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A COMPARATIVE ANALYSIS OF ANALYTICAL HIERARCHY PROCESSES (AHP) FOR SUPPLIER SELECTION

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Abstract

This paper examined the Analytical Hierarchy Process (AHP) as an established method for evaluating and selecting supliers. The literature identified the existence of a major gap in the AHP method. Two new AHP methods are presented to obviate the limitation in literature which is the main contribution of this paper. Using the same data and consitency test, it was observed that the two new AHP proposed in this paper showed better consistency result than the traditional AHP in literature. Novel results were also obtained.

1. Introduction

Supplier Selection is a multi-criteria decision making (MCDM) Problem. It requires the evaluation of both qualitative and quantitative factors. Selecting the best supplier among several alternatives is an enormous task for decision makers (DMs) and procurement managers (PMs).Since no single supplier can excel in all the attributes required by DMs. The Analytical Hierarchy process (AHP) is an established method for evaluating and selecting suppliers [1-5]

One of the drawbacks of the AHP is the compromise nature of the method which leads to inconsistency and imprecision in the method. To this end, This paper in an attempt to address this limitation in the method.

2. Analytical Hierarchy Process

The analytical hierarchy process (AHP) was first developed by [1-4]. AHP is a widely used multi-criteria decision making method which is based on the decomposition of a complex decision problem into several smaller and easier to handle sub-problems [2,3,4,6]. Since its introduction, the AHP has become one of the most widely used multi-criteria decision making (MCDM) methods in different areas of human endeavour, such as political, military, economic, industries, social, education, administration and management sciences.

In AHP a problem is structured as a hierarchy. Once the hierarchy has been constructed the decision makers begin prioritization procedure to determine the relative importance of the elements in each level. Prioritization involves eliciting judgments in response to questions about the dominance of one element over another with respect to a property. The scale used for comparisons in AHP enable DMs to indicate how many times an element dominates another with respect to the particular attribute or criterion [3,6].

The decision makers (D

Ms) can express their preference between pairs of element verbally as equally important, moderately important, strongly important, very strongly important, extremely important. These descriptive preferences would then be translated into numerical values 1,3,5,7,9 respectively with 2.4,6 and 8 as intermediate or compromise values for comparison between two successive judgments. Reciprocals of these values are used for the corresponding transposed judgment [6]. Table 1 shows the comparison scale used in AHP.

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Table 1. Comparison Scale used in AHP

INTENSITY	DEFINITION	EXPLANATION OF IMPORTANCE
1	Equal importance	Two activities contribute equally to the object
3	Moderate importance	Experience and judgement slightly favour one over another
5	Strong importance	Experience and judgement favour one over another
7	Very strong importance	Activity IS strongly favoured and it dominance is demonstrated in practice
9	Extreme importance	Importance of one over another affirmed on the highest possible order
2, 4, 6, 8	For compromise between the above values (i.e intermediate values)	Used to represent compromise judgement between the preferences above
Reciprocal of above	If activity i has one of the above non-zero members assign to it when compared with activity j, then j has the reciprocal value when compare with i	

(Source: [2,3,5,6])

3. BASIC PROCEDURES IN AHP

The basic procedures of AHP to supplier selection problem is stated in the following steps below:

Step 1: State the problem and its objective

Step 2: Structure the hierarchy from the top (which contains the objectives of DMs) through intermediate level containing the criteria or sub criteria to the lowest level which contains the alternatives or suppliers.

Step 3: Develop a pair wise comparison matrix A.

The pairwise comparison matrix A with element a_{ij} denotes the relative importance or preference of the ith factor with respect to jth factor. The pairwise comparison matrix is given as:

$$A = (a_{ij}) = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ {}^{1}a_{1n} & {}^{1}/a_{2n} & & 1 \end{pmatrix} \quad (1)$$

There are n(n-1)/2 judgments required to develop the set of matrices in step 3. Reciprocal are automatically given to each element in the pairwise comparison matrix in the rows below the first row, just before the diagonal, n is the size of the matrix [3].

Assuming we are given n criteria or attributes, $A_1 \ldots A_n$ with preference weigh $W_1, \ldots w_n$.

