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MANAGING LONG LEAD TIMES EMERGENCY ORDERS PROBLEMS USING THE CRITICAL BACKORDER CRITERION

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Abstract

This study sets to develop an emergency ordering policy where depending on the value of the backorder satisfied at the beginning of an order cycle, with reference to a critical value, the need for an emergency order can be determined early enough and placed to arrive even before the regular order is issued. The policy is unique with the integration of a backorder parameter into its framework that can help to determine the need for emergency order against previous policies of inventory level warning point.
To achieve this, a backorder parameter is introduced to the approximate cost expression method of the Lot-size reorder point system and thereafter, solving to obtain the critical backorder level. Results are presented from the numerical evaluation of the obtained data and a cost comparison test of the developed policy with that of a regular ordering policy with backorder effect is done. It is found that a critical backorder level of 52.8 units exist beyond which the developed policy has a cost saving advantage over that of the regular ordering policy with backorder effect in the range of 0.2% to 9.7% for backorder levels within the effective zone.

Keywords: Backordering, Lead time, Stock-out, Cycle Stock, Adjusted reorder level Critical backorder constant.

1. Introduction

Firms in business world of today, require small lead times on order deliveries to remain relevant competitors through adequate and prompt responses to customers demand [1]. One way by which firms can achieve small lead times is through the consolidated freight mode of order deliveries[2]. Emergency ordering policy is a stock replenishment policy in inventory control that allows for stock-out minimization within the regular order lead time based on the assumption that the emergency order lead time is shorter than that of the regular order lead time, hence emergency orders will arrive in inventory before the regular orders [3, 4,5].

In the event that the emergency order lead time is longer than expected, such orders when placed after the regular order have been issued will arrive in inventory outside the schedule period[6]. The late arrival of the emergency order will fail to restore the stock level of the order cycle where backordered excess demand was satisfied and subsequently lead to higher numbers of units stock-out at the end of the order cycle especially under wild demand variability [7].

Although previous studies on emergency ordering policies where complete backordering is allowed requires that all excess demand at the end of an order cycle be backorder and satisfied at the beginning of the next order cycle, they however do not specify how to determine the amount of backorder for which the use of the policy is more cost effective over others[8, 9]. They also fail to address the issue of the stock restoration of an order cycle stock where backordered excess demand was satisfied should the emergency order lead time be longer than expected and placed in the conventional form of issuance after the regular order has been placed.

This paper considers the peculiar nature of developing countries, including Nigeria, where epileptic power supply, increased production facility down time due to lack of proper maintenance policy, delay on delivery times due to bad road network, poor flight management system etc[10], could result in long emergency order lead times that are approximately equal to the regular order lead times, which alongside the existence of a higher order cycle demand relative to the expected demand could increase the chances of stock-out (excess demand) at the end of the order cycle. Since emergency orders with long lead time

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could initiate the arrival of such orders outside the order cycle, if issued after the regular order has been placed. Our model proposes to determine the need of emergency order early enough through the use of a critical backorder indicator so that the emergency order can be placed in time from the beginning of the order cycle. This will minimize the risk of emergency orders with longer than expected lead time arriving outside the schedule period if placed in the conventional form and effectively address the issue of stock level restoration of order cycle where backorder excess demand was satisfied.

2. Notations

The following notations will be adopted in development of the proposed model.

$\mu =$ The mean regular lead time demand rate

$\gamma =$ The mean demand rate per unit time

$\tau_1 =$ The regular lead time

$\tau_2 =$ The emergency lead time

$\tau_R =$ The time frame between the beginning of an order cycle and when the next regular order is placed

$\tau_e =$ The time period that elapses before the emergency order replenishes inventory to $R_b + l_b$

$l =$ Inventory rate

$l_b =$ The backorder level at the beginning of an order cycle $l_b = 1, 2, ,3...l_m$

$R_b =$ The adjusted regular reorder point for the emergency order cycle and the regular order with backorder effect

$\rho =$ The unit shortage cost per unit of item stock out at the end of a cycle

$C =$ Variable ordering cost per unit of the regular order quantity

$C_e =$ Variable ordering cost per unit of the emergency order quantity

F(R): = Probability that lead time demand is less than or equal to the regular reorder point

T = The regular order cycle length; a random variable

$F(x_L) =$ The distribution function at X_L

$f(x_L) =$ The probability density function p.d.f of X_L

$E(T) =$ The expected regular order cycle length

$G_b(l_b) =$ The expected regular order cycle cost for the regular ordering policy only

$G_e(l_b) =$ The expected emergency order cycle cost

$G_d =$ The difference between the expected order cycle cost for the regular ordering policy with the backorder effect and that of the emergency ordering policy

h = Holding cost per unit per unit time

$l_a =$ The apparent critical backorder level.

