^\infty(-cW_1+V(w_{m-1})+cy+L(y)+$$
$$[\alpha\lambda(\int_0^\infty(-cW_1+V(w_{m-1})+cy+L(y))e^{-\lambda t}dt)])e^{-\lambda t}dt])e^{-\lambda t}dt\}$$

so that

$$\frac{\partial G_3}{\partial y}=c\alpha-\alpha e^{-\lambda y}(h+p)+h\alpha+c\alpha^2-\alpha^2e^{-\lambda y}(h+p)+h\alpha^2+c\alpha^3-\alpha^3e^{-\lambda y}(h+p)$$
$$+h\alpha^3+c-e^{-\lambda y}(h+p)+h=0$$

$$e^{-y\lambda}=\frac{h+c+h\alpha+c\alpha+h\alpha^2+c\alpha^2+h\alpha^3+c\alpha^3}{h+p+\alpha(h+p)+\alpha^2(h+p)+\alpha^3(h+p)}$$

$$y_4=\frac{Log[\frac{h+c+h\alpha+c\alpha+h\alpha^2+c\alpha^2+h\alpha^3+c\alpha^3}{h+p+\alpha(h+p)+\alpha^2(h+p)+\alpha^3(h+p)}]}{-\lambda}$$

$$y_4=\frac{Log[\frac{(h+c)(1+\alpha+\alpha^2+\alpha^3)}{(h+p)(1+\alpha+\alpha^2+\alpha^3)}}{-\lambda}$$

y_5 *is obtained from* $G_4^1 = 0$

$$G_4(w,y) = cy + L(y) +$$

$$\alpha\lambda\{\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) + [\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) +$$

$$[\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y) + [\alpha\lambda(\int_0^\infty(-cW_1 + V(w_{m-1}) + cy + L(y))e^{-\lambda t}dt])e^{-\lambda t}dt])e^{-\lambda t}dt)e^{-\lambda t}dt\}$$

$G_4^1(w,y) = 0$, *yields*

$$y_5 = \frac{Log[\frac{h + c + h\alpha + c\alpha + h\alpha^2 + c\alpha^2 + h\alpha^3 + c\alpha^3 + h\alpha^4 + c\alpha^4}{h + p + \alpha(h+p) + \alpha^2(h+p) + \alpha^3(h+p) + \alpha^4(h+p)}]}{-\lambda}$$

$$y_5 = \frac{Log[\frac{(h+c)(1+\alpha+\alpha^2+\alpha^3+\alpha^4)}{(h+p)(1+\alpha+\alpha^2+\alpha^3+\alpha^4)}]}{-\lambda}$$

A careful observation shows that the pattern for obtaining the order quantities can be generalized using equation (4.10)

$$y_n = \frac{Log\left[\frac{h + c + \sum_{i=1}^{n-1} h\alpha^i + c\alpha^i}{h + p + \sum_{i=1}^{n-1}\alpha^i(h+p)}\right]}{-\lambda}, n = 2.3.4, \ldots m-1 \qquad (4.10)$$

Case3: $n \geq m$

If $W_1 < x$, order up to $y_n(w) > W_1$; otherwise do not order, where $y_n(w)$ *solves* $G_{n-1}^1\{w, y_n(w)\} = 0$ and $y_n(w) \leq x$. The x^* in equation (4.9) for case 2 is the same for case 3.

4.4 NUMERICAL EXAMPLES.

In this section, we give numerical examples for fixed lifetime products whose useful lifetimes are 3, 4,12 and 21periods.The results for each product is shown in Tables 4,5,6,and 7.

EXAMPLE 1

Lifetime of product is 3. Using equations (5) and (6) we obtain x^* for time era $n = 1$ *and* $1 < n < m$.

Table 4: ordering policy for a product with 3 useful lifetime.

Time era	x^*	W_1	y
$n = 1$	78.4839	78.4839	-
$1 < n < m$	79.8595		78.4839
$n \geq m$			78.4839
			78.4839
			78.4839

Analysis: we don't order in period 1 because $W_1 = x^*$. We order 78.4839 in periods 2,3,4,and 5 since $y < x^*$. The CPU time is 0.48secs.

