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DYNAMIC LOT-SIZING MODEL WITH BACKLOGGED NORMALLY DISTRIBUTED DEMAND AND MULTIPLE SUPPLY POINTS

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Abstract

This paper considers a dynamic lot size model for a producer with stochastic demand over multiple time periods when production and inventory level have upper bounds. Characterized by backlogging cost as well as supply cost to multiple supply points, an optimal cost policy that gives a minimum total cost over the multiple time periods in the planning horizon is sought. The inventory and backlogging costs are proportional to the quantity of inventory stocked and quantity backlogged respectively while production cost for each period is composed of fixed set-up and unit production costs. Demand is normally distributed while supply to the different supply points is prioritized such that centers with higher demands are satisfied before those with lower demands. The model was illustrated with data collected from a single-item company Boltzmann Nigeria Ltd. Using the backward dynamic programming algorithm, an optimal cost policy that satisfies expected demand over a 12-period time interval to 4 supply points was obtained.

Keywords*:* Lot sizing, backlogging, probabilistic demand, inventory, loss function.

1.0 Introduction

There are several types of lot-sizing techniques such as economic order quantity (EOQ), lot-for-lot periodic order quantity, silver-med algorithm, part-period algorithm as well as the Wagner-Whitin algorithm. These lot-size approaches focus on controlling the holding and ordering/production costs. None of them, with the exception of the Wagner-Whitin algorithm, assumes a time-varying demand pattern. The main concern of the lot-sizing problem is to determine production or procurement lots for the given product over a finite or infinite planning horizon so as to minimize the total cost while known demand is satisfied. A solution to the dynamic economic order model for single item demand, inventory holding charges and set up cost to vary over N periods while seeking known demand in every period was presented in [1]. This algorithm presented in [1] has been extensively studied, modified and extended by different researchers since it was introduced. Such modifications and extensions includes cases like replacing the deterministic demand in each period with a case where demand is stochastic and time varying, introducing backlogging of unfulfilled demand with penalty cost of backlogged sales and inclusion of other cost like transportation cost in the model.. The algorithm presented in [1] was extended for N periods lot-sizing model to include backorders in [2]. The problem was formulated as a transshipment network model and then reformulated as a transportation model in order to facilitate the process of getting the optimal solution by using commonly available tools. Meanwhile the stochastic dynamic lot size model with probabilistic state variable constraints was investigated in [3]. In the work the new equivalent deterministic dynamic lot size problem with new parameters using the open loop approach was solved. In [4] the multi-supplier economic lot-sizing model in which the retailer replenishes his inventory from several suppliers where each supplier is characterized by one of three of order cost structures: incremental quantity discount cost structure, multiple set-ups cost structure and all-unit quantity discount cost structure was presented. For all cases of the problem some optimality properties were proposed and optimal algorithm based on dynamic programming designed. Also in [5] a single-product lot-sizing model with constant demand and backordering over a finite planning horizon was proposed. The ordering cost was fixed while holding and backordering costs are proportional to the amount of inventory stocked or

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backlogged. Inventory models in which backorder and a finite replenishment rate are considered according to the characteristics in MRP ordering was proposed in [6]. The optimal solution of the model was obtained using general purpose linear program solver like EXCEL and LINDO. In [7] a stochastic version of the single-level, multi-product dynamic lot-sizing problem subject to a capacity constraint was presented. A production schedule was determined for random demand so that expected costs are minimized and a constraint based on anew backlog-oriented σ-service level measure was met. This led to a non-linear model that is approximately by two different linear models. On the other hand a dynamic lot-size model for cases where single-items are produced and shipped by an overseas export company was presented in [8]. An optimal production scheduling with the constraint of production and production capacity that minimizes the total cost over the planning horizon was obtained. Meanwhile in [9] a multi-item capacitated lot-sizing model with set-up times and shortage costs with demand that cannot be backlogged but has total and partial cost was presented while in [10] the loss of customer goodwill in incapacitated single-item lot sizing with a mixed integer programming model extending the well-known Wagner-Whitin model in [1] was considered. In [11] a general model for single item lot-sizing problem with multiple suppliers, quantity discount and backordering of shortages was presented. On the other hand in [12] an algorithm for determining the optimal solution over the entire planning horizon for the dynamic lot-size model where demand is stochastic, non-stationary and normally distributed with known density was presented. In the model full backlogging of unfulfilled demand is assumed with a fixed set up cost. Lead time is also assumed to be fixed and no disposal of inventory is allowed. The model was illustrated with data extending the example given in [1].

