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TUNNELING SPEED OF POINT INTERACTIONS

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Abstract

In this paper the tunneling speed of some families of point interaction are investigated.

1. INTRODUCTION

Application of point interaction model are found in areas of Physics such as quantum Physics, solid state Physics and optics. Researches in these models of point interaction have been very active in the last couple of decades due to remarkable success in the fabrication of nanoscale quantum devices.

Localized potentials are used to represent point or contact interaction. According to [1], a mathematically well defined potential can be constructed using the Schrodnger operator. If the kinetic energy operator$-\frac{h}{2m}\frac{d^2}{dx^2}$ with the interaction point switch off giving the free particle Hamiltonian at singularity points, the boundary conditions have to be imposed on the wave function $\psi(x)$ and its first derivative. Detailed consideration of operator theory on point interaction is found in the work in [2-5]. These papers dealt with the time aspect of point or contact interaction scattering properties, however to the best of our knowledge the tunneling speed has not yet been considered.

Evaluation of device performance requires the idea of the speed of tunneling. In this paper, after the introduction, consideration of the theoretical formalism of tunneling speed of some potentials are presented

2. THEORETICAL FORMALISM

Starting with the one – dimensional Schrodinger equation

$$-\frac{d^2\varphi}{dx^2}+V(x)\varphi = E\varphi \tag{1}$$

$V(x)$is a complex potential and arbitrarily shaped. h = 2m = 1 is assumed in Eq. (1). For a small infinitesimal interval $-\varepsilon < x < +\varepsilon$, $\varepsilon \to 0$ for point interaction, the wave function that satisfies Eq. (1) outside the interval is

$$\varphi_L = e^{ikx} + Re^{ikx}, \qquad -\infty < x < -\varepsilon \tag{2}$$

$$\varphi_R = Te^{ikx}, \qquad +\varepsilon < x < \infty$$

Where $T = IT|exp(i\emptyset_t)$ is the transmission coefficient and $R = IR|exp(i\emptyset_r)$ is the reflection coefficient, and E = k^2.

Differentiating Eq. (1) with respect to energy and using its complex conjugate gives

$$\int_{-\varepsilon}^{+\varepsilon}\varphi\varphi^* dx = \left(\frac{\partial\varphi}{\partial E}\frac{\partial\varphi^*}{\partial x} - \varphi^*\frac{\partial^2\varphi}{\partial E\partial x}\right)\Big|_{-\varepsilon}^{+\varepsilon} + 2i\int_{-\varepsilon}^{+\varepsilon} V_i\varphi^*\frac{\partial\varphi}{\partial E\partial x} \tag{3}$$

V_i is the imaginary part of potential $V(x)$.

Substituting Eq.(2) into Eq.(3) gives

$$\frac{1}{k}\int_{-\varepsilon}^{+\varepsilon}|\varphi(x)|^2 dx = |T|^2\frac{d\phi_o}{dE} + |R|^2\frac{d\phi_r}{dE} - \frac{1}{2E}Im(R) + \frac{1}{k}\int_{-\varepsilon}^{+\varepsilon} V_i Im\left(\varphi^*\frac{\partial\varphi}{\partial E}\right)\partial x \tag{4}$$

In [6] the relationship among relevant tunneling times goes as follows

$$\tau_d = \tau_g - \tau_i + \tau_a \tag{5}$$

Where τ_d is the dwell time, the total time the particle spends in the barrier of length2ε

$$\tau_d = \frac{1}{k}\int_{-\varepsilon}^{+\varepsilon}|\varphi(x)|^2 dx \tag{6}$$

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$$\tau_g = |T|^2 \frac{d\phi_o}{dE} + |R|^2 \frac{d\phi_r}{dE} \quad (7)$$

$$\tau_{gt} = \frac{\partial\varphi_R}{\partial E}, \quad \tau_{gr} = \frac{\partial\varphi_L}{\partial E} \quad (8)$$

τ_g is known as the bidirecting group delay $\phi_o = \phi_t + z\varepsilon k$, $\phi_o = \phi_t$, as $\varepsilon \to 0$ where phases in the transmission and reflection becomes identical for real and symmetric barriers.

$$\tau_i = -\frac{1}{2E} Im(R) \quad (9)$$

τ_i the self interaction is a consequence of overlap of the incident and reflected particle wave in front of the barrier

$$\tau_a = \frac{1}{k}\int_{-\varepsilon}^{+\varepsilon} V_i In\left(\varphi^* \frac{\partial\varphi}{\partial E}\right)\partial x \quad (10)$$

τ_a for the absorption time of the non – zero imaginary part of the potential

$\tau_d = 0$ as $\varepsilon \to 0$ for point interaction Eq (5) becomes $\tau_g = \tau_i + \tau_a$. For real potential, that is, $\tau_a = 0$ it gives

$$\tau_g = \tau_i \quad (11)$$

The point interaction are the self-adjoint extension (SAE) of kinetic energy operation subject to the following boundary condition

$$\begin{pmatrix}\varphi_+ \\ \varphi_+'\end{pmatrix} = e^{i\theta}\begin{pmatrix}a & b \\ c & d\end{pmatrix}\begin{pmatrix}\varphi_- \\ \varphi_-'\end{pmatrix} = U\begin{pmatrix}\varphi_- \\ \varphi_-'\end{pmatrix} \quad (12)$$

Where $\varphi' = \frac{\partial\varphi}{\partial x}, \varphi_\pm' = \varphi_{(\pm 0)}'$ and $\varphi_\pm = \varphi_{(\pm 0)}$ for wave function at the origin ($x = 0$).

