Transactions of the Nigerian Association of Mathematical Physics
***Volume 8,* (*January, 2019*), p131 –136**

REDUCING THE INVENTORY COST OF A DUAL DELIVER POLICY USING A SINGLE ORDERING POLICY WITH BACKORDER PARAMETER

Edokpia O.R[1*] and Owu U.F.[2]

Department of Production Engineering, University of Benin, Benin City, Edo State, Nigeria

Abstract

In this study, a Lot-size reorder point inventory policy with backorder parameter is developed as a single delivery policy and a cost comparison test carried out between the developed policy and an emergency replenishment policy for a range of backorder levels to establish the cost effectiveness of the developed policy. The developed model is sensitive enough to allow for a range of backorder levels for which it has a cost savings over a dual delivery policy and establishes a maximum backorder level for allowable cycle stock reduction, which distinguishes it from other single ordering models. To achieve this, a backorder parameter is introduced to the policy frame work of the Lot-size reorder point system and a cost function is developed following the approximation method of the cost formulation.
Numerical results are presented and it is shown that between a backorder level of 0 and 52 units, the developed single delivery policy is more cost effective over the dual delivery policy with a cost savings in the range of 0.2% to 7.5%.

Keywords: *Backorder Effect, Safety Stock, Cycle Stock, Reorder Level, Adjusted Reorder Level, Stock-Out and Lot-Size.*

1. Introduction

The dual delivery inventory policy has low cost with a longer lead time delivery mode and a more expensive but shorter lead time delivery mode [1, 2]. It is employed in inventory control in the face of imminent stock-out in an order cycle with a depleted stock beyond a particular trigger point due to wild demand variability, longer than expected lead times and the satisfaction of backorder excess demand [3, 4, 5].
When the stock level of an order cycle is depleted due to the satisfaction of backordered excess demand, which is referred to as backorder effect in this study, the stock level is either replenished through the use of emergency policies or through a regular ordering policy with an order quality that is partly for the backorder excess demand and partly for the routine demand, [4, 5].
The strategy of the dual delivery policy is to minimize the risk of higher numbers of units stock-out at the end of an order cycle with a depleted stock and its attendant high inventory cost due to increased penalty cost resulting from backordering or lost sales cost as the case maybe [6].
Previous studies have not considered, the cost effectiveness of a single delivery policy with complete backordering and no stock restoration over that of a dual delivery policy with emergency replenishment strategy at some backorder levels[2, 7, 8].
In this study, a single delivery inventory policy with a backorder parameter is developed from the frame work of the lot-size reorder point system to investigate the whether there is a cost saving with the use of the developed policy over that of a dual delivery mode at some backorder level.

2. Notations

The following notation where used in the sequel.
The following notations will be adopted in development of the proposed model.

Correspondence Author: Edokpia O.R., Email: ralphedokpia@yahoo.com, Tel: +2348023368811, +2348032774899 (OUF)

$\mu =$ The mean regular lead time demand rate

$\gamma =$ The mean demand rate per unit time

$\tau_1 =$ The regular lead time

$l =$ Inventory rate

$l_b =$ The backorder level at the beginning of an order cycle

$l_b = 1, 2, ,3 .. l_m$

$R_b =$ The adjusted regular reorder point for an order cycle with backorder effect

$\rho =$ The unit shortage cost per unit of item stock out at the end of a cycle

$C =$ Variable ordering cost per unit of the regular order quantity

F(R): = Probability that lead time demand is less than or equal to the regular reorder point

$F(x_L) =$ The distribution function at X_L

$f(x_L) =$ The probability density function p.d.f of X_L

$E(T) =$ The expected regular order cycle length

$G_b(l_b) =$ The expected regular order cycle cost for the regular ordering policy only with backorder effect

$G_e(l_b) =$ Expected order cycle cost for a lot-size ordering system with emergency replenishment policy

$G_d = G_e(l_b) - G_b(l_b)$

h = Holding cost per unit per unit time

$h_0 =$ The expected holding cost per regular order cycle without backorder effect.

$H_c =$ The expected holding cost per regular order cycle with backorder effect backordering

$l_m =$ The maximum backorder level allowable within an order cycle. $l_m = R^*$

$K_1 =$ Set-up or fixed ordering cost per regular order cycle

$C_1 =$ The ordering cost per regular order cycle with backorder effect

$S_b =$ The expected shortage per regular order cycle with backorder effect

$\overline{S}_s =$ The safety stock for regular order cycle with backorder effect

$X =$ The annual demand rate, a random variable

$x =$ A value of X i.e a realization on X

$X_L =$ The Lead time demand, a random variable

$x_L =$ A value of X_L

P (R): = The probability that lead time demand is greater than or equal to R

$Q =$ The regular order quantity

$Q^* =$ The optimal regular order quantity.

