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# COMBINED AZIMUTHAL AND AXIAL SHEAR WAVE PROPAGATION IN AN INCOMPRESSIBLE HOLLOW CYLINDER.

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***Abstract***

***The problem of determining the stresses and displacement in a hollow cylindrical material under combined telescopic and torsional shear wave propagation is considered. The cylinder under consideration is Mooney Rivlin model. The analysis of the model resulted into systems of nonlinear second order partial differential equations which are uncoupled. Monge's method was used in solving the uncoupled systems of nonlinear second order partial differential equations in order to determine the displacement in each of the equations. The Monge's subsidiary equations reduced each of the equations into a nonlinear ordinary differential equation. The method of solution adopted provided a closed form solution for the determination of both the circular and axial displacement and stresses at any cross section.***

***Keywords*****:** Angular displacement, Axial displacement, Shear stresses, Deformed radius, Displacement gradient and shear strain.

## 1.0 Introduction

The vibration and wave application characteristics of rubberlike solids are important for practical applications in sonar, ultrasound, engine mountings, seismic isolators and in modeling of some biological tissues. Erumaka et al [1] worked on Axial shear wave propagation in an incompressible cylindrical solid of a Mooney Rivlin material of order two and obtained a closed form solution for the axial displacement. Different aspect of the axial shear problem for compressible materials have been examined by Agarwal [2] and Ertepinar and Erarslanoglu [3], the latter use numerical method to find the solution of the governing equations. More recently, theoretical results have been obtained by Jiang and Knowles [4]. Haddow and Erbay [5] considered a condition which a strain energy function must satisfy for an axial shear waves to propagate. They established that a necessary condition for the propagation of a pure axial shear wave to be admitted is that static pure shear is admitted and a sufficient condition for static pure axial shear to be admitted is that propagation of a pure axial shear wave is admitted. They also investigated the possibility of the simultaneous propagation, in the radial direction of a pure longitudinal wave and a finite amplitude pure axial transverse wave. Their results correspond with that of Haddow and Mioduchowski [6], Jiang and Beatty [7,8]. Jiang and Ogden [9] also obtained the same result in their work. In the work of Akbarov and Guliev [10], axisymmetric longitudinal wave propagation in a finite prestretched compound circular cylinder made of an incompressible material, they assumed that the inner and outer cylinder were made of incompressible neo-Hookean materials. Numerical result on the influence of the prestrains in the inner and outer cylinder on wave dispersion are presented and discussed. It is established that the pretension of the cylinder increases the wave velocity. Abo-el-nour and Fatimah [11] in their work investigated some aspect of dispersion relation of flexural waves propagation in a transversely isotropic hollow circular cylinder of infinite extent placed in a primary magnetic field. The result shows that the effect of the primary magnetic field is to increase the value of the material constants. According to Selim [12] damping of the medium has strong effect in the propagation of torsional waves and the velocity of such waves depend on the presence of initial stress. Dai and Wang [13] considered the stress wave propagation in piezoelectric reinforced by inextensible fibres. Rivlin [14] obtained an exact solution for an incompressible isotropic linearly elastic materials of the Mooney-Rivlin material, his result corresponds with that of Green and Zerna [15], and Ogden [16]. The same problem for incompressible materials was examined in Ogden et al [17]. Exact solution for some linear cases have been obtained in Tao et al [18] for a class of neo-Hookean materials.
The present study is to determine both the azimuthal and telescopic displacement and stresses developed as a result of both torsional and axial wave propagating in a hollow cylindrical material.

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## 2.0 FORMULATION OF THE PROBLEM:

Let the region $D_0 = \{(\mathrm{R}, \Theta, \mathrm{Z}), a \leq R \leq b, 0 \leq \Theta \leq 2\pi, 0 \leq Z \leq h\}$ denotes the cross section of a right circular hollow cylinder with inner radius a and outer radius b in its undeformed configuration. The hollow circular cylinder is subjected to both axial and azimuthal shear force. The resulting deformation is a one-to-one axisymmetric deformation which maps the point with cylindrical polar coordinate $(\mathrm{R}, \Theta, \mathrm{Z})$ in the undeformed configuration $D_0$ to the point $(\mathrm{r}, \theta, \mathrm{z})$ in the deformed region $D$.

