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A SINGLE STOCK POINT INVENTORY MODEL FOR AMELIORATING DETERIORATING ITEMS WITH NON-LINEAR SECOND ORDER DEMAND RATE

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Abstract

In this paper, an optimization framework to derive optimal replenishment policy for ameliorating deteriorating items with non-linear second order consumption rate is developed. The present work attempts to model the situation where ameliorating rate and deteriorating rate are constant fraction of the on hand inventory. Shortages are not allowed and payment of the replenishment items is affected on the receipt of the items in the inventory. Numerical examples are provided to ascertain the validity of the model developed.

***Keywords*:** Ameliorating deteriorating items, Single stock point, Non-linear second order demand,

1.0 Introduction

Very often, mathematical ideas are being used in different areas in real life problems, especially for controlling inventory. The most important problem of inventory or stock managers is to decide when and how much to order so that the total cost associated with the inventory system is optimized. This is somewhat more important, when the inventory exhibits the dual property of amelioration and deterioration. Amelioration activation refers to the gradual increase in quality, quantity or both of inventory items during their storage period. Items such as: fruits (like orange, pineapple, mango etc.), high breed fishes in breeding yard (fish culture facility)or fast growing animals like chickens, broiler, goose, rabbit etc. in farming yard provide good example. When these items are kept in the farm or in sales counter, they will increase in value due to their growth and once grown, they are used to produce food. The inventory of such items grows faster in a first period and then starts to decrease (deteriorate) due to insufficient space, feeding expenses, diseases and/other factors. As a result, in determining the optimal replenishment policy of these types of items, the changes in the inventory due to amelioration and deterioration cannot be ignored.

Many researchers studied the inventory systems of deteriorating items, starting with Ghare and Schrader [1] who proposed a model for exponentially decaying inventory. Other relevant literature can be found in[2−8].

In the above literature, practitioners ignore or give little attention to ameliorating process of the inventory items. For the first time, [9] studied an inventory system for items with Weibull distribution ameliorating rate while at a breeding yard and deteriorate when in the distribution centre. [9] in [10]extended his earlier findings by developing three consecutive models; the Economic Order Quantity (EOQ), the Partial Selling Quantity (PSQ) and the Economic Production Quantity (EPQ) of ameliorating and deteriorating items. An EOQ model for both ameliorating and deteriorating items under the influence of inflation and time-value of money was proposed in[11]. A study of inflation effects on an EOQ model for Weibull deteriorating ameliorating items with ramp-type of demand and shortages was carried out by [12]. The model was studied under the replenishment policy, starting with shortages under two different types of backlogging rates. The model concludes that inflation definitely plays a major role on the replenishment policies and the optimum inventory cost. Here, total cost was more sensitive to inflation than optimum order quantity. A note on inventory model for ameliorating items with time dependent second order demand rate was provided by[13].Deepa and Khimya [14] came up with stochastic inventory model for ameliorating items under supplier's trade credit policy. Inventory model for Weibull ameliorating deteriorating items under the influence of inflation was presented by [15]. Recently, [16] extended the work of [11] by considering exponentially increasing demand and linear holding cost in their EOQ model for both ameliorating and deteriorating items.

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In the present article, an attempt has been made to propose an inventory model for ameliorating deteriorating items in which the ameliorating rate and deteriorating rate are constant fraction of the on-hand inventory with nonlinear second order demand rate. Shortages are not allowed.

2.0 Assumptions and Notations

Assumptions

The model is developed based on the following assumptions and notations:

(i) The inventory systems involves only one single item and one stocking point.

(ii) The replenishment rate is instantaneous, lead time is zero

(iii) Amelioration occurs when the items are effectively in stock.

(iv) Deterioration occurs when the items are effectively in stock.

(v) The demand rate $D(t)$ is nonlinear second order i.e. $D(t) = at^2$, where$a$ is the initial demand,$a > 0$.

Notations

(i) T is the planning horizon.

(ii) $I(t)$ is the inventory level at any time t.

(iii) $A_0$is the fixed ordering cost per order.

(iv) $I_0$is the initial inventory.

(v) $r$is the constant ameliorating rate.