$l_c =$ The critical backorder level

$l_e =$ The effective backorder level

$l_m =$ The maximum backorder level allowable within an order cycle. $l_m = R^*$

$K_1 =$ Set-up or fixed ordering cost per regular order cycle

$k_a =$ The apparent critical backorder constant

$k_c =$ The critical backorder constant

$S_c =$ The expected shortage per emergency order cycle with complete backordering

$S_s =$ The safety stock per regular order cycle without backorder effect

$X =$ The annual demand rate, a random variable

$x =$ Avalue of X i.e a realization on X

$X_L =$ The Lead time demand, a random variable

$x_L =$ Avalue of X_L

P (R): = The probability that lead time demand is greater than or equal to R

$Q =$ The regular order quantity

$Q^* =$ The optimal regular order quantity.

$R =$ The regular reorder point

$R^* =$ The optimal regular reorder point

$S_2 =$ The expected shortage cost per emergency order cycle with complete backordering

$t =$ Time

LTCP = Low Tension Concrete pole

EOQ = Economic Order Quantity

Assumptions

1. The demand rate per annum (X) is discrete and generally follows a Poisson process, but is approximated by a continuous process within the lead time for convenience of computation
2. The maximum allowable backorder level (l_m) is equal to the optimal regular reorder level R^*
3. The emergency order is issued at the beginning of the order cycle and replenishes inventory only at the adjusted regular reorder point (R_b).
4. The order cycle demand is greater than the expected demand and the emergency lead time is approximately equal to the regular order lead times.

Development of the Generic Model for Handling Backorder Effect

Emergency ordering polices available in literature assume that, if the emergency order will arrive later than the regular order, then it is unwise to place such an order, as its purpose of checking excess demand before the arrival of the regular order would have been defeated [9, 10, 12]. The shorter the emergency lead time, the better it will serve its objective.

In the case under consideration,

i. The emergency order lead time and that of the regular order lead time are have almost the same time frame
ii. Due to the fluctuating nature of demand, peak demand or near peak demand may occur resulting to increased excess demand, even as high as the regular reorder level.
iii. The emergency order on arrival is held in inventory up till the adjusted regular reorder point, if it arrives earlier than the regular reorder point and replenishes inventory at the regular reorder point.