The deformation equations for the combined axial and azimuthal shear wave in an incompressible hollow cylindrical material is given by

$$r = R, \theta = \Theta + \alpha(\mathrm{R}, \mathrm{t}), z = \mathrm{Z} + \beta(\mathrm{R}, \mathrm{t}) \tag{2.1}$$

where the function $\alpha(\mathrm{R}, \mathrm{t})$ and $\beta(\mathrm{R}, \mathrm{t})$ are to be determined . The deformation gradient tensor $\bar{\mathrm{F}}$ is given by

$$\bar{\mathrm{F}} = \begin{pmatrix} 1 & 0 & 0 \\ r\alpha_R & 1 & 0 \\ \beta_R & 0 & 1 \end{pmatrix} \tag{2.2}$$

The left Cauchy-Green deformation strain tensor $\bar{B}$ and $\bar{B}^{-1}$ are

$$\bar{\mathrm{B}} = \bar{\mathrm{F}}\bar{\mathrm{F}}^{\mathrm{T}} = \begin{pmatrix} 1 & r\alpha_R & \beta_R \\ r\alpha_R & (r\alpha_R)^2 + 1 & \beta_R r\alpha_R \\ \beta_R & \beta_R r\alpha_R & \beta_R^2 + 1 \end{pmatrix} \tag{2.3}$$

and

$$\bar{B}^{-1} = \begin{pmatrix} \beta_R^2 + (r\alpha_R)^2 + 1 & -r\alpha_R & -\beta_R \\ -r\alpha_R & 1 & 0 \\ -\beta_R & 0 & 1 \end{pmatrix} \tag{2.4}$$

$$\mathrm{I}_1 = 3 + \beta_R^2 + r^2\alpha_R^2$$
$$\mathrm{I}_2 = 3 + \beta_R^2 + r^2\alpha_R^2 = \mathrm{I}_1$$
$$\mathrm{I}_3 = 1$$

Here we consider a material with strain energy function given by

$$\mathrm{W} = \sum_{p,q=0}^{\infty} \mathrm{C}_{pq} (\mathrm{I}_1 - 3)^p (\mathrm{I}_2 - 3)^q \tag{2.5}$$

where $C_{00} = 0$ and $p + q \leq 2$.

$$W = A_1(I_1 - 3) + A_2(I_2 - 3) + A_3(I_1 - 3)(I_2 - 3) + A_4(I_1 - 3)^2 + A_5(I_2 - 3)^2 \tag{2.6}$$

The stress tensor for incompressible material is given by

$$\bar{\tau} = -\mathrm{PI} + 2\mathrm{W}_1\bar{B} - 2\mathrm{W}_2\bar{\mathrm{B}}^{-1} \tag{2.7}$$

where I is unit tensor and P is the hydrostatic pressure and negative sign indicates hydrostatic pressure in compression and

$$W_1 = A_1 + k_1 r^2\alpha_R^2 + k_1\beta_R^2 \tag{2.8}$$
$$W_2 = A_2 + k_2 r^2\alpha_R^2 + k_2\beta_R^2 \tag{2.9}$$

where $k_1 = A_3 + 2A_4$ and $k_2 = A_3 + 2A_5$

Putting (2.3), (2.4), (2.8) and (2.9) in (2.7) we have

The components of the stress tensors summarized as

$$\tau_{rr} = -P + k_3 + k_4 r^2\alpha_R^2 + k_4\beta_R^2 + k_6 r^2\alpha_R^2\beta_R^2 + k_7\beta_R^4 + k_7 r^4\alpha_R^4 \tag{2.10a}$$
$$\tau_{r\theta} = c_1 r\alpha_R + c_2 r^3\alpha_R^3 + c_2\beta_R^2 r\alpha_R = \tau_{\theta r} \tag{2.10b}$$
$$\tau_{rz} = c_1\beta_R + c_2 r^2\alpha_R^2\beta_R + c_2\beta_R^3 = \tau_{zr} \tag{2.10c}$$
$$\tau_{\theta\theta} = -P + k_3 + c_6 r^2\alpha_R^2 + c_8\beta_R^2 + c_9\beta_R^2 r^2\alpha_R^2 + c_9 r^4\alpha_R^4 \tag{2.10d}$$
$$\tau_{\theta z} = d_2\beta_R r\alpha_R + c_9\beta_R r^3\alpha_R^3 + c_9\beta_R^3 r\alpha_R = \tau_{z\theta} \tag{2.10e}$$
$$\tau_{zz} = -P + k_2 + c_6\beta_R^2 + c_8 r^2\alpha_R^2 + c_9 r^2\alpha_R^2\beta_R^2 + c_9\beta_R^4 \tag{2.10f}$$