(vi) $\theta$is the constant deteriorating rate.

(vii) $h$is the inventory holding cost per unit per unit time.

(viii) $c_1$is the cost of each deteriorated item.

(ix) $c_2$isthe cost of each ameliorated item.

(x) $A_m$is the total ameliorated amount in the period, $(0, T)$.

(xi) $D_m$is the total deteriorated amount in the period $(0, T)$.

(xii) $I_T$is the total number of on-hand inventory in the period $(0, T)$.

(xiii) $TVC(T)$is the total (average) inventory cost per unit time.

3.0 Formulation and Solution of the Model

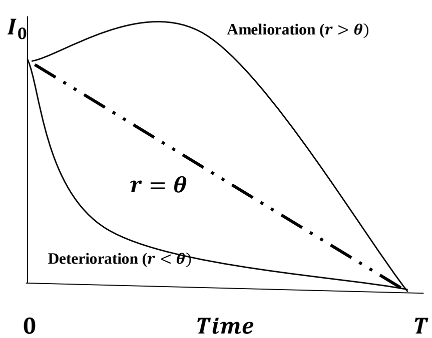

Figure 1: ***Inventory movement in a nonlinear second order demand ameliorating/deteriorating syste***

The depletion of the inventory during the interval $(0, T)$ is a function of the ameliorating rate,r, deteriorating rate, θ, demand rate, $D(t)$ and the inventory level in the system. The differential equations governing the inventory system at any time t is given by:

$$\frac{dI(t)}{dt} + (\theta - r)I(t) = -D(t), \qquad 0 \le t \le T \qquad (1)$$

Equation (1) is first order linear differential equation given by:

$$\Rightarrow \frac{dI(t)}{dt} + (\theta - r)I(t) = -at^2, \qquad 0 \le t \le T \qquad (2)$$

The integrating factor, $\rho = e^{\int(\theta - r)dt} = e^{(\theta - r)t}$.

The solution of Equation (2) is then given by:

$$e^{(\theta-r)t}I(t) = -\int at^2 e^{(\theta-r)t}dt + k$$

$$\Rightarrow I(t) = \frac{2a}{(\theta-r)^2}t - \frac{a}{(\theta-r)}t - \frac{2a}{(\theta-r)^3} + ke^{-(\theta-r)t}. \quad (3)$$

Putting the boundary condition, at $t = 0, I(t) = I_0$ in equation (3) we get:

$$I_0 = -\frac{2a}{(\theta-r)^3} + k$$

$$\Rightarrow k = I_0 + \frac{2a}{(\theta-r)^3}. \quad (4)$$

Substituting equation (4) in equation (3) we get:

$$I(t) = \frac{2a}{(\theta-r)^2}t - \frac{a}{(\theta-r)}t - \frac{2a}{(\theta-r)^3} + \left[I_0 + \frac{2a}{(\theta-r)^3}\right]e^{-(\theta-r)t}.$$

$$= \frac{2a}{(\theta-r)^2}t - \frac{a}{(\theta-r)}t - \frac{2a}{(\theta-r)^3} + \frac{2a}{(\theta-r)^3}e^{-(\theta-r)t} + I_0 e^{-(\theta-r)t} \quad (5)$$

Putting the boundary condition at $t = T,\ I(t) = 0$ in equation (5) we get:

$$0 = \frac{2a}{(\theta-r)^2}T - \frac{a}{(\theta-r)}T - \frac{2a}{(\theta-r)^3} + \frac{2a}{(\theta-r)^3}e^{-(\theta-r)T} + I_0 e^{-(\theta-r)T}$$

$$\Rightarrow I_0 e^{-(\theta-r)T} = \frac{2a}{(\theta-r)^3} - \frac{2a}{(\theta-r)^3}e^{-(\theta-r)T} - \frac{2a}{(\theta-r)^2}T + \frac{a}{(\theta-r)}T$$

$$\therefore\ I_0 = \frac{2a}{(\theta-r)^3}\left(e^{(\theta-r)T} - 1\right) + \frac{a}{(\theta-r)}T\left(1 - \frac{2}{(\theta-r)}\right)e^{(\theta-r)T} \quad (6)$$