$R =$ The regular reorder point

$R^* =$ The optimal regular reorder point

$S_1 =$ The expected shortage cost per regular order cycle with backorder effect

$t =$ Time

T = The regular order cycle length; a random variable.

$T_b =$ The expected total inventory cost per unit time for the regular order policy only with backorder effect

LTCP = Low Tension Concrete pole

EOQ = Economic Order Quantity

Assumption

1. The optimal values of R and Q i.e R^* and Q^* and the expected demand per unit time have been determined already from the set of relevant data provided using any of the known standard method [6, 11].
2. The demand per unit time is discrete and generally follows a Poisson process, but is approximated by a continuous process within the lead time for ease of computation. Also, it is assumed that the maximum backorder level allowed l_m is equal to the optimal reorder level R^*.

3. MODEL DEVELOPMENT

Integrating the Backorder Parameters into the Regular Order Policy Framework of a Lot – size Reorder Point System

The approximation method employed in the estimation of the expected average inventory held in the order cycle, assume that every order cycle must end with a positive on-hand inventory. Hence, the value of the safety stock was added to the regular order quantity at the beginning of the order cycle and also taken as the cycle ending stock [6].

In reality, this is usually not the case, suppose the demand occurring within an order cycle is higher than the expected demand per unit time, which is one of the vital parameters in the estimation of the optimal order quantity, then, the probability that the order cycle will end with a stock out will increase [11, 12, 13].
The proposed policy is of a single item multi period (R,Q) continuous review type. In the event that there is an amount of backorder l_b to be satisfied when the regular order of size Q arrives in inventory, then $(Q-l_b)$ becomes the starting inventory of the next order cycle, giving rise to an adjusted reorder level R_b and an on –hand inventory at the end of the order cycle of $\overline{S_s}$. A unit holding cost, *h*, per unit time and ρ backorder penalty cost are incurred for every backordered unit of excess demand.
Figure 1 shows how the proposed policy is structured from the Lot-size reorder point system. Line DEF represent the inventory realization of an order cycle replenished by the proposed policy, where the starting inventory position is reduced to position D due to the satisfaction of a backorder excess demand l_b from a previous order cycle.

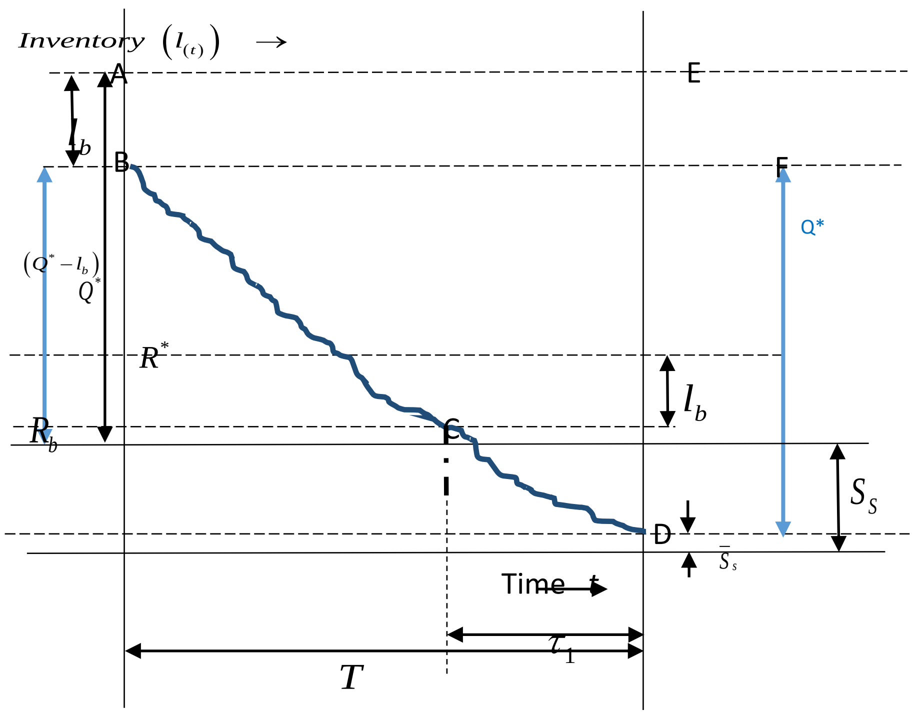

Figure 1: Inventory process for the restructured regular ordering policy for a Lot – size reorder point system with a backorder parameter.