The model in [12] considered set-up cost, inventory cost and backlogging cost. But production companies while trying to manage the lot-sizing problem also has to cope with supply cost to various demand centers. Hence in this work we present a lot-sizing model that considers a the lot-size model with fixed set-up cost, unit production cost, inventory holding cost, backlogging cost as well as supply cost to multiple points for a normally distributed demand. This model is an extension of the lot-size model in [12] to accommodate supply cost to multiple supply points but with constant penalty cost for backlogged demand throughout all time periods. So with a production and inventory level with upper bounds, a normally distributed demand and full backlogging of unfilled demand together with inventory and backlogging cost in some periods in a given planning horizon we intend to obtain a production-inventory-backlogging schedule that minimizes total expected costs.

2.0 Methodology

2.1 The Stochastic Lot sizing model with Normally Distributed demand.

The stochastic version of the dynamic lot-sizing algorithm as presented in [10] extended the standard assumptions given in [1] by replacing the deterministic demand in each period with a known demand density, stipulates full backlogging of unfulfilled normally distributed demand while introducing time-varying penalty cost for backlogged demand. The minimum production-inventory-backlogging cost policy model in [10] is presented as follows;

$$K(S,i,j) = A_{1-L} + \sum_{1-L}^{j-1} h_i\sigma_i\{z_i + I_N(z_t)\} + \sum_{i=1}^{j-1} \pi_i\sigma_i I_N(z_i) \qquad (1)$$

$for\ i = 1,2,3,\ldots H$

Where $\pi_i = ph_i \quad , \ p > 0$

$z_i = \frac{S-\mu_i}{\sigma_i} \quad , \ 1 \le i < j < H$

With $L = \text{lead time}$

$A_{1-L} = \text{fixed production set} - \text{up cost at beginning of period}$

$\pi_i = \text{backlogging cost in period i}$

$h_i = \text{holding cost in period i}$

$\sigma_i = \text{standard devaition of cumulative demand throug period i}$

$\mu_i = \text{mean of cumulative demand through period i}$

$x_i = \text{amount ordered (or manufactured) in period i}$

$I_N(z_t) = \text{normal loss function}$

2.2 Formulation of Dynamic lot-sizing model with backlogged normally distributed demand and multiple supply points.

The lot-sizing problem described here is an extension of the stochastic version of the lot-sizing model in [10] to include supply cost where products are supplied to multiple supply points with a known constant unit supply cost to each point. The total available quantity of product in each period t is supplied to the supply points in fixed proportions. The model intends to minimize the total cost in all time period which comprises of the production cost, inventory cost, backlogging cost as well as supply cost in the planning horizon. Given the following notations

$A = \text{fixed production set} - \text{up cost}$

$u_t = \text{unit production cost}$

$P_t = \text{Production quantity in period t}$

I_t = quantity of product in inventory in beginning of period t
b_t = quantity of product backlogged in period t
Q_t = Total quantity supplied in period t
π_t = backlogging cost
h_t = holding cost
σ_t = standard devaition of demand in period t
μ_t = mean of demand in period t
q_{it} = quantity of product supplied to point i in period t
c_{it} = unit cost of supplying product to point i in period t
α_{it} = proportion of total quantity available supplied to point i in period t
$I_N(z_t)$ = normal loss function

The lot-sizing model with normally distributed and fully backlogged demand and multiple supply points is presented as follows;

$$Min\, T(C) = A + u_t p_t + \sum_{t=1}^{n} h_t\sigma_t\{z_t + I_N(z_t)\} + Y_t \sum_{t=1}^{n} \pi_t\sigma_t I_N(z_t) + \sum_{t=1}^{n} q_{it}\, c_{it} \qquad (2)$$
$$for\ t = 1,2,\dots,n, \qquad i = 1,2,\dots,m$$

Where $E(I_t) = \sigma_t z_t$, $E(b_t) = \sigma_t I_N(z_t)$, $z_t = \frac{Q_t-\mu_t}{\sigma_t}$