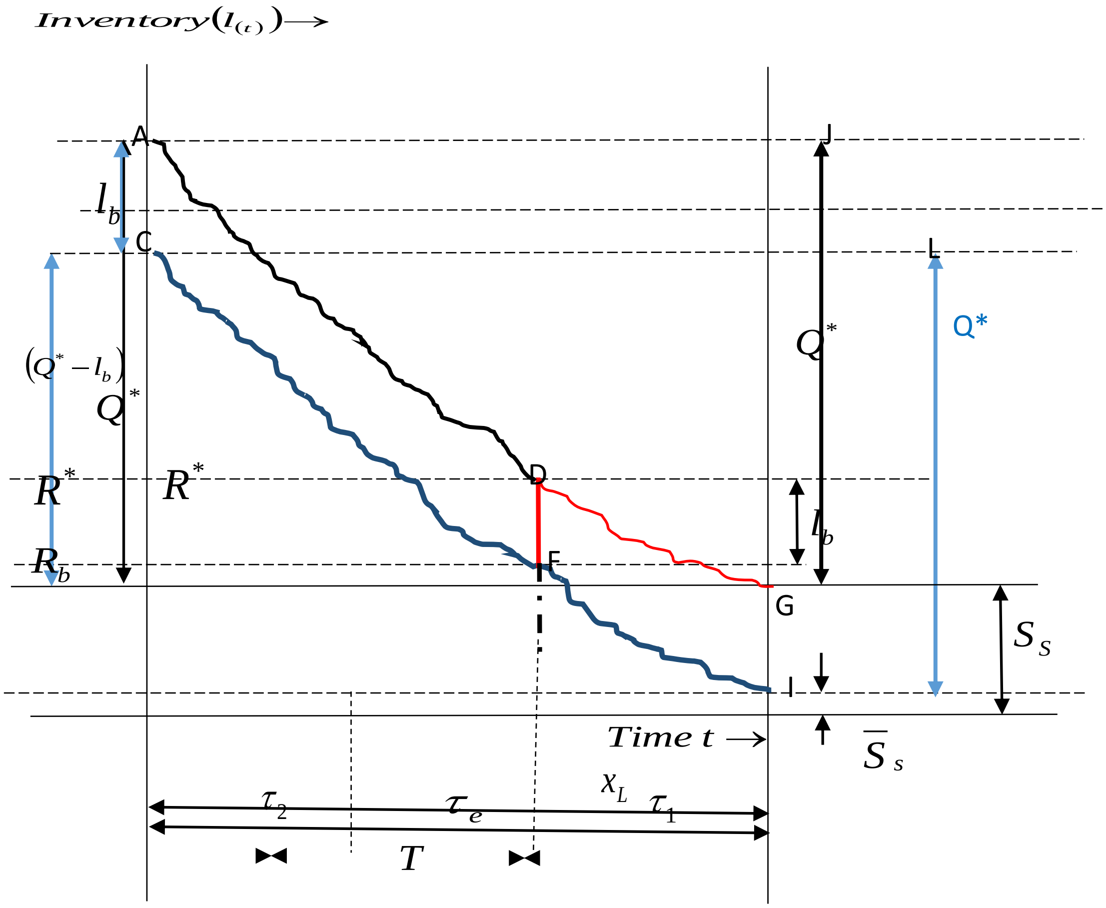

Figure 1:Shows the inventory profile of an order cycle with backorder effect that is corrected through complete emergency ordering

The line CFDG in Figure 1 captures the inventory profile described. At the beginning of the order cycle, a backorder of an amount l_b placed at the end of the previous order cycle is cleared, thereby reducing the cycle stock of the current order cycle by the amount of the backorder cleared [13, 14]. In the event that $l_b > l_c$, then an emergency order of size l_b is placed at the beginning of order cycle to arrive in inventory after a lead time of τ_2. The order is held in inventory for a time frame of τ_e if it arrives before the adjusted regular reorder point. It replenishes inventory from this point to the optimal reorder level through line FD. This will make the inventory profile of the regular order with backorder effect represented by line CFI end through FDG as though there was no reduction in the stock level of $Q^* - l_b$ at the beginning of order cycle.

Formulation of the Cost Function of the Generic Emergency Ordering Policy under Complete Backordering Regime

Suppose an emergency order is placed in a regular order cycle with backorder effect, the costs components will be that of the expected ordering, holding and shortage costs, as usual, but their composition will differ due to the presence of the emergency order costs.

The expected ordering cost will comprise the regular order ordering cost and the emergency ordering cost. The expected ordering cost per emergency order cycle C_c is express as follows;

$$C_c = \left(CQ^* + K_1\right) + C_e l_b \qquad (1)$$

where C_e is the variable unit cost per emergency order quantity.

The expected holding cost per emergency order cycle has the component of the holding cost due to the average inventory held in the order cycle and the cost incurred for keeping the emergency order in inventory for time τ_e before replenishing inventory to the regular reorder point. The inventory level of the inventory process represented by the line CFDG in Figure (1) varies linearly between the starting inventory of $Q^* - l_b + \bar{S}_s + l_b$ and the ending inventory of $\bar{S}_s + l_b$. Hence, the average inventory held within the order cycle is $\frac{1}{2}\left[Q^* - l_b + 2\left(\bar{S}_s + l_b\right)\right]$. Therefore, the expected holding cost per emergency order cycle, H_e, is expressed as follows

$$H_e = h\left[\frac{Q^* + l_b}{2} + \bar{S}_s\right] + h\tau_e l_b \qquad (2)$$

where τ_e is the time period before the emergency order replenishes inventory after its arrival, $\overline{S_s}$ is the safety stock of the emergency order cycle with complete backordering.

$$\bar{S}_s = \left(R^* - l_b\right) - \gamma\tau_1 \text{ [18]} \qquad (3)$$

Hence, H_e can be rewritten in terms of R^* as

$$H_e = h\left[\frac{Q^* - l_b}{2} + R^* - \gamma\tau_1\right] + h\tau_e l_b \qquad (4)$$

The component $h\tau_e l_b$ is the holding cost incurred for keeping the emergency order of size l_b in inventory for a time period of τ_e after its arrival before replenishing inventory.

