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RIEMANNIAN EQUATION OF MOTION FOR A STATIC HOMOGENEOUS SPHERICAL DISTRIBUTION OF MASS WHOSE TENSOR FIELD VARIES WITH RADIAL AND POLAR ANGLE

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Abstract

Einstein's General Theory of Relativity (EGTR) has been credited as the greatest intellectual achievement of the 20th Century. According to this theory gravitation is not due to a force rather a manifestation of curved space and time. In this paper, we constructed the golden Riemannian affine connection for a body whose tensor field varies with radial and polar angle using golden Riemannian metric tensor. The Riemannian affine connection were applied to the well known Einstein's general relativistic equation of motion for test particle of non zero rest masses in the gravitational field to obtain the corresponding golden Riemannian relativistic equation of motion for test particles of non zero rest masses (acceleration tensors). The Riemannian relativistic equation of motion for test particles of non zero rest masses (acceleration tensors) was substituted into the well known Riemannian tensorial geodesic equation of motion to obtain the corresponding golden Riemannian tensorial geodesic equation of motion for test particles of non-zero rest masses. The results are that the generalized affine connection, golden Riemannian relativistic equation of motion for test particles of non zero rest masses and the golden Riemann's tensorial geodesic equation of motion are augumented with additional correction terms of the order c^{-2} which are not found in Schwarzschild's metric tensors, Einstein's gravitational field equation for test particle and the Riemann's tensorial geodesic equation of motion and are uncovered for theoretical development and experimental verification and application.

Keywords: Motion, Test Particle, Golden Riemannian Metric Tensor, Riemann's tensorial geodesic equation, Radial Angle and Polar Angle.

1.0 Introduction

Einstein's geometrical theory of gravitation (general relativity) was published in the year 1915. Einstein's General Theory of Relativity (EGTR) has been credited as the greatest intellectual achievement of the 20th Century. According to this theory gravitation is not due to a force rather a manifestation of curved space and time, with the curvature being produced by the mass-energy and momentum content of the space time [1]. It unifies special relativity and sir Isaac Newton's law of universal gravitation [1,2]. After the publication of Einstein's geometrical field equation in 1915, the search for their exact and analytical solutions for the entire gravitational field in nature began [3].

More than any other theory of modern physics, general relativity is usually seen as the work of one man, Albert Einstein. In taking this point of view, however, one tends to overlook the fact that gravitation has been the subject of controversial discussion since the time of Newton [3,4]. That Newton's theory of gravitation assumes action at a distance, i.e., action without an intervening mechanism or medium, was perceived from its earliest days as being problematical. Around the turn of the last century, in the classical physics, the problems of Newtonian gravitation theory had become more acute, also due to the rise of field theory suggesting alternative perspectives. Consequently, there was a proliferation of alternative theories

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of gravitation which were quickly forgotten after the triumph of general relativity. Yet in order to understand this triumph, it is necessary to compare general relativity to its contemporary competitors. General relativity owes much to this competition. The proliferation of theories of gravitation provides an exemplary case for studying the role of alternative pathways in the history of science. Thus, from this perspective, the emergence of general relativity constitutes an ideal topic for addressing longstanding questions in the philosophy of science on the basis of detailed historical evidence [5].

The extension of Einstein's planetary theory of gravitation (general relativity) has continued till today without satisfactory resolution to its numerous objections. A number of physicists have continued to hold on to the view that Einstein's planetary theory of gravitation can be generalized in such a way as to account satisfactorily for the principle of reciprocity, mathematical difficulty, inconsistency with quantum mechanics and violation of principle of equivalence. On the basis of this, we shall show in this paper one way of extending Einstein's planetary theory using Riemannian golden metric tensors and whose tensors field varies with radial and polar angle only. Schwarzschild's metric is the solution of Einstein's gravitational field equation exterior to a static homogeneous spherical body [6].