where $k_3 = 2A_1 - 2A_2$, $k_4 = 2k_1 - 2A_2 - 2k_2$, $k_6 = -4k_2$, $k_7 = -2k_2$, $c_1 = 2A_1 + 2A_2$, $c_2 = 2k_1 + 2k_2$, $c_6 = 2A_1 + 2k_1 - 2k_2$, $c_8 = 2k_1 - 2k_2$, $c_9 = 2k_1$, $and\ d_2 = 2A_1\ etc.$

**3 Equations of motion**: The equations of motion in cylindrical polar coordinates (r,θ,z) is given by

$$\frac{\partial\tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\partial\tau_{rz}}{\partial z} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) + \rho b_r = \rho a_r \tag{3.1a}$$
$$\frac{\partial\tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta} + \frac{\partial\tau_{\theta z}}{\partial z} + \frac{2}{r}\tau_{r\theta} + \rho b_\theta = \rho a_\theta \tag{3.1b}$$
$$\frac{\partial\tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{z\theta}}{\partial\theta} + \frac{\partial\tau_{zz}}{\partial z} + \frac{1}{r}\tau_{zr} + \rho b_z = \rho a_z \tag{3.1c}$$

Where $b_r, b_\theta$ and $b_z$ are the components of the body forces and $a_r$ , $a_\theta$ and $a_z$ are the components of the acceleration and $\rho$ is the material mass density. The non-zero equations of motion which in the absence of body force are given by

$$\frac{\partial\tau_{rr}}{\partial R}\frac{\partial R}{\partial r}+\frac{1}{r}(\tau_{rr}-\tau_{\theta\theta})=0 \quad (3.2a)$$

$$\frac{\partial\tau_{\theta r}}{\partial R}+\frac{2}{R}\tau_{r\theta}=\rho a_\theta \quad (3.2b)$$

$$\frac{\partial\tau_{zr}}{\partial R}+\frac{1}{R}\tau_{zr}=\rho a_z \quad (3.2c)$$

Substituting the components of the stress tensors in (3.2) and simplifying we obtain the equations of motion as

$$\delta_6 r^2 g_R^2+\delta_8 r^3 g_R g_{RR}+\delta_8 r w_R w_{RR}+\delta_9 r w_R^2 w_{RR}+\delta_7 r^4 g_R^4+\delta_9 r^5 g_R^3 g_{RR}+\delta_2 w_R^2+\delta_5 w_R^4=0 \quad (3.3a)$$

$$3c_1\alpha_R+5c_2R^2\alpha_R^3+c_1R\alpha_{RR}+3c_2R^3\alpha_R^2 g_{RR}=\rho a_\theta \quad (3.3b)$$

$$3c_2R\beta_R^2\beta_{RR}+c_1R\beta_{RR}+c_1\beta_R+c_2\beta_R^3=\rho R a_z \quad (3.3c)$$

where $\delta_1=-c_1,\ \delta_2=-2A_2,\ \delta_2=-4k_2-2k_1,\ \delta_4=-c_2,\ \delta_6=2k_4+\delta_1,\ \delta_7=4k_7+\delta_4,$
$\delta_8=2k_4,\qquad \delta_9=4k_7,\qquad \delta_5=k_7$

Equation (3.3b) and (3.3c) are uncoupled systems which can be solved separate to determine the circular displacement and axial displacement.

Solving equation $(3.3b)$ first, using monge method of solving second order partial differential equation. From $(3.3b)$, we have

$$\alpha_{RR}(c_1R+3c_2R^3\alpha_R^2)-\rho\alpha_{tt}=-3c_1\alpha_R-5c_2R^2\alpha_R^3 \quad (3.4)$$

where $a_\theta=\alpha_{tt}$

We apply Monge method to solve (3.4)

The standard form of the Monge equation is given as [19]

$$R^\star r+Ss+Tt^\star=V \quad (3.5)$$

where $R^\star$,S,T and V are functions of R, t, w, p, and q.