The quantity of stock available to meet demand within the regular lead time region in Figure 1 for line CFDG is $R_b + l_b$. Shortage can only occur, if the lead time demand exceeds this amount. Hence the expected amount of stock out at the end of the order cycle S_c can be expressed as

$$S_c = \int_{\left(R_b^* + l_b\right)}^{\infty}\left[x_L - \left(R_b + l_b\right)\right] f\left(x_L\right) dx_L \qquad (5)$$

Hence, the expected shortage cost incurred at the end of the order cycle S_2 is given as follows:

$$S_2 = \rho\int_{\left(R_b + l_b\right)}^{\infty}\left[x_L - \left(R_b + l_b\right)\right] f\left(x_L\right) dx_L, \qquad (6)$$

From operational policy statement the adjusted reorder point R_b is given as;

$$R_b = R^* - l_b \qquad (7)$$

Hence, substituting the value of R^* from equation (7) gives S_c as

$$S_c = \int_{R^*}^{\infty} (x_L - R^*) f(x_L) dx_L \quad (8)$$

$$S_2 = \rho \int_{R^*}^{\infty} (x_L - R^*) f(x_L) dx_L \quad (9)$$

From [18], the evaluation of the amount shortage within the order cycle S_c is given as,

$$S_c = n(R^*) \quad (10)$$

where *n(R*)* is the expected unit stock out at the end of the order cycle.

$$n(R^*) = \mu P(R^*) - R^* P(R^* + 1) \quad (11)$$

$P(R^*) = P\{X_L \geq R^*\}$ and $P(R^*+1) = P\{X_L \geq (R^*+1)\}$ respectively. The reduction in cycle stock caused by the cleared backorder of size l_b was restored by the emergency order of the same size, which replenished inventory from R_b to R^* as indicated by the line FD in Figure (1). Hence, the inventory process of a regular order with backorder effect represented by the line CFI ends with the line DG on the same part with the line ADG, which defined the inventory profile of a regular order without backorder effect.

The expected inventory cost per emergency order cycle for the corrected backorder effect $G_e(l_b)$ is the sum of the ordering cost C_c, the holding cost H_e and the shortage cost S_2. The expression of $G_e(l_b)$ is given in equation (12) as;

$$G_e(l_b) = (CQ^* + K_1) + C_e l_b + \tau_e h l_b + h\left[\frac{Q^* - l_b}{2} + R^* - \gamma\tau_1\right] + \rho\int_{R^*}^{\infty}(x_L - R^*) f(x_L) dx_L \quad (12)$$

The expected total inventory cost per unit time *(T_e)* of the emergency order with corrected backorder effect with full backordering is given as;

$$T_e = \frac{\gamma}{Q^*}\left[(CQ^* + K_1) + C_e l_b\right] + \frac{\gamma h \tau_e l_b}{Q^*} + h\left[\frac{Q^* - l_b}{2} + R^* - \gamma\tau_1\right] + \frac{\gamma}{Q^*}\rho\int_{R^*}^{\infty}(x_L - R^*) f(x_L) dx_L \quad (13)$$

Features of the Expected Cost Function Curve

Figure 2 shows the shape of the expected inventory cost per emergency order cycle for the corrected backorder effect $G_e(l_b^{\cdot})$ and that of the regular order only $G_b(l_b^{\cdot})$ against various backorder value l_b .