The well known Schwarzschild's covariant metric tensors in the gravitational field is given explicitly as [6,7]

$$g_{11} = \left\{1 + \frac{2}{c^2} f(r)\right\}^{-1} \tag{1.1}$$

$$g_{22} = -r^2 \tag{1.2}$$

$$g_{33} = -r^2 sin^2\theta \tag{1.3}$$

$$g_{00} = -\left\{1 + \frac{2}{c^2}(r)\right\} \tag{1.4}$$

$$g_{\mu\nu} = 0; \qquad otherwise \tag{1.5}$$

$$x^0 = ct \tag{1.6}$$

The contra-variant metric tensor is given as

$$g^{11} = -\left\{1 + \frac{2}{c^2} f(r)\right\}^{-1} \tag{1.7}$$

$$g^{22} = -\frac{1}{r^2} \tag{1.8}$$

$$g^{33} = -\frac{1}{r^2 sin^2\theta} \tag{1.9}$$

$$g^{00} = 1 + \frac{2}{c^2} f(r) \tag{1.10}$$

$$g^{\mu\nu} = 0; \qquad otherwise \tag{1.11}$$

Theoretical Analysis

2.0 Construction of Golden Riemannian Affine Connection

The coefficients of affine connection for any gravitational field in terms of the metric tensors is given explicitly as [6,7,8].

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\nu}\left(g_{\mu\nu,\lambda} + g_{\nu\lambda,\mu} - g_{\mu\lambda,\nu}\right) \tag{2.1}$$

where the comma as in usual notation denotes partial differentiation with respect to $x^{\lambda}, x^{\mu}, x^{\nu}$.

$g^{\sigma\nu}$ and $g_{\sigma\nu}$ are the covariant and contravariant metric tensors.

The golden Riemannian covariant metric tensor for a body whose tensor field varies with radial and polar angle is given explicitly as [8,9]

$$g_{11}(r,\theta) = \left\{1 + \frac{2}{c^2} f(r,\theta)\right\}^{-1} \tag{2.2}$$

$$g_{22}(r,\theta) = r^2\left\{1 + \frac{2}{c^2} f(r,\theta)\right\}^{-1} \tag{2.3}$$

$$g_{33}(r,\theta) = r^2 sin^2\theta\left\{1 + \frac{2}{c^2} f(r,\theta)\right\}^{-1} \tag{2.4}$$

$$g_{00}(r,\theta) = -\left\{1 + \frac{2}{c^2}(r,\theta)\right\} \tag{2.5}$$

$$g_{\mu\nu}(r,\theta) = 0; \qquad otherwise \tag{2.6}$$

$$x^0 = ct \tag{2.7}$$

The contra-variant metric tensor is given as

$$g^{11}(r,\theta) = \left[1 + \frac{2}{c^2} f(r,\theta)\right] \tag{2.8}$$

$$g^{22}(r,\theta) = \frac{1}{r^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right] \quad (2.9)$$
$$g^{33}(r,\theta) = \frac{1}{r^2 sin^2\theta}\left[1 + \frac{2}{c^2}f(r,\theta)\right] \quad (2.10)$$
$$g^{00}(r,\theta) = -\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1} \quad (2.11)$$
$$g^{\mu\nu}(r,\theta) = 0; \quad otherwise \quad (2.12)$$

We substituted equations (2.2) to (2.12 into equation (2.1) to obtain the coefficient of affine connections which are found to be given in terms of (r,θ) as