With $$I_{t+1} = P_t + E(I_t) - \sum_{i=1}^{m} q_{it} = b_t \qquad (3)$$
$$\sum_{i=1}^{m} q_{it} = Q_t = P_t + xI_t + yb_t \qquad (4)$$
$$q_{it} = \alpha_{it} Q_t \quad , for\ all\ i \qquad (5)$$
$$x = \begin{matrix} 0 & , \text{ if there is no inventory at beggining of period t} \\ 1 & , \text{ if thre is available inventory at beginning of period t} \end{matrix} \qquad (6)$$
$$y = \begin{matrix} 0 & , \text{ if there is no backlogged quantity at beggining of period t} \\ 1 & , \text{ if there is backlogged quantity at beginning of period t} \end{matrix} \qquad (7)$$

In the model, the objective function (2) includes the sum of the production costs (i.e. fixed set-up cost and variable production cost), inventory holding cost, backlogging cost and supply costs. Constraint (3) represents the fact that inventory at the beginning of a new period or quantity to be backlogged at the beginning of a new period is gotten after supply has been made to the different points from the available inventory and production in the previous period. Constraint (4) indicates that the quantity distributed to the different points is made up of the quantity produced in that period and or its sum with the available inventory or backlogged quantity in that period. Meanwhile constraint (5) indicates that a constant proportion of total available quantity in a given period is supplied to each supply point while (6) and (7) presents the binary variables indicating that at the beginning of each period there can either be available inventory in which case there will be no backlog demand or there can be no available inventory in which case there might be backlogged demand.

2.3 Constraints and Assumptions of the model.

Some constraints and assumptions of the model are stated as follows;
(i) There is a fixed set-up cost associated with production in every period.
(ii) The unit production cost is known and constant throughout all periods.
(iii) The unit inventory cost is known and constant throughout all periods.
(iv) There is an upper bound on inventory level in all periods.
(v) There is an upper bound production level and inventory level throughout all periods.
(vi) Full Backlogging of demand is done in a given period for the immediate past period only.
(vii) Backlogging cost is constant throughout all periods and is a proportion of unit inventory cost.
(viii) Unfulfilled backlogged demand at end of planning horizon is left unfulfilled.
(ix) Demand is normally distributed.
(x) A fixed proportion of total available product in period t is supplied to each point *i*.
(xi) Unit supply cost to each point is known and constant throughout all periods.

3.0 Data illustration of the model.

3.1 Data presentation.

The lot-sizing model with backlogged normally distributed demand and multiple supply points will be illustrated using data collected from a single-product manufacturing company, Boltzman Nigeria Ltd Aba, that produces and distributes biscuits to four supply points which includes Umuahia, Port-Harcourt, Uyo and Benin City for a 12 time period planning horizon. Data was collected from the production and distribution department of the factory. The data used in the illustration is as shown in table 1 and 2. He data in table 1 includes the average demand for the 12 periods in the planning horizon. In table 2 we have the fixed production set-up cost, unit production cost, unit inventory and backlogging costs, upper bound for inventory and production, supply cost to the multiple points as well as the proportion of available product in each period supplied to each of the supply points.

Table 1: Average demand in each period

Periods	1	2	3	4	5	6	7	8	9	10	11	12
Average Demand	11200	12700	12100	12600	9300	9000	9300	9200	8900	12500	12200	12000

Table 2: Costs, production and inventory bounds and supply proportions.

Unit inventory cost		₦ 120
Unit production cost		₦ 300
Unit backlogging cost		₦ 75
Set-up cost		₦ 300,000
Inventory bounds in each month		$0 \le I_t \le 300$
Production bounds in each month		$0 < P_t \le 12300$
Supply cost per carton of product to each supply point.	Owerri	₦ 70
	Uyo	₦ 65
	Umuahia	₦ 40
	Port-Harcourt	₦ 50
Proportion of total available product supplied to each supply point.	Owerri	33%
	Uyo	28%
	Umuahia	20%
	Port-Harcourt	19%

3.2 Results and Discussion

The optimal production-inventory-backlogging-supply cost throughout the planning horizon (from period 1 through 12) using the forward dynamic programming algorithm with EXCEL software package is as given in table 3.

Table 3: Optimal production- inventory-backlogging-supply policy.