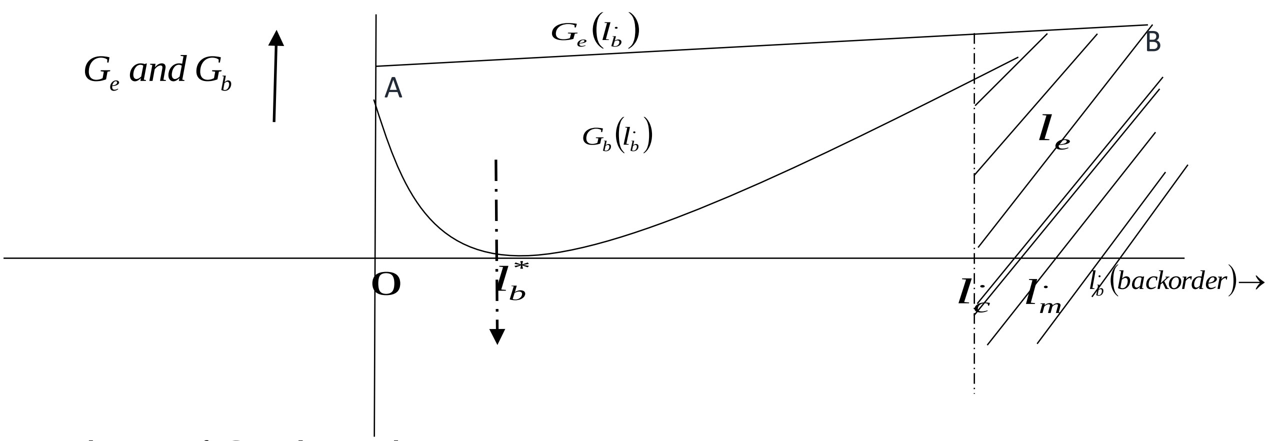

Fig 2: The curve of G_b and G_e with respect to $l_b^{\cdot}$

K_c is a constant known as the critical backorder level constant and it is estimated from equation (14).

$$K_c = h[\tau_e + 1] + C_e \quad (14)$$

There exit a level of backordered (l_a) that when its values replaces l_c, it will give a value K_a closest to (K_c). The backorder level l_a is the apparent backorder level and K_a is apparent critical backorder constant. They are both express in equation (15).

$$K_a = \frac{\rho\left[n(R^* - l_a) - n(R^*)\right]}{l_a} \quad (15)$$

The apparent critical backorder level is that backorder level to which an increment by a unit of backorder will produce the effective backorder level (l_e). The effective backorder level is that where the expected inventory cost per emergency order cycle for an emergency order placed at the beginning of an order cycle for complete backordering is less than that of a regular order only with backorder effect, it is given in equation (16).

$$l_e = l_a + 1 \qquad (16)$$

The critical backorder level is defined by the point of interception of the two curve as shown in figure 2 and it is somewhere between the apparent backorder level and the lower bound of the effective backorder level.

Computational Result

Table 1: Monthly Demand of Low Tension Concrete Electric Poles (LTCP) 2010 – 2016 from O and O Technical Company

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
2010	80	96	98	114	120	128	128	136	136	136	144	148
2011	100	104	90	62	55	55	44	40	39	42	50	54
2012	52	52	40	32	28	37	43	50	56	56	60	63
2013	40	35	42	38	34	47	59	60	67	78	78	80
2014	71	73	60	60	60	55	48	53	58	69	78	88
2015	95	112	121	125	130	130	132	136	140	125	108	108
2016	60	60	54	51	50	48	45	30	48	55	61	70

The monthly demand of low tension concrete poles (LTCP) for seven years (2010 - 2016) obtained from O and O Technical Company is shown in table 1.
The following information is given about the firm's inventory process.

K_1 = ₦200,000, C = ₦90,000, ρ = ₦170,000, h = ₦9,000, τ_1 = 1 month

The estimated values includes γ =865units/yr,Q^*= 198 units,R^* = 93 units, $\tau_e = 0.0882$yrs, $l_b^* = 8$ units K_c = 101,793.8, K_a = 101,259.29, l_e = 53 units, l_a = 52 units and l_c = 52.8 units

Table 2: The apparent critical backorder constant (K_a) values for the various backorder levels