$$\Gamma^1_{00} = -\frac{1}{2}g^{11}g_{00,1} \quad (2.13)$$
$$\Gamma^1_{11} = \frac{1}{2}g^{11}g_{11,1} \quad (2.14)$$
$$\Gamma^1_{12} \equiv \Gamma^1_{21} = \frac{1}{2}g^{11}g_{11,2} \quad (2.15)$$
$$\Gamma^1_{22} = -\frac{1}{2}g^{11}g_{22,1} \quad (2.16)$$
$$\Gamma^1_{33} = -\frac{1}{2}g^{11}g_{33,1} \quad (2.17)$$
$$\Gamma^2_{00} = -\frac{1}{2}g^{22}g_{00,2} \quad (2.18)$$
$$\Gamma^2_{11} = -\frac{1}{2}g^{22}g_{11,2} \quad (2.19)$$
$$\Gamma^2_{12} \equiv \Gamma^2_{21} = \frac{1}{2}g^{22}g_{22,1} \quad (2.20)$$
$$\Gamma^2_{33} = -\frac{1}{2}g^{22}g_{33,2} \quad (2.21)$$
$$\Gamma^0_{01} \equiv \Gamma^0_{10} = \frac{1}{2}g^{00}g_{00,1} \quad (2.22)$$
$$\Gamma^0_{02} \equiv \Gamma^0_{20} = \frac{1}{2}g^{00}g_{00,2} \quad (2.23)$$
$$\Gamma^3_{13} \equiv \Gamma^3_{31} = \frac{1}{2}g^{33}g_{33,1} \quad (2.24)$$
$$\Gamma^3_{23} \equiv \Gamma^3_{32} = \frac{1}{2}g^{33}g_{33,2} \quad (2.25)$$
$$\Gamma^2_{22} = \frac{1}{2}g^{22}g_{22,2} \quad (2.26)$$

Where the comma denotes partial differentiation w.r.t $(r,\theta) = (1,2)$.
Equation (2.13) to (2.26) can be given more explicitly as

$$\Gamma^1_{00} = \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]\frac{\partial f(r,\theta)}{\partial r} \quad (2.27)$$
$$\Gamma^1_{11} = -\frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r} \quad (2.28)$$
$$\Gamma^1_{12} \equiv \Gamma^1_{21} = -\frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial \theta} \quad (2.29)$$
$$\Gamma^1_{22} = -r + \frac{r^2}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r} \quad (2.30)$$
$$\Gamma^1_{33} = -r sin^2\theta + \frac{r^2 sin^2\theta}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r} \quad (2.31)$$
$$\Gamma^2_{00} = \frac{1}{r^2c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]\frac{\partial f(r,\theta)}{\partial \theta} \quad (2.31)$$
$$\Gamma^2_{11} = \frac{1}{r^2c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial \theta} \quad (2.32)$$
$$\Gamma^2_{12} \equiv \Gamma^2_{21} = \frac{1}{r} - \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r} \quad (2.33)$$

$$\Gamma_{33}^{2} = -sin\theta cos\theta + \frac{sin^2\theta}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta} \quad (2.34)$$

$$\Gamma_{01}^{0} \equiv \Gamma_{10}^{0} = \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r} \quad (2.35)$$

$$\Gamma_{02}^{0} \equiv \Gamma_{20}^{0} = \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta} \quad (2.36)$$

$$\Gamma_{13}^{3} \equiv \Gamma_{31}^{3} = \frac{1}{r} - \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta} \quad (2.37)$$

$$\Gamma_{23}^{3} \equiv \Gamma_{32}^{3} = cot\theta - \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta} \quad (2.38)$$

$$\Gamma_{22}^{2} = -\frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta} \quad (2.39)$$

$$\Gamma_{\alpha\beta}^{\mu} = 0; \qquad otherwise \quad (2.40)$$

Equation (2.27) to (2.40), are the golden Riemannian affine connections.The golden Riemannian affine connections (2.27) to (2.40)contains 14 non-zero affine connection coefficients while Schwarzschild's contains only 9 non zero affine connection due to the inclusion of rotational effects. These coefficients are very instrumental in the construction of general relativistic equations of motion for test particles of non-zero rest mass. Also the golden Riemannian affine connections can be used to construct Einstein's equation of motion, Riemann's Christoffel tensors and Ricci tensors.