Period	I_t	P_t	b_t	Q_t	COST (₦)					
					SET-UP	PRODUCTION	INVENTORY	SHORTAGE	SUPPLY	TOTAL
1	300	10900	0	11200	₦ 300,000	₦ 3570000	0	0	₦ 632,240	₦ 4,517,240
2	0	12300	0	12300	₦ 300,000	₦ 3,990,000	0	0	₦ 716,915	₦ 5,006,915
3	0	12300	400	12700	₦ 300,000	₦ 3,990,000	0	₦ 30,000	₦ 683,045	₦ 5,003,045
4	0	12300	200	12300	₦ 300,000	₦ 3,990,000	0	₦ 15,000	₦ 711,270	₦ 5,016,270
5	0	9800	500	10,100	₦ 300,000	₦ 3,240,000	0	₦ 37,500	₦ 524,985	₦ 4,102,485
6	0	9000	0	9000	₦ 300,000	₦ 3,000,000	0	0	₦ 508,050	₦ 3,808,050
7	0	9300	0	9300	₦ 300,000	₦ 3,090,000	0	0	₦ 524,985	₦ 3,914,585
8	0	9200	0	9200	₦ 300,000	₦ 3,060,000	0	0	₦ 519,340	₦ 3,879,340
9	0	8900	0	8900	₦ 300,000	₦ 2,970,000	0	0	₦ 502,405	₦ 3,772,405
10	0	12300	0	12300	₦ 300,000	₦ 3,990,000	0	0	₦ 717,610	₦ 5,007,610
11	0	12300	200	12300	₦ 300,000	₦ 3,990,000	0	₦ 15,000	₦ 688,690	₦ 4,993,690
12	0	12100	100	12100	₦ 300,000	₦ 3,990,000	0	₦ 7,500	₦ 677,400	₦ 4,914,900
OPTIMAL TOTAL COST										**₦ 53,936,535**

From the optimal production-inventory-backlogging-supply policy given in table 3, the production-inventory-backlogging plan is such that we have starting inventory of 300 cartons of biscuits at the beginning of the planning horizon and with a production of 10900 cartons the average demand of 11200 in the first period is satisfied without any shortage. In period two even though we had a demand of 12700 cartons yet because of the upper bound on production we could only produce 12300 with a shortage of 400 cartons. Hence in period three with the upper bounded production of 12300, from the 12300 cartons produced we backlog the 400 cartons of shortage of period two from that with only 11900 available for satisfying the period three demand. So with the demand of 12100 in the period three we only satisfied 11900 with a balance of 200. Now in period four we have a demand of 12600. From the production of 12300 cartons and with 200 cartons backlogged we have 12100 cartons left for satisfaction of period's four demand. Hence a shortage of 500 cartons.

So the balance of 500 was taken into period five. In the fifth period with a demand of 9300 cartons of biscuits, a total quantity of 9800 was produced with 500 backlogged for the shortage of period four and 9300 used to satisfy the demand of period five. So no shortage at the end of period five. In period six though nine we have demands of 9000 , 9300, 9200 and 8900 respectively which are all satisfied without shortage since they are below the production's level upper bound. In period ten, with a demand of 12500, a total of 12300 was produced with a balance of 200 taken to the next period. Hence in the eleventh period, with a demand of 12200 and a maximum period's production of 12300, the balance of 200 cartons for period ten was backlogged with 12100 remaining to satisfy the period eleven's demand. So the balance of 100 cartons was taken into the last period twelve. In period twelve with a demand of 12000 a total of 12100 is produced with 100 cartons backlogged and 12000 used to satisfy the demand for period twelve to end the planning horizon with no shortage. The optimal minimum cost policy is ₦ 53,906,535 at the end of the planning horizon.

4.0 Conclusion

In this paper we extend the stochastic version of the dynamic lot-size model in [12] to include supply cost where products are supplied to multiple supply points. The cost structure of the model is made up of a fixed set-up cost, a variable production cost, an inventory holding cost, backlogging cost as well as supply cost. With a production and inventory with upper bounds, a normally distributed demand with full backlogging of unfilled demand, a constant inventory and backlogging cost through all time periods we intend to obtain a production-inventory policy throughout the planning horizon that minimizes total expected costs. The formulated lot-sizing model was illustrated using data collected from a single-product manufacturing company, Boltzman Nigeria Ltd Aba, which produces and distributes biscuits to four supply points which includes the cities Umuahia, Port-Harcourt, Uyo and Benin City, for a 12 time period planning horizon. Data of the average demand for each period, fixed production set-up cost, unit production cost, unit inventory and backlogging costs, upper bound for inventory and production, supply cost per carton of product to the multiple points as well as the proportion of available product in each period supplied to each of the supply points was collected from the production and distribution departments of the factory. Using the forward dynamic programming algorithm on the EXCEL software package, an optimal production-inventory-backlogging-supply policy was obtained. The optimal minimum costs policy for the planning horizon which comprises of twelve periods was ₦ 53,906,535.

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