$$R_b = R^* - l_b \quad (1)$$

The safety stock due to backorder effect $\overline{S_s}$ is shown in equation (2)

$$\overline{S_s} = R_b - \gamma\tau_1 \qquad [9, 10] \quad (2)$$

Equation 2 can be rewritten in term of can *R** in equation (3)

$$\overline{S}_s = (R^* - l_b) - \gamma\tau_1 \quad (3)$$

Any regular ordering (single mode) policy where excess demand at the end of a previous order cycle is cleared at the beginning of a current order cycle thereby reducing its cycle stock without subsequent restoration of the stock is referred to as a regular order with back order effect in this study.

4. Cost Formulation for the Restructured Regular Ordering Policy with Back Order Parameter

The cost formulation of the regular ordering with back order parameter for complete back ordering will follow the argument in the development of the cost formulation of the regular ordering without backorder effect treated in open literature, expect that it is modified in the introduction of a backorder parameter l_b and an adjusted reorder point R_b^*.

5. The Expected Ordering Cost

The expected ordering cost per regular order cycle with back order effect for full back ordering C_1 is given as follows:

$$C_1 = CQ^* + K_1 \tag{3}$$

6. Expected Holding Cost

For the inventory process represented with the line BCD in Figures 1, assuming the approximation method, is adhered to as in this case, the inventory level within the order cycle will fluctuate linearly between the total stock at the beginning of the order cycle $\left(Q - l_b + \bar{S}_s\right)$ and the on-hand inventory at the end of the cycle $\left(\bar{S}_s\right)$. The average inventory held in the order cycle will be $\frac{1}{2}\left[\left(Q - l_b + \bar{S}_s\right) + \bar{S}_s\right]$. Hence, the expected inventory cost per order cycle, H_c ,is express in equation (4):

$$H_c = h\left(\frac{Q - l_b}{2} + \bar{S}_s\right) \tag{4}$$

From equations (2) and (4), we have

$$H_c = h\left(\frac{Q - 3l_b}{2} + R^* - \gamma\tau_1\right) \tag{5}$$

7. Expected Shortage Cost

The expected shortage within the order cycle is only possible within lead time region if the lead time demand X_L is greater than R_b . Hence, the expected shortage at the end of the order cycle S_b is given in equation (6);

$$S_b = \int_{R_b}^{\infty} (x_L - R_b) f(x_L) dx_L \quad [6] \tag{6}$$

The expected shortage cost per order cycle, S_1 , is expressed in equation (7):

$$S_1 = \rho \int_{R_b}^{\infty} (x_L - R_b) f(x_L) dx_L \tag{7}$$

where ρ is the shortage cost per unit of item short at the end of the order cycle.

From equations (1) and (7), we have

$$S_1 = \rho \int_{\left(R^* - l_b\right)}^{\infty} \left[x_L - \left(R^* - l_b\right)\right] f(x_L) dx_L \tag{8}$$

But $S_0 = n(R)$ [10] (9)

$$\therefore S_1 = \rho\left[n\left(R^* - l_b\right)\right] \tag{10}$$

where $n\left(R^* - l_b\right) = \mu P\left(R^* - l_b\right) - \left(R - l_b\right) P\left(R^* - l_b + 1\right)$ (11)

But $P\left(R^* + l_b - 1\right) = P\left\{X_L \geq \left(R^* - l_b + 1\right)\right\}$

$P\left(R^* - l_b\right) = P\left\{X_L \geq \left(R^* - l_b\right)\right\}$

8. Expected Order Cycle Cost and Total Cost Per Unit Time For The Regular Ordering Policy With Backorder Parameter

The expected order cycle cost for the regular ordering with back order effect for complete back ordering $G_b(l_b)$ is expressed in equation (12)

$$G_b(l_b) = \left(CQ^* + K_1\right) + h\left[\frac{Q^* - 3l_b}{2} + R^* - \gamma\tau_1\right] + \rho \int_{\left(R^* - l_b\right)}^{\infty} \left[x_L - \left(R^* - l_b\right)\right] f(x_L) dx_L \tag{12}$$

Equation (13) gives the expected total inventory cost per unit time T_b

$$T_b = \frac{\gamma}{Q^*}\left(CQ^* + K_1\right) + h\left[\frac{Q^* - 3l_b}{2} + R^* - \gamma\tau_1\right] + \frac{\gamma}{Q^*} \int_{\left(R^* - l_b\right)}^{\infty} \left[x_L - \left(R^* - l_b\right)\right] f(x_L) dx_L \tag{13}$$

9. Features of the Cost Function of the Regular Order only with Backorder Parameter

Figure 2 shows the shape of the expected inventory cost per order cycle $G_b(l_b)$ for the regular ordering policy only with backorder effect against various backorder value l_b .

