Transactions of the Nigerian Association of Mathematical Physics
Volume 7, **(***March, 2018***), pp33 – 40**
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EFFECT OF DAMPING COEFFICIENTS ON EULER-BERNOULLI BEAM SUBJECTED TO PARTIALLY DISTRIBUTED LOAD

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Abstract

This work examines the effects of damping coefficients on Euler-Bernoulli beam subjected to uniform partially distributed load. The partial differential equation of order four governing the system was transformed using method of series solution to Ordinary differential equation of order two. The Finite difference method was used to solve the resulting second order ordinary differential equation. It was found that the dynamic response of the amplitude of the beam decreases as the speed of the load increases with the effect of damping coefficient and flexural rigidity.

Keywords: Euler-Bernoulli beam**,** Damping coefficients, flexural rigidity, lateral deflection, partially distributed load.

1.0 Introduction

The investigation of the dynamic response of beams on Winkler foundation subjected to partially distributed moving loads has been of great significance in railway engineering. These structures have been designed to support partially distributed loads.

In the analysis of elastically supported beams, the elastic support is provided by a load – bearing medium referred to as the 'foundation' along the length of the beam [1]. Such conditions of support can be found in large variety of geotechnical problems. There are two basic types characterized by the fact that the pressure in the foundation is proportional at every point to the deflection occurring at that point and is independent of pressure or deflection produced at other point [2-3].

Beam is a piece of horizontal structure that is usually supported at both ends. It can be in form of wood, metal or plastic, this is concern with the theory describing the response of elastic structure under the influence of partially distributed moving loads [4-6]. The most obvious example of structure subjected to partially distributed moving loads in history are railways bridges. Furthermore, there is a form of interaction between the motion of the bridge and that of the vehicle [7].

In general, there are two types of motion of elastic structure.

(i) The Thick-structure theory which account for the effect of shear deformation and rotatory inertia while,

(ii) The classical thin structures neglect the effects of shear deformation and rotary inertia.

Loads are generally forces acting on a structure. When loads act on a structure they produce stress and deformations.

The problem of carrying out a dynamic analysis of the reactions of structures under moving loads is known as moving load problem such moving load problems are of practical important [8]. The most obvious example of structures subjected to moving loads is highway and railway bridges. The pertinent analysis is, however, complicated by the fact that the mass of a moving vehicle or locomotive is usually large compare with that of the bridge itself.[9-11].

Moving load problems may be grouped into two main classes. The first class consist of problems involving concentrated forces (or point masses) moving with a specified or an unspecified velocity while the other class deals with the problem of vibration analysis of structures due to partially distributed uniformly moving forces (or masses) [12-13].

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One obvious application for the analysis of the second class is the study of vibration of a bridge under a travelling train. Also, since no point or concentrated mass exists physically, consideration of a load distribution interval enhances the reality of problem formulation involving the second class [14],

Euler- Bernoulli beams are the simplest and most common ones. They are known not to possess the effect of both shear deformation and rotatory inertia. Shear beams are beams in which only the effects of shear deformation are retained while Rayleigh beams take into consideration the effect of rotatory inertial only. For Timoshenko beams, the effects of both shear deformation and rotatory inertia are retained. Beams vibrations described by the Timoshenko model have been studied over the years by many authors [15].

The Timoshenko model is an extension of the Euler-Bernoulli model by taking into account two additional effects; shearing force effect and rotatory motion effect. In any beam except one subjected to pure bending only, a deflection due to the shear stress occurs. The exact solution to the beam vibration problem requires this deflection to be considered. Some numerical solutions to the oscillator and Bernoulli equations were also studied, simulated and discussed in [16].

In engineering, deflection is the degree to which a structural element is displaced under a load. It may refer to an angle or a distance and is inversely proportional to moment of inertia, modulus of elasticity is constant for all structural steels, so the larger the moment of inertia, the smaller the deflection. Beams are traditionally descriptions of building or civil engineering structural elements but smaller structures like trucks or automobile frames, machine frames and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion.

