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# GROUP VELOCITY IN A DISSIPATIVE MEDIUM

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***Abstract***

***Group velocity in a fluid medium that is dissipative is studied. Two solutions were obtained based on two realistic assumptions. Incidentally, each of the solutions appears singular in the neighbourhood of the origin. A different inverse conclusion appears in the case of high (in magnitude) dissipative coefficient.***

## 1. Introduction

This paper is on the effects of dissipation on the evolving wave group velocity. The characteristic behaviour of group velocity in wave physics was studied in detail [1]. In this consideration, group velocity is proved to be associated with hyperbolic and thus, kinematic behavior in an ideal fluid. Kinematic phenomenon is associated with other involving processes such as motor traffic flow on the highway, flow in an estuary [2]. In this consideration, a more generalised definition of group velocity is established [3].

In this study, we consider the evolution of group velocity in a fluid with dissipative behaviour. Asymptotic behaviours of these evolutional processes will be analysed.

## 2. Group velocity related to the concept of parabolic events

Take x as the spatial variable with t as the corresponding time, V(x, t) is a function representing group velocity. Let q(x,t) be related to V(x,t) through the equation

$$\frac{\partial q}{\partial x} = \frac{\partial v}{\partial t} \quad (1)$$

Equation (1) can be explained by the relationship

$$q = -Q(V) - k\frac{\partial V}{\partial x} \quad (2)$$

$k$ = dissipation coefficient characteristics of physical phenomenon andusually constant

From equation (2),

$$\frac{\partial q}{\partial x} = -\frac{\partial Q}{\partial V}\frac{\partial V}{\partial x} + \frac{\partial^2 V}{\partial x^2} = -V\frac{\partial v}{\partial x} + k\frac{\partial^2 V}{\partial x^2} \quad (3)$$

$$\partial Q = V\partial V, = \partial\left(\frac{V^2}{2}\right), Q = \frac{V^2}{2} + R, R = constant\ of\ integration$$

Thus, from (1) and (3) [3, 4]

$$\frac{\partial V}{\partial t} + \frac{V\partial V}{\partial x} - \frac{k\partial^2}{\partial x^2}V = 0 \quad (4)$$

Equation (4) is a familiar equation of mathematical physics, for example in water wave mechanics and magneto hydrodynamics phenomena. It expresses the relationship between the time variation, non linearity and physical dissipation in an evolving process. Hence, the equation (4) is quite familiar. However, the interesting development in this consideration is that the evolution of group velocity is described by this equation (4) and thus can be so studied.

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## 3. The Periodic Wave Solution

Let V = V(X) where $X = x + Ut$

U is a constant that describes the processes of phase change in the evolving wave field.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial X}\frac{\partial X}{\partial x} = \frac{\partial}{\partial X}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial X^2}, \frac{\partial}{\partial t} = U\frac{\partial}{\partial x}$$

Thus, equation (4) is now of the form that is ordinary. Thus,

$$U\frac{dV}{dx} + V\frac{dV}{dX} = k\frac{d^2V}{dX^2} = U\frac{dV}{dX} + \frac{1}{2}\frac{dV^2}{dX} = k\frac{d^2V}{dX^2}$$

That is

$$\frac{d}{dX}\left[UV + \frac{v^2}{2}\right] = k\frac{d^2V}{dx^2}$$

Integrating both sides, we obtain

$$UV + \frac{V^2}{2} = k\frac{dV}{dX} + J_0 \qquad (4)$$

$J_0 = constant\ of\ integration$

As $V \to 0, \frac{dV}{dX} \to 0$

Thus, $J_0 = 0\ and$

$$\frac{dV}{dX} - \frac{UV}{k} = \frac{V^2}{2k} \qquad (5)$$

Equation (5) is a simple nonlinear equation that is a form of Benoullis. The appearance of this important equation in this consideration, though interesting, is quite surprising.

From

$$V^{-2}\frac{dV}{dX} - \frac{UV^{-1}}{k} = \frac{1}{2k}, let\ W = V^{-1}, \frac{dW}{dX} = -V^{-2}\frac{dV}{dX}, V^{-2}\frac{dV}{dX} = -\frac{dW}{dX},$$

Thus

$$\frac{dW}{dX} + \frac{UW}{k} = -\frac{1}{2k}\ \ ie,\ \ \frac{d}{dX}\left[We^{Ux/k}\right] = -\frac{1}{2k}e^{Ux/k}$$

That

$$We^{Ux/k} = -\frac{1}{2U}e^{Ux/k} + R_1$$

$$W = -\frac{1}{2U} + R_1e^{-UX/k},\ \ X = 0\ when\ W = 0, R_1 = \frac{1}{2U}$$

$$W = \frac{1}{2U}\left[e^{-Ux/k} - 1\right] \qquad (6a)$$

From (6a)

$X \to 0, \quad V \to \infty. \quad X \to \infty, e^{-UX/k} \to 0, \qquad V \to -2U.$