$r=\alpha_{RR}$ , $s=\alpha_{Rt}$, $\qquad t^\star=\alpha_{tt}$

V is only first order derivative and constant.

$$R^\star dpdt+TdqdR-VdRdt=0 \quad (3.6a)$$

$$R^\star(dt)^2-SdRdt+T(dR)^2=0 \quad (3.6b)$$

The equations $(3.6a)$ and $(3.6b)$ are called Monge's subsidiary equations and the relations which satisfy these equations are called intermediate integrals.

Comparing (3.4) and (3.5), we have the following

$$R^\star=c_1R+3c_2R^3\alpha_R^2,\ T=-\rho,\ S=0,\qquad V=-3c_1\alpha_R-5c_2R^2\alpha_R^3 \quad (3.7a)$$

From $(3.6a)$ and $(3.6b)$

where $p=\alpha_R$ , $dp=\alpha_{RR}dR$, $q=\alpha_t$ , $dq=\alpha_{tt}dt$ (3.7b)

Substituting $(3.7a)$ in $(3.6a)$ and $(3.6b)$, we have

$$(c_1R+3c_2R^3\alpha_R^2)dpdt-\rho dqdR+(3c_1\alpha_R+5c_2R^2\alpha_R^3)dRdt=0 \quad (3.8)$$

$$(c_1R+3c_2R^3\alpha_R^2)(dt)^2-\rho(dR)^2=0 \quad (3.9)$$

From (3.9), we have

$$dR=\pm\left(\sqrt{\frac{c_1R+3c_2R^3\alpha_R^2}{\rho}}\right)dt \quad (3.10)$$

Substituting (3.10) in the second term of the left hand side of (3.8) for dR in order to introduce dt.

$$(c_1R+3c_2R^3\alpha_R^2)dpdt-\rho dq\left(\sqrt{\frac{c_1R+3c_2R^3\alpha_R^2}{\rho}}\right)dt$$
$$+(3c_1\alpha_R+5c_2R^2\alpha_R^3)\,dRdt=0 \quad (3.11)$$

$$(c_1R+3c_2R^3\alpha_R^2)dpdt+\rho dq\left(\sqrt{\frac{c_1R+3c_2R^3\alpha_R^2}{\rho}}\right)dt$$
$$+(3c_1\alpha_R+5c_2R^2\alpha_R^3)dRdt=0 \quad (3.12)$$

It implies that $t=c$

Incompressible materials should not be used in a dynamic analysis because the speed of elastic pressure waves is infinite.

$t=c$ is satisfied by the incompressible condition that is $\det(\overline{F})=1$.

(3.11) + (3.12) reduces to

$$(c_1R+3c_2R^3\alpha_R^2)dp+(3c_1\alpha_R+5c_2R^2\alpha_R^3)dR=0 \quad (3.13)$$

*From* $(3.7b)$ wher $\alpha_R=p$ then (3.13) becomes

$$R(c_1+3c_2R^2p^2)dp+p(3c_1+5c_2R^2p^2)dR=0$$

Which is equivalent to

$$(c_1 + 3c_2 a^2)dp.R + (3c_1 + 5c_2 a^2)dR.p = 0 \quad (3.14)$$

Equation (3.14) can be expressed as

$$8c_2 a^2 da + 4c_1 da = 0 \quad (3.15)$$

where $\delta = 8c_2$ and $\eta = 4c_1$

integrating equation (3.15), we have

$$\frac{\delta R^2}{2} - \frac{\eta p^{-2}}{2} = \mu_2 \quad (3.16)$$

Equation (3.16) simplifies to

$$p = \pm\sqrt{\frac{\mu}{(R^2 - \lambda^2)}} \quad (3.17)$$

Where $\mu$ and $\lambda^2$ are constants

Equation (3.17) implies

$$\frac{d\alpha}{dR} = \pm\frac{\psi}{\sqrt{R^2 - \lambda^2}}$$

which integrate to

$$\alpha = \pm\psi Intan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) - 0 = \pm\psi log_e(sec\theta + \tan\theta)$$

Since $R = \lambda\sec\theta$

From the boundary conditions $R(0) = 1$ and $\alpha(0) = 0$

Then $\lambda = 1$ *and* $\psi = 1$

Consequently,

$$R = \sec\theta \text{ and } \alpha = \pm log_e(sec\theta + \tan\theta) \quad (3.18)$$