2.0 Governing Equation

Consider Euler-Bernoulli beam of Length L resting on a Winkler foundation and traversed by uniform partially distributed moving load. The resulting vibrational behaviour of the system is described by the following partial differential equation.

$$EI\frac{\partial^4 W(x,t)}{\partial x^4}+\rho A\frac{\partial^2 W(x,t)}{\partial t^2}+K_L W(x,t)+C\frac{\partial W(x,t)}{\partial t}=\left(\frac{1}{\varepsilon}\right)\left\{-Mg-M\frac{d^2W}{dt^2}\right\} \quad (1)$$

$$\left[H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)\right]$$

Where

E = Young's modulus, I = Moment of inertia of the cross section

ρ = Density of the mass, A = Area of the cross section of the beam

EI = Rigidity of the beam, K_L = Winkler's Foundation

M = Mass of the load, C=Damping Coefficient

Boundary Conditions

The pertinent boundary conditions for the problem under consideration can be any of the following of the classical boundary conditions.

$$W(x,t)=\frac{\partial W(x,t)}{\partial x}=0, \quad at \quad x=0 \ or \ x=L$$

$$W(x,t)=\frac{\partial^2 W(x,t)}{\partial x^2}=0, \quad at \ x=0 \ or \ x=L \quad (2)$$

$$\frac{\partial^2 W(x,t)}{\partial x^2}=\frac{\partial^3 W(x,t)}{\partial x^3}=0, \quad at \ x=0 \ or \ x=L$$

Finally, the initial conditions are

$$W(x,0)=0$$

$$\frac{\partial W(x,0)}{\partial x}=0 \quad (3)$$

Method of Solution

To obtain a solution for the fourth order partial differential equation (1), we used series solution to reduce the equation from fourth order partial differential equation to a second order ordinary differential equation.

The finite difference method is used to solve the resulting second order differential equation.

We assume a solution of the form.

$$W(x,t)=\sum_{j=1}^{\infty}X_j(x)\gamma_j(t) \quad (4)$$

$$EI\sum_{j=1}^{\infty}X^{(iv)}(x)\gamma_j(t)+\rho A\sum_{j=1}^{\infty}X(x)\ddot{\gamma}_j(t)+K\sum_{j=1}^{\infty}X(x)\gamma_j(t)+C\sum_{j=1}^{\infty}X(x)\dot{\gamma}_j(t)=P_0(x,t) \quad (5)$$

Where $P_0(x,t)=\left(\frac{1}{\varepsilon}\right)\left\{-Mg-M\frac{d^2W}{dt^2}\right\}\left[H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)\right]$ (6)

and $$\frac{d^2W}{dt^2}=\frac{\partial^2W}{\partial x^2}+2V\frac{\partial^2W}{\partial t\partial x}+V^2\frac{\partial^2W}{\partial t^2} \quad (7)$$

$$\sum_{j=1}^{\infty}\gamma_{fj}(t)X_j(x)=\left(\frac{1}{\varepsilon}\right)\left\{-Mg\left[H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)\right]\right\}$$
$$-M\left\{\sum_{j=1}^{\infty}X_j^{''}(x)\gamma_j(t)+2V\sum_{j=1}^{\varepsilon}X^{'}(x)\dot{\gamma}(t)+V^2\sum_{j=1}^{\infty}X_j(x)\ddot{\gamma}_j(t)\right\} \quad (8)$$
$$\left[H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)\right]$$

$$\sum_{j=1}^{\infty}\gamma_{fj}(t)X_j(x)=-\frac{Mg}{\varepsilon}\left[H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-H(x-\varepsilon-\frac{\varepsilon}{2}\right]$$
$$-\frac{M}{\in}\sum_{j=1}^{\infty}X_j^{''}(x)\gamma_j(t)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H\left(x-\varepsilon-\frac{\in}{2}\right)\right] \quad (9)$$
$$-\frac{2VM}{\in}\sum_{j=1}^{\infty}X_j^{'}(x)\dot{\gamma}_j(t)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H\left(x-\varepsilon-\frac{\in}{2}\right)\right]$$
$$-\frac{MV^2}{\in}\sum_{j=1}^{\infty}X_j(x)\ddot{\gamma}(t)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H\left(x-\varepsilon-\frac{\in}{2}\right)\right]$$

Multiply equation (9) by $X_i(x)$ and then integrate

$$\sum_{j=1}^{\infty}\gamma_{fj}(t)\int_0^L X_i(x)X_j(x)=-\frac{Mg}{\in}\int_0^L X_i(x)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H(x-\varepsilon-\frac{\in}{2}\right]dx$$
$$-\frac{M}{\in}\sum_{j=1}^{\infty}\int_0^L X_j^{''}(x)X_i(x)\gamma_j(t)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H\left(x-\varepsilon-\frac{\in}{2}\right)\right]dx$$
$$-\frac{2VM}{\in}\sum_{j=1}^{\infty}\int_0^L X_j^{'}(x)X_i(x)\dot{\gamma}_j(t)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H\left(x-\varepsilon-\frac{\in}{2}\right)\right]dx \quad (10)$$
$$-\frac{MV^2}{\in}\sum_{j=1}^{\infty}\ddot{\gamma}(t)\int_0^L X_j(x)X_i(x)\left[H\left(x-\varepsilon+\frac{\in}{2}\right)-H\left(x-\varepsilon-\frac{\in}{2}\right)\right]dx$$