Substituting equation (6) in equation (5) and simplifying we get:

$$I(t) = \frac{2a}{(\theta-r)^3}\left(e^{(\theta-r)(T-t)} - 1\right) + \frac{a}{(\theta-r)}\left(Te^{(\theta-r)(T-t)} - t\right)\left(1 - \frac{2}{(\theta-r)}\right). \quad (7)$$

The total on-hand inventory in the period $(\mathbf{0}, \boldsymbol{T})$ is given by:

$$I_t = \int_0^T I(t)dt$$

$$= \int_0^T \frac{2a}{(\theta-r)^3}\left(e^{(\theta-r)T} - 1\right)dt + \int_0^T \frac{a}{(\theta-r)}\left(Te^{(\theta-r)(T-t)} - t\right)\left(1 - \frac{2}{(\theta-r)}\right)dt$$

$$= \frac{2a}{(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - T\right)\right] + \frac{a}{(\theta-r)}\left(1 - \frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - \frac{T^2}{2}\right] \quad (8)$$

Ameliorated amount

The ameliorated, $A_m = rI_t$

$$= \frac{2ar}{(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - T\right)\right] + \frac{ar}{(\theta-r)}\left(1 - \frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - \frac{T^2}{2}\right] \quad (9)$$

Deteriorated amount

Deteriorated amount, $D_m = \theta I_t$

$$= \frac{2a\theta}{(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - T\right)\right] + \frac{a\theta}{(\theta-r)}\left(1 - \frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - \frac{T^2}{2}\right] \quad (10)$$

Inventory holding cost per cycle

$$H_c = h\int_0^T I(t)dt$$

$$= \frac{2ah}{(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - T\right] + \frac{ah}{(\theta-r)}\left(1 - \frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - \frac{T^2}{2}\right] \quad (11)$$

The total inventory cost per unit time

$$TVC(T) = \frac{1}{T}\left[Ordering\ cost,\ A_0 + Holding\ cost, H_c + \ Cost\ of\ deteriorated\ items, D_m - Cost\ of\ ameliorated,\ A_m\right]$$

$$= \frac{1}{T}\left[A_0 + H_c + c_1 D_m - c_2 A_m\right].$$

$$\Rightarrow TVC(T) = \frac{A_0}{T} + \frac{2ah}{T(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - T\right] + \frac{ah}{(\theta-r)}\left(1 - \frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - \frac{T^2}{2}\right]$$

$$+ \frac{2ac_1\theta}{T(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - T\right)\right] + \frac{ac_1\theta}{T(\theta-r)}\left(1 - \frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T} - 1\right) - \frac{T^2}{2}\right]$$

$$-\frac{2ac_2r}{T(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T}-1\right)-T\right)\right]+\frac{ac_2r}{(\theta-r)}\left(1-\frac{2}{(\theta-r)}\right)\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T}-1\right)-\frac{T^2}{2}\right]$$

$$\Rightarrow TVC(T)=\frac{A_0}{T}+\frac{2a}{T(\theta-r)^3}\left[\frac{1}{(\theta-r)}\left(e^{(\theta-r)T}-1\right)-T\right)\right](h+c_1\theta-c_2r)$$

$$+\frac{a(\theta-r-2)}{T(\theta-r)^3}\left[\left(e^{(\theta-r)T}-1\right)-\frac{T^2}{2}(\theta-r)\right](h+c_1\theta+c_2r). \qquad (12)$$

Differentiating equation (12) with respect to T and equating to zero we get:

$$\frac{dTVC(T)}{dT}=0\Rightarrow-\frac{A_0}{T^2}+\frac{2a}{T^2(\theta-r)^4}\left[(T(\theta-r)-1)e^{(\theta-r)T}+1-(\theta-r)T^2\right](h+c_1\theta-c_2r)$$

$$+\frac{a(\theta-r-2)}{T^2(\theta-r)^3}\left[(T(\theta-r)-1)e^{(\theta-r)T}+1-\frac{T^2}{2}\right](h+c_1\theta+c_2r)=0. \qquad (13)$$

Multiplying equation (13) by T^2 we get:

$$-A_0+\frac{2a}{(\theta-r)^4}\left[(T(\theta-r)-1)e^{(\theta-r)T}+1-(\theta-r)T^2\right](h+c_1\theta-c_2r)$$

$$+\frac{a(\theta-r-2)}{(\theta-r)^3}\left[(T(\theta-r)-1)e^{(\theta-r)T}+1-\frac{1}{2}T^2\right](h+c_1\theta+c_2r)=0. \qquad (14)$$

The solution of equation (14) gives us the optimal cycle length $T=T^*$.

