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NOTE ON 'AN EXTENSION OF THE PROPERTIES OF INVERSE α-CUTS TO FUZZY MULTISETS'

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Abstract

In this paper, we showed that an alleged proposition in the literature does not hold in general by providing an example.

1.0 Introduction

The notion of α-cut (or α-level set) of a fuzzy set [1], and inverse α-cut of a fuzzy set and its properties as an extension of alpha-cut [2], were introduced to establish a bridge between fuzzy set theory and classical set theory. Proposition 3.2, items (iii) and (iv), parts (a) and (b) in [3], states that;

${}^{\alpha}(A\cap B)^{-1} = {}^{\alpha}A^{-1} \cup {}^{\alpha}B^{-1}$, ${}^{\alpha}(A\cup B)^{-1} = {}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1}$

and

${}^{\alpha^-}(A\cap B)^{-1} = {}^{\alpha^-}A^{-1} \cup {}^{\alpha^-}B^{-1}$, ${}^{\alpha^-}(A\cup B)^{-1} = {}^{\alpha^-}A^{-1} \cap {}^{\alpha^-}B^{-1}$.

respectively. However, these do not hold in general and we provide an example to show that the propositions indeed fails.

2.0 Preliminaries

Definition 2.1: (Inverse α-Cut [2])

Let $A \in F(X)$, and $\alpha \in [0,1]$. Then, the non-fuzzy set;

${}^{\alpha}A^{-1} = \{x \in X | \mu_A(x) < \alpha\}$.

is called an *inverse α-cut* (or *inverse α-level set*) of A. If the strict inequality is replaced by the weak inequality $\leq$, then it is called a weak *inverse α-cut* of A, denoted by ${}^{\alpha^-}A^{-1}$. That is;

${}^{\alpha^-}A^{-1} = \{x \in X | \mu_A(x) \leq \alpha\}$.

3.0 Example

The example below shows that the alleged proposition 3.2, items (iii) and (iv), parts (a) and (b) in [3], do not hold in general.

Let $\boldsymbol{X = \{x, y, z, w, t\}}$, $\boldsymbol{A = \{(x, o.4), (y, 0.8), (z, 0.5), (w, 0.9)\}}$,

$X = \{x, y, z, w, t\}$, $A = \{(x, o.4), (y, 0.8), (z, 0.5), (w, 0.9)\}$,

and $B = \{(x, 0.7), (y, 1), (w, 0.5), (t, 0.3)\}$.

Then, $(A\cup B) = \{(x, 0.7), (y, 1), (z, 0.5), (w, 0.9), (t, 0.3)\}$

and $(A\cap B) = \{(x, o.4), (y, 0.8), (w, 0.5)\}$.

Now, for $\alpha = 0.7$, we have the following;

${}^{\alpha}A^{-1} = \{x, z\}$, ${}^{\alpha}B^{-1} = \{w, t\}$, ${}^{\alpha}(A\cup B)^{-1} = \{z, t\}$,

${}^{\alpha}(A\cap B)^{-1} = \{x, w\}$, ${}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1} = \emptyset$, and

${}^{\alpha}A^{-1} \cup {}^{\alpha}B^{-1} = \{x, z, w, t\}$.

Therefore, ${}^{\alpha}(A\cap B)^{-1} \neq^{\alpha} A^{-1} \cup^{\alpha} B^{-1}$,

and ${}^{\alpha}(A\cup B)^{-1} \neq^{\alpha} A^{-1} \cap^{\alpha} B^{-1}$.

Also, for the weak *inverse α-cut*, we have;

${}^{\alpha^-}A^{-1} = \{x, z\}$, ${}^{\alpha^-}B^{-1} = \{x, w, t\}$, ${}^{\alpha^-}(\mathrm{A}\cup\mathrm{B})^{-1} = \{x, z, t\}$, ${}^{\alpha^-}(\mathrm{A}\cap\mathrm{B})^{-1} = \{x, w\}$,

${}^{\alpha^-}A^{-1} \cap {}^{\alpha^-}B^{-1} = \{x\}$, and ${}^{\alpha^-}A^{-1} \cup {}^{\alpha^-}B^{-1} = \{x, z, w, t\}$.