3.0 Derivation of the Golden Riemannian Equation of motion for Test Particle

The well known Einstein's equation of motion for test particle of non zero rest masses in the gravitational field is given explicitly as [9,10]

$$\ddot{x}^{\alpha} + \Gamma_{\mu\nu}^{\alpha}\dot{x}^{\mu}\dot{x}^{\nu} = 0 \quad (3.1)$$

where,

$\Gamma_{\mu\nu}^{\alpha}$ is the Christoffel's symbol

$\ddot{x}^{\alpha}$ is the Riemann acceleration tensor

$\dot{x}^{\mu}\dot{x}^{\nu}$ is the Riemann velocity tensors

The generalized Einstein's general relativistic equations of motion for test particles of non-zero rest masses in the gravitational field is given explicitly as [9,11]

$$\frac{d^2x^{\alpha}}{d\tau^2} + \Gamma_{\mu\nu}^{\alpha}\big(f(r,\theta)\big)\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \quad (3.2)$$

where,

$\Gamma_{\mu\nu}^{\alpha}\big(f(r,\theta)\big)$ is the generalized Einstein's affine connection and x^{α} is the space time coordinates tensors

Substituting the generalized affine connections (2.27) to (2.40) into equation (3.2) we obtain equations

For $\alpha = 1$

$$a^1 = \ddot{r} + \frac{1}{c^2}\left[1 + \frac{1}{c^2}f(r,\theta)\right]\frac{\partial f(r,\theta)}{\partial r}c\dot{t}^2 - \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{r}^2$$
$$-\frac{2}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}\dot{r} - r\dot{\theta}^2 + \frac{r^2}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\theta}^2$$
$$-rsin^2\theta\dot{\phi}^2 + \frac{r^2sin^2\theta}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\phi}^2 \quad (3.3)$$

For $\alpha = 2$

$$a^2 = \ddot{\theta} + \frac{1}{r^2c^2}\left[1 + \frac{1}{c^2}f(r,\theta)\right]\frac{\partial f(r,\theta)}{\partial\theta}c\dot{t}^2 + \frac{1}{r^2c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{r}^2$$
$$\frac{2}{r}\dot{\theta}\dot{r} - \frac{2}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\theta}\dot{r} - \frac{1}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}^2$$
$$-sin\theta cos\theta\dot{\phi}^2 + \frac{sin^2\theta}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\phi}^2 \quad (3.4)$$

For $\alpha = 3$

$$a^3 = \ddot{\phi} + \frac{2}{r}\dot{\phi}\dot{r} - \frac{2}{c^2}\left[1 + \frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\phi}\dot{r} + 2cot\theta\dot{\phi}\dot{\theta}$$

$$-\frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}\dot{\phi} \quad (3.5)$$

For $\alpha = 0$

$$a^0 = c\ddot{t} + \frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{r}c\dot{t}$$
$$+\frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}c\dot{t} \quad (3.6)$$

Equation (3.3) to (3.6) is the golden Riemannian equations of motion for test particles of non zero rest masses. The instantaneous speed of a particle of non zero rest mass in this gravitational field can be obtained from equation (3.3).

4.0 Derivation of Golden Riemannian Tensorial Geodesic Equation of Motion

The Riemann's tensorial geodesic equation of motion for particles of non-zero rest masses in a gravitational field is given explicitly as [11]

$$m_0 a^\mu = f_{ng}^\mu \quad (4.1)$$

where,

m_0 is the rest mass of the particles

a^μ is the Riemann's acceleration vector

f_{ng}^μ is the force tensor corresponding to all the non-gravitational interaction on the particles.