EXAMPLE 2

Lifetime of product is 4. Using equations (5) and (6) we obtain x^* for time era $n = 1$ *and* $1 < n < m$.

Table 5: ordering policy for a product with 4 useful lifetime.

Time era	x^*	W_1	y
$n = 1$	69.1485	69.1485	-
$1 < n < m$	84.4766		69.1485
			69.1485
$n \geq m$			69.1485
			69.1485
			69.1485
			69.1485

Analysis: we don't order in period 1 because $W_1 = x^*$. We order 69.1485 in periods 2,3,4,5,6 and 7 respectively. The CPU time is 0.532secs.

Example 3.

Lifetime of product is 12. Using equations (5) and (6) we obtain x^* for time era $n=1$ *and* $1<n<m$.

Table 6: ordering policy for a product with 12 useful lifetime.

Time era	x^*	W_1	y
$n=1$	95.3011	95.3011	-
$1<n<m$	97.6289		95.3007
			95.3007
			95.3007
			95.3007
			95.3007
			95.3007
			95.3007
			95.3007
			95.3007
			95.3007
$n \geq m$			95.3007
			95.3007
			95.3007
			95.3007
			95.3007

Analysis: we don't order in period 1 because $W_1 = x^*$. We order 95.3007 in periods 2,3,4,5,6,7,8,9,10,11,12,13,14,15 and 16 respectively. The CPU time is 0.63 secs.

Example 4.

Lifetime of product is 21. Using equations (5) and (6) we obtain x^* for time era $n=1$ *and* $1<n<m$.

Table 7: ordering policy for a product with 21 useful lifetime.

Time era	x^*	W_1	y
$n=1$	95.2799	95.2799	-
$1<n<m$	97.6067		95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
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			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
$n \geq m$			95.2799
			95.2799
			95.2799
			95.2799
			95.2799
			95.2799

Analysis: we don't order in period 1 because $W_1 = x^*$. We order 95.2799 in periods 2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22.23.24,25 and 26 respectively. The CPU time is 0.685 secs.

Computer Time.

Table 8 shows the CPU time for some values of m. Clearly, the CPU time increases with increasing *m*. for m =21, the computer time is 0.685secs. Figure1 shows the computer time reported in Table 8.

Table 8: lifetime and CPU time for some fixed lifetime products

s/n	Lifetime of product	CPU Time(secs)
1	3	0.485
2	4	0.532
3	5	0.554
4	12	0.63
5	21	0.685

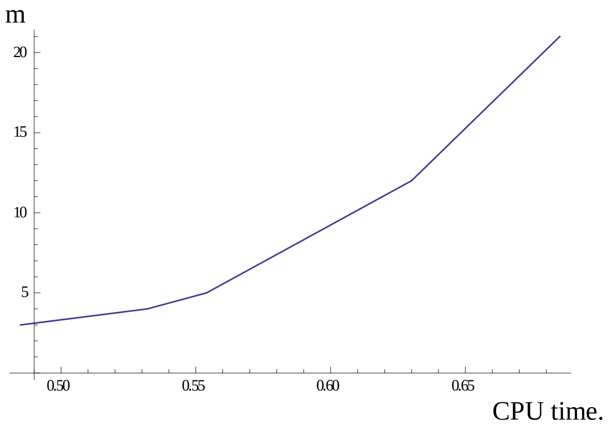

Figure 1: lifetime vs computer time

Conclusion

For $m > 2$, the dimension of the dynamic programing approach is high and difficult to handle. This is why the prevailing view on the dynamic programing approach is that it is unrealizable in practice when the dimension exceed 2periods. However, the use of computer programmesmakes it easy and the computing time is fast. For $f = \lambda e^{-\lambda t}$ which is the widely used distribution in the literature, we found out that the computer time is in seconds for products with fixed lifetime between 3 and 21 periods. The computing process is routine and does not require special techniques. Using the model in [3], we computed ordering policy for products with lifetime 3,4,12, and 21periods. For the four products considered, we observed that the quantity ordered is the same in all the periods except period one. The implication of this is that outdating may be high when the demand is low, since the same quantity of items arrives inventory every period.

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