Integrating the above equation (10) by parts, we have

$$A_{11}=\frac{Mg}{\in}\left[X_1(\varepsilon)+\frac{\in^2}{24}X_i^{''}(\varepsilon)\right] \quad (11)$$

$$B_{11}=M\sum_{i=1}^{\infty}Y_j(t)\left[X_i(\varepsilon)X_j(\varepsilon)+\frac{\in^2}{24}X_i(\varepsilon)\right] \quad (12)$$

$$C_{11}=2MV\sum_{j=1}^{\infty}\dot{\gamma}_j(t)\left[X_I(\varepsilon)'X_j(\varepsilon)+\frac{\in^2}{24}X_i(\varepsilon)X^{''}{}_J(\varepsilon)\right]' \quad (13)$$

$$D_{11}=MV^2\sum_{j=1}^{\infty}\gamma_j(t)\left[X^{''}(\varepsilon)X_j(\varepsilon)+\frac{\in^2}{24}\left\{X_i^{(iv)}(\varepsilon)X_j(\varepsilon)+2X_i^{'''}(\varepsilon)X_j{}'+X_i^{''}(\varepsilon)X_j^{''}(\varepsilon)\right\}\right] \quad (14)$$

By using orthorgonality condition

$$E=\gamma_{fj}(t) \quad (15)$$

Now substituting equation A, B, C, D and E into equation (9)

$$\gamma_{fj}(t)=Mg\left[X_i(\varepsilon)+\frac{\in^2}{2}X_t^{''}(\varepsilon)\right]-M\sum_{j=1}^{\infty}\gamma_j(t)\begin{bmatrix}X_j(\varepsilon)X_i(\varepsilon)+\frac{\in^2}{24}X_j^{''}(\varepsilon)X_i(\varepsilon)\\+2X_j^{'}(\varepsilon)X_i(\varepsilon)+X_j(\varepsilon)X_i^{''}(\varepsilon)\end{bmatrix}$$
$$-2MV\sum_{i=1}^{\infty}\dot{\gamma}_i(t)\left[X_j(\varepsilon)X_i^{'}(\varepsilon)+\frac{\in^2}{24}\begin{Bmatrix}X_j^{'''}(\varepsilon)X_i(\varepsilon)+2X_j^{''}(\varepsilon)X_i^{'}(\varepsilon)\\+X_j^{'}(\varepsilon)X_i^{''}(\varepsilon)\end{Bmatrix}\right] \quad (16)$$
$$-MV^2\sum_{j=1}^{\infty}\gamma_j(t)\left[X_j^{''}(\varepsilon)X_i(\varepsilon)+\frac{\in^2}{24}\begin{Bmatrix}X_j^{(iv)}(\varepsilon)X_i(\varepsilon)+2X_j^{'''}(\varepsilon)X_i^{'}(\varepsilon)\\+X_j^{''}(\varepsilon)X_i^{''}(\varepsilon)\end{Bmatrix}\right]$$

Next substituting equation (16) into (1)

$$EI\sum_{j=1}^{\infty} X_j^{(iv)}(x)\gamma_j(t) + \rho A\sum_{j=1}^{\infty} X_j(x)\ddot{\gamma}_j(t) + K_L\sum_{j=1}^{\infty} X_j(x)\gamma_j(t)$$
$$+ C\sum_{j=1}^{\infty} X_j(x)\dot{\gamma}(t) =$$
$$X_j(\varepsilon)\left[\begin{array}{l} Mg\left\{X_i(\varepsilon) + \frac{\in^2}{24}X_i^{''}(\varepsilon)\right\} \\ -M\sum_{j=1}^{\infty}\dot{\gamma}_j(t)\left[\begin{array}{l} X_j(\varepsilon)X_i(\varepsilon) + \frac{\in^2}{24}X_j^{''}(\varepsilon)X_i(\varepsilon) + 2X_J^{'}(\varepsilon)X_i(\varepsilon) \\ X_j(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right] \\ -2MV\sum_{J=1}^{\infty}\begin{array}{l}\dot{\gamma}_j(t)\left[X_j(\varepsilon)X_i^{''}(\varepsilon) + \frac{\in^2}{24}\left\{\begin{array}{l} X_J^{'''}(\varepsilon)X_I(\varepsilon) + 2X_j^{''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j^{''}(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}\right] \\ -MV^2\sum_{J=1}^{\infty}\gamma_J(t)\left[X_j^{''}(\varepsilon)X_i(\varepsilon)\right]\end{array} \\ + \frac{\in^2}{24}\left\{\begin{array}{l} X_j^{(iv)}(\varepsilon)X_i(\varepsilon) + 2X_i^{'''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j^{''}(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}\end{array}\right] \quad (17)$$