Then, let the entries or elements of matrix A be given as $a_{ij} = W_i/W_j$ implies

$$A = \begin{pmatrix} W_1/W_1 & W_1/W_2 & \cdots & W_1/W_n \\ W_2/W_1 & W_2/W_2 & \cdots & W_2/W_n \\ \vdots & \vdots & \ddots & \vdots \\ W_n/W_1 & W_n/W_2 & \cdots & W_n/W_n \end{pmatrix} \quad (2)$$

Step 4. Calculate for the rank of the priority vectors and normalize. This is done for the criteria and each of the alternative with respect to each of the criteria.

Step 5. Carryout a consistency test of the comparison matrix is given by the consistency ratio (CR) to assess the consistency of the comparison matrix, this is given as

$$CR = CI / RI \quad (3)$$

Where the consistency index(CI) is

$$CI = \frac{\lambda \max - n}{n-1} \quad (4)$$

i.e. λ Max = (cell value 1 x obtained weight 1) + (Cell value 2 x obtained weight 2) + … + Cell value (n – 1) x obtained weight (n – 1) + (cell value n x obtained weight n) [5]. (5)

CI is the consistency intensity which shows the entire consistency judgment for each comparison matrix and the hierarchy structure [2.7]. And $\lambda_{\max}$ is the highest eigen value of the judgment matrix. The random indicators developed for the matrices of size n, where $1 \leq n \leq 15$ is given below in Table 2

Table 2 Random index (indicators)
(Source: [7])
The C.R. is accepted if
$CR \leq 0.10$, OTHERWISE the judgment matrix is inconsistent [5,7].

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Random indicator	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

6. If λmax; CI and CR are satisfactory, then the decision is taken based on the normalized values.
OTHERWISE the process is repeated until these values lies in the desired range (Saravanan, et al, 2012)

3.1 Eigen vector(Normalized Priority weight) Algorithm

To solve for the eigenvector (priority vectors) of the pairwise comparison matrix, we follow the algorithm (Saaty,1990) below:

1. Square the pairwise comparison matrix A
2. Calculate the row sums and normalized. By normalizing we mean
   $\frac{Rn}{RT}$
   Where Rn is the row sums of matrix size n and RT is the now total
3. Then stop. If the difference between successive iterations is insignificant.

A computer program in MATLAB was developed to solve (1) using the Eigen vector algorithm [2] in subsection 3.1. The data used throughout this paper is adopted from literature [8].

4. Materials and Methods

One of the problems of the AHP method in literature is the compromise nature of the method which often leads to inconsistency and imprecision in the model. In order to obviate this limitation, This paper develops and presents two New Analytical Hierarchy process (NAHP) methods as follows:

(i) The null Compromised Analytical Hierarchy Process (NCAHP) Method
(ii) The Zero Compromised Analytical Hierarchy Process (ZCAHP) Method

4.1 The null Compromised **Analytical Hierarchy Process (NCAHP) Method**

The NCAHP method presented in this paper is easier to use than the AHP method [2,3] in literature. The NCAHP method is a compromise not permitted method. It allows for the max[w1, w2] to be used in the interval, where the element between the interval is a compromise factor. Therefore, the compromise nature of the AHP method [2,3] is eliminated and by so doing removing the inconsistency of the AHP method in literature. This proposed method (NCAHP) provides a better solution to the supplier selection problem than the AHP method [3] in literature as we shall see later in this work.

The pairwise comparison matrix of NAHP is given below

$$A = (a_{ij}) = \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \cdots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix} \qquad (6)$$

Provided w_n is not a compromise value, otherwise, we replace with the max (w_1, w_2) if w_n is a compromise value.
To test for consistency of the pairwise matrix, we use equations (3), (4) and (5).

The steps for implementing the NCAHP are as follows:

1. State the objective of the problem
2. identify the criteria
3. Identify the alternatives or suppliers

4. Develop a pairwise comparison matrix by subsisting max (w_1, w_2) in place of the compromise factors in the existing AHP in literature, where w_1 , w_2 are elements in the pairwise comparison matrix.
5. Evaluate the criteria
6. Evaluate the suppliers with each of the criteria.
7. Rank and select the alternatives or suppliers.