Substituting equations (3.7) to (3.10) into the well known Riemannian tensorial geodesic equation of motion equation (4.1) we obtain the following corresponding golden Riemannian tensorial geodesic equations of motion given as

$$f_{ng}^1 = m_0\left\{\ddot{r} + \frac{1}{c^2}\left[1+\frac{1}{c^2}f(r,\theta)\right]\frac{\partial f(r,\theta)}{\partial r}c\dot{t}^2 - \frac{1}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{r}^2\right.$$
$$-\frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}\dot{r} - r\dot{\theta}^2 + \frac{r^2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\theta}^2$$
$$\left.-rsin^2\theta\dot{\phi}^2 + \frac{r^2sin^2\theta}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\phi}^2\right\} \quad (4.2)$$

$$f_{ng}^2 = m_0\left\{\ddot{\theta} + \frac{1}{r^2c^2}\left[1+\frac{1}{c^2}f(r,\theta)\right]\frac{\partial f(r,\theta)}{\partial\theta}c\dot{t}^2 + \frac{1}{r^2c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{r}^2\right.$$
$$\frac{2}{r}\dot{\theta}\dot{r} - \frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\theta}\dot{r} - \frac{1}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}^2$$
$$\left.-sin\theta cos\theta\dot{\phi}^2 + \frac{sin^2\theta}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\phi}^2\right\} \quad (4.3)$$

$$f_{ng}^3 = m_0\left\{\ddot{\phi} + \frac{2}{r}\dot{\phi}\dot{r} - \frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{\phi}\dot{r} + 2cot\theta\dot{\phi}\dot{\theta}\right.$$
$$\left.-\frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}\dot{\phi}\right\} \quad (4.4)$$

$$f_{ng}^0 = m_0\left\{c\ddot{t} + \frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial r}\dot{r}c\dot{t}\right.$$
$$\left.+\frac{2}{c^2}\left[1+\frac{2}{c^2}f(r,\theta)\right]^{-1}\frac{\partial f(r,\theta)}{\partial\theta}\dot{\theta}c\dot{t}\right\} \quad (4.5)$$

Equations (4.2) to (4.5) are the golden Riemannian geodesic equation of motion for a test particle of non zero rest masses.

5.0 Remarks and Conclusion

The Riemannian equation of motion for test particle of static homogeneous spherical distribution of mass whose tensor field varies with radial and polar angle were obtained as equation (3.3), (3.4), (3.5) and (3.6). Expressions for the golden Riemannian tensorial geodesic equations of motion were obtained as equations (4.2), (4.3), (4.4) and (4.5) respectively.

The immediate theoretical, physical and astrophysical consequences of the results obtained in this paper are

Firstly the golden Riemannian affine connections (2.27) to (2.40) reduces to pure Schwarzschild's affine connections $f(r)$ and contains unknown Post Schwarzschild's additional correction terms when the gravitational field depends on $f(r,\theta)$, has

$\left(1+\frac{2}{c^2}f\right)$ which are not found in the existing well known Schwarzchild's metrictensors and gives us a better understanding of gravitational field effect.

Secondly, the golden Riemannian equations of motion for test particle (3.3) to (3.6) reduce to the corresponding pure Einstein's equation of motion and to the order of c^{-2}contains post Einstein's correction terms, equation (3.3) to (3.6) has$\left(1+\frac{2}{c^2}f\right)$ which are not found in the existing Einstein's equation of motion implying that it will predict correction terms in the gravitational field of all massive body. These results contain the time component as Einstein's equation which implies that it will predict the existence of gravitational wave also contains additional rotational correction terms which are not found in Einstein's equation of motion for test particle.

Thirdly, the golden Riemannian tensorial geodesic equations of motion (4.2) to (4.5) reduces to the corresponding pure Riemannian tensorial geodesic equation of motion and to the order of c^{-2} contains post Einstein's correction terms and contains $\left(1+\frac{2}{c^2}f\right)$ in the φ-component which are not found in the existing Riemannian tensorial geodesic equation of motion implying that it will predict correction terms to the spinning effect in the gravitational field of all massive body.

Equations (4.2) to (4.5) has$\left(1+\frac{2}{c^2}f\right)$ in the θ-component which are not found in the existing Riemannian tensorial geodesic equation of motion implying that it will predict correction terms to the rotation effect in the gravitational field of all massive body. This results can be used to determine the energy density of the universe, temperature of the universe and to explain gravitational red shift.

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