For free vibration of Euler-Bernoulli beam

$$X_j^{(iv)}(x) - \alpha_j^4 X_j(x) = 0 \quad (18)$$

Where

$$\alpha_j^4 = \frac{M\rho_j^2}{EI} \quad (19)$$

and ρ_j^2 is the square of the j[th] national frequency of the beam

$$M\sum_{j=1}^{\infty}\gamma_j(t)\rho_j^2 X_j(x) + \rho A\sum_{j=1}^{\infty} X_j(x)\ddot{\gamma}(t) + K_L\sum_{j=1}^{\infty} X_j(x)\gamma_j(t) + C\sum_{j=1}^{\infty} X_j(x)\dot{\gamma}_j(t)$$
$$= X_j(x)\Big[Mg\left\{X_i(\varepsilon) + \frac{\in^2}{24}X_i^{''}(\varepsilon)\right\}$$
$$-M\sum_{j=1}^{\infty}\dot{\gamma}_j(t)\Big[X_j(\varepsilon)X_i(\varepsilon)$$
$$+\frac{\in^2}{24}\left\{\begin{array}{l} X_j^{''}(\varepsilon)X_i(\varepsilon) + 2X_i^{''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}$$
$$-2MV\sum_{j=1}^{\infty}\dot{\gamma}(t)\left[X_j(\varepsilon)X_i^{'}(\varepsilon)\right.$$
$$-MV^2\sum_{J=1}^{\infty}\gamma_J(t)\left[X_j^{''}(\varepsilon)X_i(\varepsilon)\right]$$
$$+\frac{\in^2}{24}\left\{\begin{array}{l} X_j^{(iv)}(\varepsilon)X_i(\varepsilon) + 2X_i^{'''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j^{''}(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}\Big] \quad (20)$$

$$\sum_{j=1}^{\infty}\left\{M\rho_j^2\gamma_j(t) + \rho A\ddot{\gamma}_j(t) + K_L\gamma_j(t) + C\dot{\gamma}_j(t) - \left[-Mg\left\{X_j(\varepsilon) + \frac{\in^2}{24}X_j^{''}(\varepsilon)\right\}\right\}\right.$$
$$-M\sum_{j=1}^{\infty}\dot{\gamma}_j(t)\left[X_j(\varepsilon)X_i(\varepsilon) + \frac{\in^2}{24}\left\{\begin{array}{l} X_j^{''}(\varepsilon)X_i(\varepsilon) + 2X_i^{''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j^{''}(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}\right. \quad (21)$$
$$-2MV\sum_{j=1}^{\infty}\dot{\gamma}(t)\left[X_j(\varepsilon)X_i^{'}(\varepsilon) + \frac{\in^2}{24}\left\{\begin{array}{l} X_j^{'''}(\varepsilon)X_i(\varepsilon) + 2X_i^{''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j^{'}(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}\right.$$
$$-MV^2\sum_{J=1}^{\infty}\gamma_J(t)\left[X_j^{''}(\varepsilon)X_i(\varepsilon)\right] + \frac{\in^2}{24}\left\{\begin{array}{l} X_j^{(iv)}(\varepsilon)X_i(\varepsilon) + 2X_i^{'''}(\varepsilon)X_i^{'}(\varepsilon) \\ + X_j^{''}(\varepsilon)X_i^{''}(\varepsilon)\end{array}\right\}\Bigg]\Bigg] = 0$$

For arbitrary $X_j(x)$, equation (21) becomes:

$$\ddot{\gamma}_j(t) + \frac{c}{\rho A}\dot{\gamma}_j(t) + \frac{(M\rho_j^2 + K_L)}{\rho A}\gamma_j(t)$$
$$= \frac{1}{\rho A}\left[-Mg\left\{X_j(\varepsilon) + \frac{\in^2}{24}X_J^{'''}(\varepsilon)\right\}\right.$$

$$-M\sum_{j=1}^{\infty}\dot{\gamma}_j(t)\Big[X_j(\varepsilon)X_i(\varepsilon)+\frac{\in^2}{24}\left\{\begin{matrix}X_j''(\varepsilon)X_i(\varepsilon)+2X_i''(\varepsilon)X_i'(\varepsilon)\\+X_j(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}$$