4.2 The Zero Compromise Analytical Heparchy Process (ZCAHP) Method

The ZCAHP method is also a compromise not permitted method. In order to address the problem of inconsitency associated with the compromise nature of the AHP method in literature [3]. This paper also develop and present the ZCAHP method. In the (ZCAHP) method the compromise elements are substituted as zero in the numerator of the scores in the problem comparison matrix of the AHP in literature. However, the compromise factors is the denominator is taking as the exponential of compromise (zero), implies $e^0=1$.

That is for ZCAHP if

$$aij = \begin{cases} 0, & \text{for } aij = w_n/w_m \text{ provided} \\ e^o = 1, & \text{if } w_m \text{ is a compromise factor} \end{cases} \qquad (7)$$

Therefore, the steps for implementing ZCAHP is as follow:

1. State the objective of the problem
2. Identify the criteria
3. Identify the supplier
4. Develop a pairwise comparison matrix by subtitling the following in place of compromise element in (7).
5. Evaluate and weight the criteria
6. Evaluate the supplier with each of the criteria
7. Rank and select the suppliers

5. Discussion of Results

Using same data in literature[8] in implementing both the NCAHP and ZCAHP we obtained the decision matrices for the two new methods in Table 4 and Table 5. Table 3 is adopted from literature[8]. The NPW values (results) for the supplier are also shown in Tables(3-5). The NPW results are used for the ranking of the suppliers. The summary of the results using the three methods, AHP, NCAHP and ZCAHP are shown in Table 6.

Table 3. Decision Matrix Using AHP Method

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	NPW
S_1	0.1898	0.1672	0.1834	0.1415	0.2114	0.1320	0.1974	0.0463	0.1500	0.1071	0.1627	0.1180	0.1727
S_2	0.0764	0.2155	0.1128	0.0533	0.1702	0.0717	0.1465	0.1184	0.1637	0.2029	0.1483	0.1053	0.1395
S_3	0.1289	0.0506	0.1582	0.1654	0.1285	0.0741	0.0805	0.0853	0.2020	0.1612	0.0663	0.0719	0.1074
S_4	0.2304	0.1106	0.0590	0.1177	0.0919	0.1131	0.0961	0.2134	0.0484	0.1460	0.0981	0.2263	0.1422
S_5	0.1448	0.1275	0.1861	0.1911	0.1281	0.0770	0.1105	0.1811	0.1706	0.0481	0,0805	0.0848	0.1291
S_6	0.1028	0.0538	0.0963	0.1213	0.1151	0.1375	0.107O	0.1181	0.0774	0.1327	0.1664	0.1036	0.0918
S_7	0.1076	0.0683	0.0669	0.1434	0.0936	0.1869	0.1742	0.0908	0.1108	0.0989	0.1648	0.1959	0.1116
S_8	0.0194	0.2065	0.1373	0.0663	0.0613	0.2078	0.0878	0.1466	0.0770	0.1031	0.1129	0.0942	0.105

(Source:[8])

Table 4. Decision Matrix NCAHP Method

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	NPW
S_1	0.1898	0.1672	0.1834	0.1415	0.2114	0.1320	0.1974	0.0463	0.1500	0.1071	0.1627	0.1180	0.1709
S_2	0.0764	0.2155	0.1128	0.0533	0.1702	0.0717	0.1465	0.1184	0.1637	0.2029	0.1483	0.1053	0.1325
S_3	0.1289	0.0506	0.1582	0.1654	0.1285	0.0741	0.0805	0.0853	0.2020	0.1612	0.0663	0.0719	0.1079
S_4	0.2304	0.1106	0.0590	0.1177	0.0919	0.1131	0.0961	0.2134	0.0484	0.1460	0.0981	0.2263	0.1457
S_5	0.1448	0.1275	0.1861	0.1911	0.1281	0.0770	0.1105	0.1811	0.1706	0.0481	0,0805	0.0848	0.1289
S_6	0.1028	0.0538	0.0963	0.1213	0.1151	0.1375	0.107O	0.1181	0.0774	0.1327	0.1664	0.1036	0.0869
S_7	0.1076	0.0683	0.0669	0.1434	0.0936	0.1869	0.1742	0.0908	0.1108	0.0989	0.1648	0.1959	0.0869
S_8	0.0194	0.2065	0.1373	0.0663	0.0613	0.2078	0.0878	0.1466	0.0770	0.1031	0.1129	0.0942	0.0968