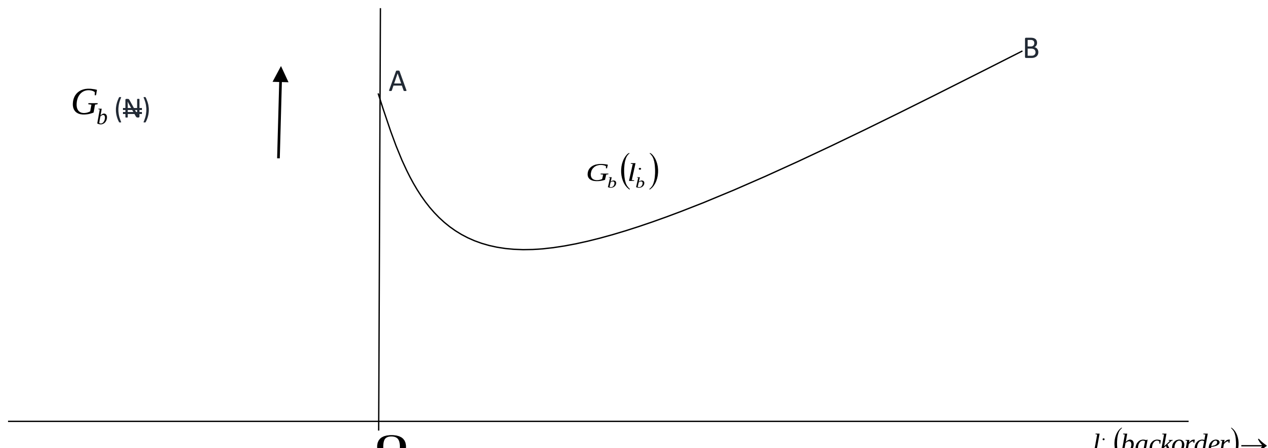

Fig 2: The curve of G_b against l_b for the regular ordering policy only with backorder effect.

10. Computation Result

The monthly demand of low tension concrete poles (LTCP) for seven years (2010 - 2016) obtained from O and O Technical Company is shown in Table 1.

Table 1: Monthly Demand of Low Tension Concrete Electric Poles (LTCP) 2010 – 2016 from O and O Technical Company

Year	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sept**	**Oct**	**Nov**	**Dec**
2010	80	96	98	114	120	128	128	136	136	136	144	148
2011	100	104	90	62	55	55	44	40	39	42	50	54
2012	52	52	40	32	28	37	43	50	56	56	60	63
2013	40	35	42	38	34	47	59	60	67	78	78	80
2014	71	73	60	60	60	55	48	53	58	69	78	88
2015	95	112	121	125	130	130	132	136	140	125	108	108
2016	60	60	54	51	50	48	45	30	48	55	61	70

The following information is given about the firm's inventory process.

K_1 = ₦200,000, C = ₦90,000, ρ = ₦170,000, h = ₦9,000, τ_1 = 1 month

The value of the mean demand per year was estimated from Table 1 as γ =865units/yr. Q^*= 198 units and R^* = 93 units.

Table 2 shows the various expected order cycle cost with the use of the regular or single ordering policy with backorder effect at different backorder levels.

Table 2: Cost comparison of the Expected Order Cost for the Regular Ordering Policy with Backorder Effect $G_b(l_b)$ and that of the Developed Complete Emergency Ordering Policy $G_e(l_b)$

Backorder values $l_b(Unit)$	Expected Order Cycle Costs for the Emergency Ordering Policy $G_e(l_b)$₦$\times 10{,}000$	Expected Order Cycle Costs for the Regular Ordering Policy $G_b(l_b)$₦$\times 10{,}000$	Cost Different in the Policies G_d ₦$\times 10{,}000$ $G_e(l_b)-G_b(l_b)$	Percentage Costs Saving
0	1,910.5	1,910.5	0	0
4	1,945.8	1,906.1	39.7	2.0
8	1,981.1	1,903.6	77.5	3.9
16	2,051.7	1,913.7	138	6.7
24	2,122.4	1,963.7	158.7	7.5
32	2,193.0	2,059.7	133.3	6.1
40	2,263.6	2,179.4	84.2	3.7
48	2,334.3	2,304.2	30.1	1.3
51	2,360.8	2,351.3	9.5	0.4
52	2,369.6	2,366.8	2.8	0.1
53	2,378.4	2,382.5	-4.1	
56	2,404.9	2,429.4	-24.5	
64	2,475.5	2,554.6	-79.1	
72	2,546.2	2,679.8	-133.6	
80	2,616.8	2,805	-188.2	
88	2,687.4	2,941	-253.6	
93	2,731.6	3,026	-294.4	