Then, it follows that;

${}^{\alpha^-}(A\cap B)^{-1} \neq {}^{\alpha^-}A^{-1} \cup {}^{\alpha^-}B^{-1}$.

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and ${}^{\alpha^-}(A \cup B)^{-1} \neq {}^{\alpha^-}A^{-1} \cap {}^{\alpha^-}B^{-1}$.

Lemma 3.1. Let $A \in F(X)$, if $\alpha = 0$, then ${}^{\alpha}A^{-1} = \emptyset$ and ${}^{\alpha^-}A^{-1} = \emptyset$.

Lemma 3.2. Let $A \in F(X)$, and let $S(A)$ be the support for A, $\forall \alpha \in [0,1]$, then ${}^{\alpha}A^{-1} \subseteq S(A)$ and ${}^{\alpha^-}A^{-1} \subseteq S(A)$.

Proposition 1. Let $A, B \in F(X)$ such that $S(A) \neq S(B)$, then

(i). ${}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1} \subseteq {}^{\alpha}(A \cup B)^{-1}$ \and

(ii). ${}^{\alpha}(A \cap B)^{-1} \subseteq {}^{\alpha}A^{-1} \cup {}^{\alpha}B^{-1}$.

Proof

(i). Suppose $S(A) \neq S(B)$, then $\exists x \in X$ such that $x \in S(A)$ and $x \notin S(B)$.

It has already been shown in [3] that ${}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1} \subseteq {}^{\alpha}(A \cup B)^{-1}$, we only need to show that ${}^{\alpha}(A \cup B)^{-1} \nsubseteq {}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1}$.

Suppose on the contrary that ${}^{\alpha}(A \cup B)^{-1} \subseteq {}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1}$,

now let $x \in {}^{\alpha}(A \cup B)^{-1}$, then $\mu_{(A \cup B)}(x) < \alpha$,

$\Rightarrow max\{\mu_A(x), \mu_B(x)\} < \alpha;$

$\Rightarrow \mu_A(x) < \alpha$ and $\mu_B(x) < \alpha$,

$\Rightarrow x \in {}^{\alpha}A^{-1}$ and $x \in {}^{\alpha}B^{-1}$.

But by **Lemma 3.2.**, ${}^{\alpha}B^{-1} \subseteq S(B)$, hence its a contradiction since $x \notin S(B)$.

Therefore, ${}^{\alpha}(A \cup B)^{-1} \nsubseteq {}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1}$.

(ii). This is similar to (I).

Remark 1. In the case of strong inverse α-*Cut*, the result is also similar.

Proposition 2. Let $A, B \in F(X)$, if $S(A) = S(B)$, then

i. ${}^{\alpha}A^{-1} \cap {}^{\alpha}B^{-1} = {}^{\alpha}(A \cup B)^{-1}$

ii. ${}^{\alpha}(A \cap B)^{-1} = {}^{\alpha}A^{-1} \cup {}^{\alpha}B^{-1}$

iii. ${}^{\alpha^-}(A \cap B)^{-1} = {}^{\alpha^-}A^{-1} \cup {}^{\alpha^-}B^{-1}$

iv. ${}^{\alpha^-}(A \cup B)^{-1} = {}^{\alpha^-}A^{-1} \cap {}^{\alpha^-}B^{-1}$.

Proof. See [3].

4.0 Conclusion

In this paper, we showed that parts (a) and (b) of items (iii) and (iv) of proposition 3.2 in [3] which states that the inverse ***α-cut*** of the union of two fuzzy sets is equivalent to the intersection of their inverse ***α-cuts*** and the inverse ***α-cut*** of the intersection of two fuzzy sets equivalent to the union of their inverse ***α-cuts*** respectively, does not hold in general and we provided an example to show this. We have also provided a modification to the said proposition by including a condition for it to hold.

5.0 References

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