Table 5. Decision Matrix For ZCAHP Method

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	NPW
S_1	0.1851	0.1804	0.1359	0.1128	0.1906	0.1495	0.1805	0.0936	0.1524	0.0508	0.1854	0.1629	0.1593
S_2	0.0959	0.1626	0.1286	0.1286	0.1618	0.0990	0.1432	0.1199	0.1752	0.2275	0.2026	0.1261	0.1499
S_3	0.1533	0.0806	0.0931	0.0931	0.1795	0.1445	0.1224	0.1129	0.1952	0.0989	0.0586	0.1037	0.1378
S_4	0.1940	0.0835	0.0494	0.0494	0.1069	0.1055	0.0916	0.0774	0.0749	0.1690	0.0395	0.0632	0.0725
S_5	0.1672	0.1820	0.2158	0.2158	0.1025	0.1100	0.0907	0.3162	0.0848	0.1035	0,2004	0.1650	0.1895
S_6	0.1237	0.0682	0.1255	0.2155	0.1259	0.1246	0.0650	0.1100	0.0991	0.1347	0.0982	0.1313	0.0824
S_7	0.0557	0.0846	0.0592	0.0592	0.0408	0.0175	0.2555	0.0867	0.1110	0.01245	0.1425	0.1560	0.1029
S_8	0.0271	0.1582	0.1923	0.1923	0.0920	0.2494	0.0510	0.0833	0.1074	0.0911	0.0728	0.0919	0.1057

Table 6. Summary of Results

Supplies	NPW (AHP)	NPW (NCAHP)	NPW ZCAHP)	RANK (AHP)	RANK (NCAHP)	RANK (ZCAHP)
S_1	0.1727	0.1709	0.1593	1	1	2
S_2	0.1395	0.1325	0.1499	3	3	3
S_3	0.1074	0.1079	0.1378	6	6	4
S_4	0.1422	0.1457	0.0725	2	2	8
S_5	0.1291	0.1289	0.1895	4	5	1
S_6	0.0918	0.0869	0.0824	8	8	7
S_7	0.1116	0.1305	0.1029	5	4	6
S_8	0.1056	0.0968	0.1057	7	7	5

5.1 Consistency Test (AHP) Method

We analysis the consistency test of the AHP method using eqs.(3), (4) and (5). We have

CR $= \frac{CI}{RI} = -0.6441 \leq 0.1$

Which show that the method is consistency

Where $CI = -0.9533, RI = 1.48$ Table 2.

5.2 Consistency Test (NCAHP) Method

We compute the consistency test using eq (3), (4) and (5)

We have, $CR = -0.6428 \leq 0.1$ which means the method is consistent.

Where, $CI = -0.9514 \leq$ and $RI = 1.48$ (Table2)

5.3 Consistency Analysis

For consistency, using eg.(3) we have

$$CR = \frac{CI}{RI}$$

Using eq.(4), $CI = \frac{\sum aijwj - n}{n-1}$

Let RI = K (Constant)

Then, $CR = \frac{\frac{\sum aijwj - n}{n-1}}{\mathbf{k}}$

$$\therefore\ CR = K\frac{\left(\sum aijwj - n\right)}{n-1}$$

Limit as $n \to \infty$ CI tends to -1 and CR tends to –K. This means that as the size of the matrix increases the value of CI gets closer to -1 and the value of CR gets closer to –k where –k is the negative value of RI.

5.4 COMPARISON OF THE METHODS USING DIFFERENT RANDOM INDEX(RI) COMPUTATION METHODS

The formula[5] for the compution of RI, different from the table value commonly used in literature [7,9]. The formula for the computation of RI [5] is as follows.

$$RI = 1.98\frac{n-2}{n}$$

To this end we make a comparison of the AHP method in literature [2,3] with the two new methods NCAHP and ZCAHP presented in this paper. Using the two different RI approaches, the results and the ordering(rank) of the methods are given in Table 7.