Now $\frac{d\alpha}{dR} = \frac{d\alpha}{d\theta}\frac{d\theta}{dR} = \frac{d\alpha}{d\theta} * \frac{1}{\frac{dR}{d\theta}}$

$$Displacement\ gradient\ \ \frac{d\alpha}{dR} = \cot\theta \quad (3.19)$$

Azimuthal Shear Strain $\Upsilon = R\alpha_R$

Therefore $\Upsilon = R\alpha_R = \mathrm{cosec}\theta$ (3.20)

Substituting $\alpha$ as calculated in equation (3.18) the stress components are readily calculated using equation (2.10).

From equation (3.3c), we have

$$\left(c_1 R + 3c_2 R\beta_R^2\right)\beta_{RR} - \rho R\beta_{tt} = -c_1\beta_R - c_2\beta_R^3 \quad (3.21)$$

Where $a_z = \beta_{tt}$

Similarly from (3.21) we obtain the Monge two subsidiary equations as

$$(c_1 R + 3c_2 R\beta_R^2)dpdt - \rho RdqdR + (c_1\beta_R + c_2\beta_R^3)dRdt = 0 \quad (3.22)$$

$$(c_1 R + 3c_2 R\beta_R^2)(dt)^2 - \rho R(dR)^2 = 0 \quad (3.23)$$

From (3.23), we have

$$dR = \pm\left(\sqrt{\frac{c_1 + 3c_2\beta_R^2}{\rho}}\right)dt \quad (3.24)$$

Substituting (3.24) in the second term of the left hand side of (3.22) for dR in order to introduce dt.

$$(c_1 R + 3c_2 R\beta_R^2)dpdt - \rho Rdq\left(\sqrt{\frac{c_1 + 3c_2\beta_R^2}{\rho}}\right)dt$$

$$+(c_1\beta_R + c_2\beta_R^3)dRdt = 0 \quad (3.25)$$

$$(c_1 R + 3c_2 R\beta_R^2)dpdt - \rho Rdq\left(\sqrt{\frac{c_1 + 3c_2\beta_R^2}{\rho}}\right)dt$$

$$+(c_1\beta_R + c_2\beta_R^3)dRdt = 0 \quad (3.26)$$

It implies that $t = c$

Incompressible materials should not be used in a dynamic analysis because the speed of elastic pressure waves is infinite.

$t = c$ is satisfied by the incompressible condition that is $\det(\bar{F}) = 1$.

(3.25) + (3.26) reduces to

$$2\left(c_1 R + 3c_2 R\beta_R^2\right)dp + 2\left(c_1\beta_R + c_2\beta_R^3\right)dR = 0$$

$$\frac{(c_1 + 3c_2\beta_R^2)dp}{(c_1\beta_R + c_2\beta_R^3)} + \frac{dR}{R} = 0 \qquad (3.27)$$

which integrate to

$$F(R, t, w, p, q, t_1) = (c_1p + c_2p^3)R - \boldsymbol{\sigma} = 0 \qquad (3.28)$$

where $(c_1p + c_2p^3)R = \boldsymbol{\sigma}$ (3.29)

Therefore equation (3.27) is written as

$(c_1\beta_R + c_2\beta_R^3)R = \boldsymbol{\sigma}$

Applying Euler's method of solution [19] we obtain the solution of equation as

$$\beta = \frac{\boldsymbol{\varphi}}{2}[2\theta + sin2\theta] \qquad (3.30)$$

$Let \ \frac{\boldsymbol{\varphi}}{2} = A \ \ and \ \ 2\theta = \boldsymbol{\Theta}$ then (3.30) becomes

$$\beta = A(\Theta + sin\Theta) \qquad (3.31)$$

$R = \boldsymbol{\varphi} sin^2\theta$ can be written as

$$R = A(1 - cos\Theta) \qquad (3.32)$$

From the boundary condition $R\left(\frac{\pi}{2}\right) = 1$, then $A = 1$

$$R = (1 - cos\Theta) \ \ and \ \ \beta = (\Theta + sin\Theta) \qquad (3.33)$$

Now $\frac{d\beta}{dR} = \frac{d\beta}{d\Theta}\frac{d\Theta}{dR} = \frac{d\beta}{d\Theta}\frac{1}{\frac{dR}{d\Theta}} = Acos\Theta\frac{1}{Asin\Theta} = cot\ \boldsymbol{\Theta}$ (3.34)

The combined effect or the resultant displacement $\Omega$ at any point is the sum of the separate displacements due to two waves (principle of superposition).