From the entries in Table 2, the developed single ordering policy without replenishment shows cost saving advantage over that with emergency replenishment from a backorder level of 4 units up to 52 units. The pattern of the cost savings is such that it increased from a backorder of 4 units to 32 units and started decreasing until a backorder level of 52 units when it losses it effective.

Conclusion

In this paper, a Lot-size reorder point inventory system with an integrated backorder parameter where stock level is not replenished after the satisfaction of backordered demand is developed. It has been shown that at some backorder levels, the policy is more cost effective than the use of emergency or dual delivery replenishment policy which could be more difficult to implement due to the challenges in determining the optimal policy parameters resulting from the difference in the frequencies of deliveries. The unique nature of this study that distinguishes it from other studies on single ordering policies is the robust nature of the developed model which is capable of utilizing stock reduction strategy to an acceptable level in achieving cost minimization as shown in the results presented.

In the application of the dual delivery policy to inventory control, the range of backorder level where the policy is more cost effective should be determine before its deployment as shown in this study, otherwise, its purpose of cost minimization due to stock-out reduction will be defected at some level of backorders.

REFERENCES

[1] Steffen, K., Gudrun, K. and Stefan, M. (2011): A comparison of the constant-order and dual-index policy for dual sourcing. *International Journal of Production Economics***133**:1, 302-311.

[2] Schmdit, C. (2015): Multiple supplier/ delivery inventory models. A review from an after – market perspective. Master of Science Thesis in industrial engineering and management specialization, optimization and logistics Department of Mathematics and Mathematical Statistics, UMEA UNIVERSITY SVUEDEN.

[3] Chiang,C. (2001), A Note on Optimal Policies for a Periodic Inventory System with Emergency Orders. Computers and Operations Research. Vol. 28, pp. 93 – 103.

[4] Bhowmick, J., and Samanta G.P (2012). Optimal inventory policies for imperfect inventory with price dependent stochastic demand and partially back logged shortages. Yugoslav journal of operations research vol. 22 pp. 199-223.

[5] Ren-Qian Zhang, Yan-Liang Wu, Wi-Guo Fang and wen-Hui. Zhou. (2016). Inventory model with partial backordering when backordered customers delay purchase after stock-out. Restoration mathematical problems in Engineering, vol. 2016, article ID 6425403 pages.

[6] Chiang, C. (2010). An order expediting policy for Continuous Review Systems with Manufacturing Lead Time. European Journal of Operational Research. Vol. 203 PP. 526-531.

[7] Robert, W.B. and Jaw, A.V.M. (2015): Global Dual Sourcing and Order Smoothing: The Impact of Capacity and Lead Times. *Management Science***61**:9, 2080-2099.

[8] Joachim, A., Rob, B. and Geert-Jan, V.H. (2016): Repairable Stocking and Expediting in a Fluctuating Demand Environment: Optimal Policy and Heuristics. *Operations Research***64**:6, 1285-1301.

[9] Liang-Yuh, O. and Hung-Chi, C. (2001): The Variable Lead Time Stochastic Inventory Model with a Fuzzy Backorder Rate, Journal of the Operations Research of Japan. Vol. 44 (1) M.

[10] Nahmias, S. (2009): Production and Operations Analysis. 8th edition. McGraw Hill International

[11] Heijden M.C. and Kok T. (1998), Estimating stock levels in periodic review inventory system, journal of operation research letters. Vol. 22, pp. 79-182

[12] Dohi, T., Kaw, N. and Osaki, S. (1995). Continuous review cyclie inventory models with emergency order. Journal of the operations research. Vol. 38(2)

[13] Iwu. C.H., Ogbonna J.C, Jude O. and Onuma. G.K(2014):. Application of inventory model in determining stock control in an organization. American journal of applied mathematics and statistic, vol 2(5): pp. 307 – 317.

[14] Jaksic. M, and Franson, J. (2015): Optimal Inventory management Supply backordering. International Journal of Production Economic. Vol. 159 pp 254 – 264.

[15] Sidney, B. and Paul, Z. (1991): Inventory models with continuous stochastic demand, the Annuals of Applied Probability Vol. 1 pp 419 – 435.