Table.7 comparison of the methods using different RI approaches

S/N	Method	Matrix size (n)	CI	RItab	RI chty	CRtab	CR chty	Rank
1.	AHP	12	-0.9533	1.48	1.80	-0.6641	-0.5296	3
2.	NCAHP	12	-0.9514	1.48	1.80	-0.6428	-0.5286	2
3.	ZCAHP	12	-0.9477	1.48	1.80	-0.6403	-0.5265	1

In view of the above, it is obvious that our ZCAHP and NCAHP methods are ranked first and second and are better methods than the traditional AHP is literature [1,2,3,5]. Irrespective of the RI values used in computing CR for consitency test the results (rank) is the same. Our proposed methods eliminate the compromise elements of the AHP method and by so doing remove the inconsistency of the AHP method in literature [1,3] and provide a more consistent solution to supplier selection problem. The proposed methods (NCAHP and ZCAHP) provide better consistent alternative methods than the AHP [1,2,3,5] method in literature.

6. Findings

This paper proposed two new analytical hierarchy process (NAHP) (i) the null compromise analytical hierarchy process (NCAHP) and (ii) the zero compromise analytical hierarchy process (ZCAHP) methods for supplier selection. Our proposed methods show a better consistency performance than the AHP method in literature. The methods AHP, NCHP, ZCAHP were subjected to consistency test using the same set of data and both the NCAHP and ZCAHP have better consistency ratios (CR) of -0.6428 and -0.6403 respectively, while AHP has a consistency ratio of -0.6441. Results from this work shows that the ranking of suppliers by the three methods are different. However, the AHP and NCHP have the same ranking in six of the suppliers ie, suppliers S_1, S_2, S_3, S_4, S_6 and S_8 as 1st, 3rd, 6th, 2nd, 8th, and 7th respectively. Their rankings are only different with suppliers S_5 and S_7 where the AHP ranked them 4th and 5th and NCAHP ranked them 5th and 4th respectively. But the result is totally different with ZCAHP method where suppliers S_5, S_1, S_2, S_3, S_8, S_7, S_6 and S_4 are ranked 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th respectively. We also went further to rank the three methods based on their consistency ratio (CR) test and the ZCAHP method ranked top and adjudged to have a better result of the three methods. While the NCAHP and ranked AHP are ranked 2nd and 3rd respectively. It is concluded that the closer the CR value of a method to the consistency bound $CR \leq 0.1$, the better the method. This work also looked into the area of random index computation. Our literature search shows two RI computation methods [5]which proposed $RI = 1 - 98\frac{n-2}{n}$ and the value as shown in Table 2 [7.9] give different CR values for RI computation. The proposed new methods, ZCAHP and NCAHP were ranked as first and second best preferred methods ahead of the traditional AHP methods [2,3]) in literature. It observed from this paper that Irrespective of the RI values used in computing CR for consitency test the ordering (rank) is the same.

7. Contribution

We proposed the new analytical hierarchy process (NAHP) (i) the null compromise analytical hierarchy process (NCAHP) and (ii) the zero compromise analytical hierarchy process (ZCAHP) methods for supplier selection. Our proposed methods show a better consistency performance than the AHP method in literature. The methods AHP, NCAHP, ZCAHP were subjected to consistency test using the same set of data(problems) and both the NCAHP and ZCAHP gave a better consistency ratios (CR) of -0.6428 and -0.6403 respectively, while AHP has a consistency ratio of -0.6441 which is our major contribution to supplier selection literature.

8. Conclusion

This paper proposed the NCAHP and the ZCAHP methods for supplier selection. This proposed method helps to address the inconsistency limitation of the AHP method [1,3,5]) in literature and by so doing, providing a more consistent methodologies and better alternatives methods than the AHP method. We therefore recommend our methods as a better alternatives than the AHP for researchers, decision makers and experts in industries for supplier selection.

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