$\Omega = \alpha + \beta$

The resultant displacement $\Omega = \pm log_e(sec\theta + \tan\theta) + (\Theta + sin\Theta)$

where $\Theta = 2\theta$

$$\Omega = \pm log_e(sec\theta + \tan\theta) + (2\theta + sin2\theta) \qquad (3.35)$$

Substituting $\beta$ and $\alpha$ as calculated in equation (3.33) the stress components are readily calculated using equation (2.10)

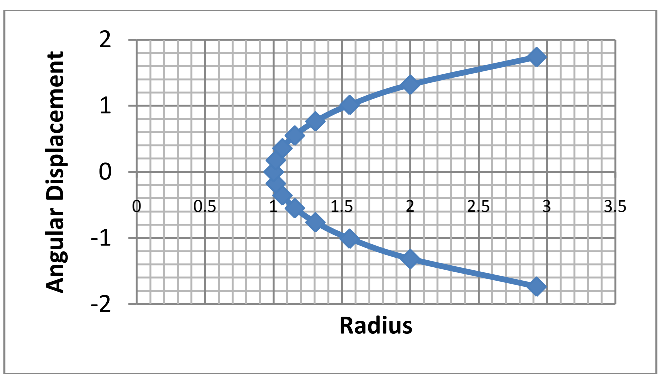


Fig 1: A graph of the equation (3.18) with Angular displacement plotted against radius.

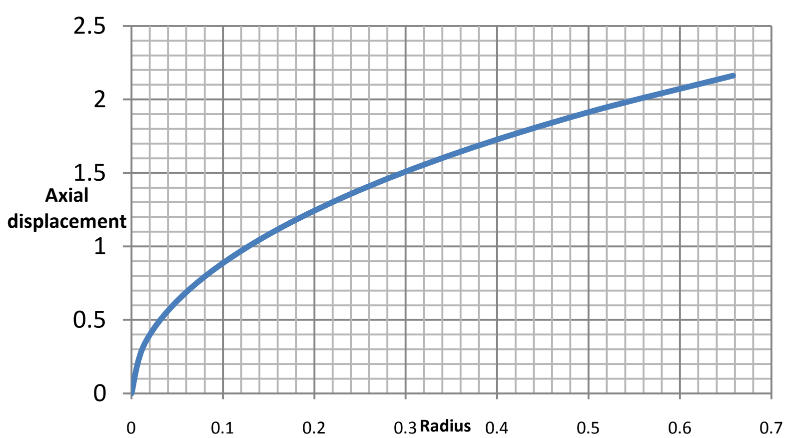


Fig 2: A graph of the equation (3.33) with Axial displacement plotted against radius.

**CONCLUSION**: We were able to obtain an analytical solutions for both angular and axial displacement given as equation (3.18) and (3.33) respectively which resulted as a result of torsional and axial force, propagating a wave in an incompressible hollow cylinder whose strain energy function is given as (2.1). Equation (3.18) has intrinsic importance in kinematics. It arises when the acceleration of a particle is proportional to its distance from a fixed point. The equation (3.33) is an equation of a Cycloid where A is the radius of a rolling circle which implies the Amplitude of the wave motion. Equation (3.33) is the solution to the problem of the brachistochrone. Thus, the brachistochrone is a cycloid. We obtained the shear strain with respect to Torsional force as the product of the radius of the cylinder and the angular displacement gradient given as (3.20) and shear strain with respect to Axial force given as (3.34). Finally, we obtained the components of the stresses at any cross section. Figure one shows that the material is isotropic since the graph is symmetric which proved the symmetric nature of the stress tensor for the isotropic material and that angular of rotation of outer boundary can be positive or negation. It also shows that Circular displacement increases with increase in radius of the cylinder. Figure two shows that the axial displacement increases with increase in radius of the cylinder. This work can be extended to the case
$p + q > 2$, however the resulting equation may be so highly nonlinear and analytic solution might be impossible.

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