The optimal Total Variable Cost per unit is obtained from equation (12) with $T=T^*$.

The Economic Order Quantity (EOQ)

$EOQ=I_0=\frac{2a}{(\theta-r)^3}\left(e^{(\theta-r)T}-1\right)+\frac{a}{(\theta-r)}T\left(1-\frac{2}{(\theta-r)}\right)e^{(\theta-r)T}$, evaluated at $T=T^*$.

4.0 Numerical examples

The following are 10 numerical examples with different input of parameters values. The output obtained (using Maple 2015 Mathematical software) gives the optimal cycle length $T=T^*$, Optimal Total Variable Cost TVC and the Economic Order Quantity EOQ.

Table 1: EOQ, Total variable cost and optimal cycle length for ameliorating/deteriorating items with nonlinear second order demand rate.

S/N	A_0	a	θ	r	h	c_1	c_2	T	TVC	AOQ
1	**1000**	**100**	**0.15**	**0.35**	**1.20**	**2.50**	**3.50**	**0.1945 = (71 days)**	**11494**	**75**
2	**1000**	**100**	**0.25**	**0.35**	**1.20**	**2.50**	**3.50**	**0.0601 = (22 days)**	**46491**	**56**
3	**1000**	**100**	**0.35**	**0.45**	**1.20**	**2.50**	**3.50**	**0.0592 = (21.6 days)**	**52499**	**55**
4	**1000**	**100**	**0.40**	**0.50**	**1.20**	**2.50**	**3.50**	**0.0587 = (21 days)**	**55524**	**54**
5	1500	150	0.10	0.20	2.50	1.50	2.50	0.0406 = (15 days)	12539	80
6	1500	150	0.10	0.25	2.50	1.50	2.50	0.0753 = (27 days)	25460	78
7	1500	150	0.10	0.30	2.50	1.50	2.50	0.1174 = (43 days)	41821	69
8	1500	150	0.10	0.40	2.50	1.50	2.50	0.2228 = (81 days)	84448	58
9	**2000**	**300**	**0.25**	**0.55**	**2.50**	**3.50**	**4.50**	**0.1950 = (71 days)**	**77772**	**147**
10	**2000**	**300**	**0.25**	**0.55**	**1.50**	**3.50**	**4.50**	**0.0829 = (30 days)**	**27870**	**150**

5.0 Discussions of Results

The results obtained from the numerical examples shows that both the amelioration and deterioration rates affect the optimal cycle length, total variable cost and the EOQ. The results reveal that, the increase in the deteriorating rate resulted in the considerable decrease in the cycle period. This is obvious as more items are removed from the inventory due to the combining effect of demand and deterioration thus decreasing the cycle period. The rise in total variable costs is consequent of rise in the cost of maintenance to reduce the deteriorating effect. Obviously, EOQ decreases as the stock owner is prompted to buy less items to that effect. However, the increase in ameliorating rate resulted in the considerable increase in the cycle period and increase in total variable cost (due to rise in the cost of maintenance) since more items may be introduced into the inventory due to ameliorating activation, hence extending the cycle period(as more items stay longer in the inventory).Consequently, EOQ diminishes to avoid over stocking.

6.0 Conclusion

In this paper we derived an inventory model for some special items that exhibits amelioration/deterioration property. Both the amelioration and deterioration rates were assumed to be constant. Shortages were not allowed in this model and the optimal solution of total variable cost per unit time and Economic Order Quantity (EOQ) were obtained. The results obtained from the numerical examples indicated the validity and stability of the model.

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