Backorder l_b	Order Cycle Shortage for Regular Ordering policy $n(R^* - l_b)$	Order cycle shortage for the emergency ordering policy $n(R^*)$	Difference in the policies order cycle shortage $n(R^* - l_b) - n(R^*)$	Shortage Costs $\rho[n(R^* - l_b) - n(R^*)]$	Apparent backorder constant (K_a) $\rho\frac{[n(R^* - l_b) - n(R^*)]}{l_b}$
8	0.26076	0.02658	0.23418	39,810.6	4,976.33
16	1.48601	0.02658	1.45943	248,103.1	15,506.44
24	5.06826	0.02658	5.04168	857,085.6	35,711.9
32	11.34925	0.02658	11.32267	1,924,853.9	60,151.68
45	19.02496	0.02658	18.99838	3,229,724.6	80,743.12
48	27.00063	0.02658	26.97405	4,585,588.5	95,533.09
51	30.00672	0.02658	29.98014	5,096,623.8	99,933.8
52	31	0.02658	30.97342	5,265,481.4	101,259.29
56	35	0.02658	34.97342	5,945,481.4	106,169.31
64	43	0.02658	42.97342	7,305,481.4	114,148.15
72	51	0.02658	50.97342	8,665,481.4	120,353,91
80	59	0.02658	58.97342	10,025,481.4	125,318.52
88	67	0.02658	77.97342	11,385,481.4	129,380.47
93	72	0.02658	71.97342	12,235,481.4	131,564.32

Determination of $G_b(l_b)$ and $G_e(l_e)$

Table 3 shows the expected order cycle costs for the regular order only with backorder effect $G_b(l_b)$ and that of the developed emergency ordering with the corrected backorder effect $G_e(l_b)$ for the various amount of backorders and the resulting cost savings. A negative cost difference show a more cost effective regular ordering only with backorder effect. A positive cost difference shows a more cost effective emergency ordering policy.

Table 3: Cost comparison of the Expected Order Cost for the Regular Ordering Policy with Backorder Effect $G_b(l_b)$ and that of the Developed Complete Emergency Ordering Policy $G_e(l_b)$

Backorder values $l_b(Unit)$	Expected Order Cycle Costs for the Regular Ordering Policy $G_b(l_b)$₦ $\times 10{,}000$	Expected Order Cycle Costs for the Emergency Ordering Policy $G_e(l_b)$₦ $\times 10{,}000$	Cost Different in the Policies G_d ₦ $\times 10{,}000$	Percentage Costs Saving
0	1,910.5	1,910.5	0	
4	1,906.1	1,945.8	-39.7	
8	1,903.6	1,981.1	-77.5	
16	1,913.7	2,051.7	-138	
24	1,963.7	2,122.4	-158.7	
32	2,059.7	2,193.0	-133.3	
40	2,179.4	2,263.6	-84.2	
48	2,304.2	2,334.3	-30.1	
51	2,351.3	2,360.8	-9.5	
52	2,366.8	2,369.6	-2.8	
53	2,382.5	2,378.4	4.1	0.2
56	2,429.4	2,404.9	24.5	1.0
64	2,554.6	2,475.5	79.1	3.1
72	2,679.8	2,546.2	133.6	5.0
80	2,805	2,616.8	188.2	6.7
88	2,941	2,687.4	253.6	8.6
93	3,026	2,731.6	294.4	9.7

From the entries in Table 2, the apparent backorder constant occurs at a backorder level of 52 units, hence the effective backorder level is 53 units. This is supported by the entries in Table 3, as the developed policy started giving cost advantage over the regular ordering policy with backorder effect from a backorder level of 53units. The pattern of the cost saving in Table 3 also shows that the develop policy becomes more attractive to use when the value of the backorder excess demand is high which tend to solve the wild demand variability problems. In the determination of the time to hold the emergency order before replenishment, it was found that the longer the emergency order lead time the smaller the holding cost and the resulting expected total inventory cost. This also has resolved the longer than expected emergency order lead time problems.

Conclusion

In this paper, an emergency ordering policy that can handle the challenges of approximately equal emergency and regular and lead time, through the use of a critical backorder indicator has been developed. It was shown that, at some backorder level that the developed policy is more cost effective to use over the regular ordering policy with backorder effect. We showed also that, in the presence of wild demand variability and longer than expected emergency order lead time, that the developed policy can perform effectively. The determination of the critical backorder level is crucial to the application of the emergency ordering policy for cost savings as shown by the developed model. This study shows a great departure from those of previous policies where all excess demand at the end of an order cycle are backordered for complete backordering policy as only backorder levels within the effective zone are allowed for cost savings as shown in the results obtained.

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