From Eq. (2), the transmission and reflection coefficient are

$$T(k) = \frac{2k}{(a+d)k + i(c - bk^2)}$$

$$R(k) = \frac{k(a-d) + i(c+bk^2)}{(a+d) + i(c+bk^2)} \quad (13)$$

3. FOR REAL POTENTIAL

$$|T(k)|^2 + |R(k)|^2 = 1 \quad (14)$$

For Eq.(13) and Eq.(7) the derivation gives

$$\tau_{gt} = \frac{1}{2k}\frac{(a+d)(c+bk^2)}{(a+d)^2k^2 + (c-bk^2)^2} \quad (15)$$

$$\tau_{gr} = \tau_{gt} + \frac{1}{2k}\frac{(a-d)(bk^2 - c)}{(a-d)^2k^2 + (c+bk^2)^2}$$

Substituting Eq (15) and Eq (13) into Eq (7) gives

$$\tau_g = \frac{1}{k}\frac{abk^2 + cd}{(a+d)^2k^2 + (c-bk^2)^2} \quad (16)$$

Since $2m = t = 1$

$$V_g^2 \tau_g = 1 \quad (17)$$

That is

$$\frac{V_g^2}{k}\frac{abk^2 + cd}{(a+d)^2k^2 + (c-bk^2)^2} = 1$$

$$V_g = \left(\frac{k[(a+d)^2k^2 + (c-bk^2)^2]}{abk^2 + cd}\right)^{1/2} \quad (18)$$

V_g is the tunneling speed

4. FOR DIRAC DELTA FUNCTIONS (δ) POTENTIAL

$$V(x) = c\delta(x), \ \ c > 0 \quad (19)$$

$V(x)$ in Eq.(19) is a real potential of a point interaction in one dimension

$$U = \begin{pmatrix}1 & 0 \\ c & 1\end{pmatrix}, ac - bd = 1 \quad (20)$$

Upon substitution in Eq.(13) gives

$$T(k) = \frac{2k}{2k+ic}, \quad R(k) = \frac{ic}{2k+ic} \tag{21}$$

The group time τ_g is

$$\tau_g = \frac{c}{k}\frac{1}{4k^2+c^2} \tag{22}$$

Using Eq.(17), tunneling speed $V_g$is gotten from

$$V_g^2 = \frac{k(4k^2+c^2)}{c} \tag{23}$$

$$V_g = \left[\frac{k}{c}(4k^2+c^2)\right]^{1/2} \tag{24}$$

5. FOR FIRST DERIVATIVE OF DIRAC DELTA FUNCTION(δ') POTENTIAL

$$V(x) = b\delta'(x) \tag{25}$$

$V(x)$in Eq (25) is a real potential

Kocinac and Milanovic [6] present two considerations for δ' function.

First consideration treating δ function potential as a dipole potential given as

$$V(x) = \curlywedge \lim_{\varepsilon\to 0}\frac{1}{2\varepsilon}[\delta(x+\varepsilon) - \delta(x-\varepsilon)] \tag{26}$$

$\curlywedge$is the limiting interaction. The group tunneling time τ_g obtained is

$$\tau_g = \tau_i = -\frac{1}{2E}Im(R) = -\frac{1}{2k}\frac{\curlywedge^2\varepsilon^{1-2v}}{k^2+(\curlywedge^2\varepsilon^{1-2v})^2} \tag{27}$$

The tunneling speed for this is

$$V_g = \left(-\frac{2k\left[k^2+(\curlywedge^2\varepsilon^{1-2v})^2\right]}{\curlywedge^2\varepsilon^{1-2v}}\right) \tag{28}$$

Second consideration

$$\delta'(x) = \lim_{\varepsilon\to 0}\lim_{l\to\varepsilon}\Delta_{\varepsilon,l}(x) \tag{29a}$$

and

$$\delta'(x) = \lim_{l\to 0}\lim_{\varepsilon\to 0}\Delta_{\varepsilon,l}(x) \tag{29b}$$

Where the interaction,$\Delta_{\varepsilon,l}(x) = \pm(\varepsilon l)^{-1}$ for $-(\varepsilon \pm l)/2 < x < (\varepsilon \pm l)/2$ and zero elsewhere

The boundary condition which define the interaction in Eq. (25) is

$$U = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \tag{30}$$

Imposing the boundary condition, the group tunneling time $\tau_g$is obtained as

$$\tau_g = \frac{b}{k}\frac{1}{4+k^2b^2} \tag{31}$$

The tunneling speed for this is

$$V_g = \left(\frac{k(4+k^2b^2)}{b}\right)^{1/2} \tag{32}$$

6. FOR POTENTIAL OF THE FORM $V(x) = \alpha\delta(x) + \beta\delta'(x)$

$$\tau_g = \frac{1}{k}\frac{\alpha(2-\beta)^2}{4\alpha^2+(4+\beta^2)^2k^2} \tag{33}$$

The tunneling speed for this is

$$V_g = \left(\frac{k\left[4\alpha^2+(4+\beta^2)^2k^2\right]}{\alpha(2-\beta)^2}\right)^{1/2} \tag{34}$$

7. CONCLUSION

The tunneling speed of various points or contact interaction have been considered in this paper. For point interaction due to real potential, it is only one relevant tunneling time namely the group tunneling time which measure the delay in the appearance of the wave packet at the front and the end of the potential barrier. On this tunneling time, the tunneling speed is obtained, which is the speed of the wave packet from the front to the end of the potential barrier.

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