$Again\ \ X \to -\infty, \qquad V \to 0$

$$V(X) = 2U\left[e^{-Ux/k} - 1\right]^{-1}$$

$$V(x - Ut) = V(x,t) = -\frac{Ue^{UX/2k}}{\sinh(UX/k)} \qquad (6b)$$

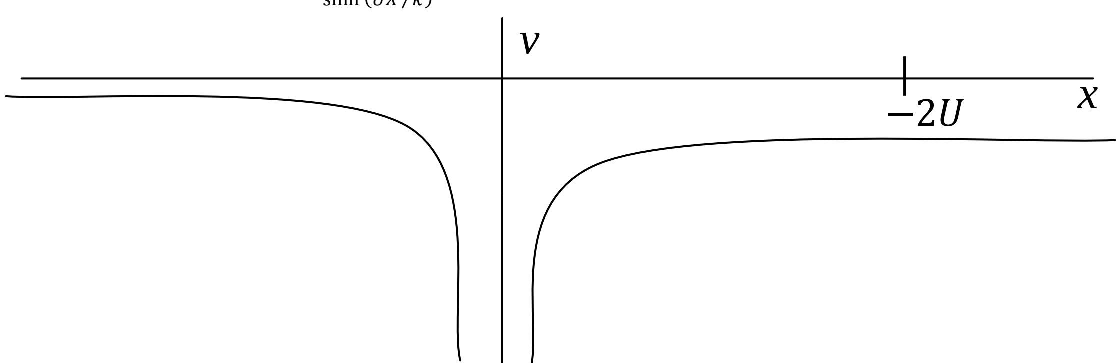


**Fig. 1.**The group velocity profile**.**

## 4. Alternative Approach

In equation (5) $J_0 \neq 0, J_1 = 2J_0$

$$2\frac{dV}{dX} = V^2 + 2UV - J_1$$

$$If\ V^2 + 2UV - J_1 = (V - V_1)(V - V_2) = V^2 - V(V_1 + V_2) + V_1V_2$$

Thus, $V_1 + V_2 = -2U, \qquad V_1V_2 = -J_0$

$$2\frac{dV}{dX} = (V - V_1)(V - V_2)$$

$$\frac{kdV}{(V - V_1)(V - V_2)} = \frac{dX}{2} = \frac{k}{V_1 - V_2}\left[\frac{dV}{V - V_1} - \frac{dV}{V - V_2}\right]$$

$$X = \frac{2k}{V_1 - V_2}\left[\ln\left(\frac{V - V_1}{V - V_2}\right)\right], \ln\left[\frac{V - V_1}{V - V_2}\right] = \frac{(V_1 - V_2)}{2k}X = R_0$$

$$\frac{V - V_1}{V - V_2} = e^{R_0}, V = \frac{V_1 - V_2e^{R_0}}{1 - e^{R_0}} = \frac{V_2e^{\frac{R_0}{2}} - V_1e^{\frac{R_0}{2}}}{2\ sinh\left(\frac{R_0}{2}\right)}$$

Obviously, as $X \to \infty, \quad R_0 \to \infty \quad V_1 \neq V_2,\ V = V_2, \quad X \to -\infty, \quad V = V_1$

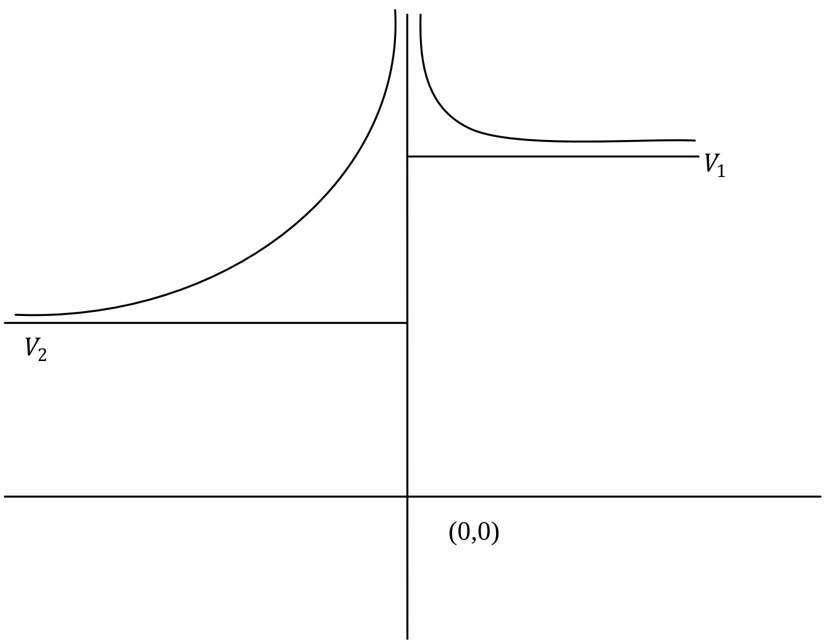


**Fig. II**.–Singular behaviour associated with the evolution of group velocity profile

## 5. Conclusion

Both fig I and fig II describe the singular behaviour when dissipative forces are moderate [2]. The methods of approach suggest identical profile though on the opposite side of x-axis eventually [1]. We may also consider the solution in an environment that is highly dissipative i.e k is large. This indicates that $\sinh(R_0/2) \to \frac{R_0}{2}, e^{R_0/2} \to 1 + \frac{R_0}{2}$. Thus, Solution (6), gives V = -U and (7) gives $V = V_1 + V_2$. This suggests that periodic oscillatory behaviour of characteristic waveforms can be completely eliminated by intense, dissipative forces in the system.

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