$$-2MV\sum_{j=1}^{\infty}\dot{\gamma}(t)\Big[X_j(\varepsilon)X_i'(\varepsilon)+\frac{\in^2}{24}\left\{\begin{matrix}X_j'''(\varepsilon)X_i(\varepsilon)+2X_i''(\varepsilon)X_i'(\varepsilon)\\+X_j'(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}$$

$$-MV^2\sum_{J=1}^{\infty}\gamma_J(t)\Big[X_j''(\varepsilon)X_i(\varepsilon)\Big]+\frac{\in^2}{24}\left\{\begin{matrix}X_j^{(iv)}(\varepsilon)X_i(\varepsilon)+2X_i'''(\varepsilon)X_i'(\varepsilon)\\+X_j''(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\Bigg]\qquad(22)$$

$$-MV^2\sum_{J=1}^{\infty}\gamma_J(t)\Big[X_j''(\varepsilon)X_i(\varepsilon)\Big]+\frac{\in^2}{24}\left\{\begin{matrix}X_j^{(iv)}(\varepsilon)X_i(\varepsilon)+2X_i'''(\varepsilon)X_i'(\varepsilon)\\+X_j''(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\Bigg]$$

$$\ddot{\gamma}_j(t)+\frac{c}{\rho A}\dot{\gamma}_j(t)+\frac{(M\rho_j^2+K_L)}{\rho A}\gamma_j(t)$$

$$=-\frac{Mg}{\rho A}\left[\left\{X_j(\varepsilon)+\frac{\in^2}{24}X_J'''(\varepsilon)\right\}\right.$$

$$-\frac{M}{\rho A}\sum_{j=1}^{\infty}\dot{\gamma}_j(t)\Big[X_j(\varepsilon)X_i(\varepsilon)+\frac{\in^2}{24}\left\{\begin{matrix}X_j''(\varepsilon)X_i(\varepsilon)+2X_i''(\varepsilon)X_i'(\varepsilon)\\+X_j(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}$$

$$-\frac{2MV}{\rho A}\sum_{j=1}^{\infty}\dot{\gamma}(t)\Big[X_j(\varepsilon)X_i'(\varepsilon)+\frac{\in^2}{24}\left\{\begin{matrix}X_j'''(\varepsilon)X_i(\varepsilon)+2X_i''(\varepsilon)X_i'(\varepsilon)\\+X_j'(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\qquad(23)$$

$$-\frac{MV^2}{\rho A}\sum_{J=1}^{\infty}\gamma_J(t)\Big[X_j''(\varepsilon)X_i(\varepsilon)\Big]+\frac{\in^2}{24}\left\{\begin{matrix}X_j^{(iv)}(\varepsilon)X_i(\varepsilon)+2X_i'''(\varepsilon)X_i'(\varepsilon)\\+X_j''(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\Bigg]$$

Putting

$$A_1=-\frac{Mg}{\rho A}\left[\left\{X_j(\varepsilon)+\frac{\in^2}{24}X_J'''(\varepsilon)\right\}\right.\qquad(24)$$

$$B=\sum_{J=1}^{\infty}\Big[X_j(\varepsilon)X_i(\varepsilon)+\frac{\in^2}{24}\left\{\begin{matrix}X_j''(\varepsilon)X_i(\varepsilon)+2X_i''(\varepsilon)X_i'(\varepsilon)\\+X_j(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\qquad(25)$$

$$C=\sum_{j=1}^{\infty}\Big[X_j(\varepsilon)X_i'(\varepsilon)+\frac{\in^2}{24}\left\{\begin{matrix}X_j'''(\varepsilon)X_i(\varepsilon)+2X_i''(\varepsilon)X_i'(\varepsilon)\\+X_j'(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\qquad(26)$$

$$D=\sum_{J=1}^{\infty}\Big[X_j''(\varepsilon)X_i(\varepsilon)\Big]+\frac{\in^2}{24}\left\{\begin{matrix}X_j^{(iv)}(\varepsilon)X_i(\varepsilon)+2X_i'''(\varepsilon)X_i'(\varepsilon)\\+X_j''(\varepsilon)X_i''(\varepsilon)\end{matrix}\right\}\Bigg]\qquad(27)$$

Substituting A1, B, C, D into (23)

$$\ddot{\gamma}_j(t)+\frac{c}{\rho A}\dot{\gamma}(t)+\frac{(M\rho_j^2+K_L)}{\rho A}\gamma_j(t)=-\frac{Mg}{\rho A}A_1-\frac{M}{\rho A}\ddot{\gamma}(t)B-\frac{2MV}{\rho A}\dot{\gamma}_j(t)C-\frac{MV^2}{\rho A}\gamma_J(t)D\qquad(28)$$

$$\ddot{\gamma}(t)+\left(\frac{C+MB+2MVC}{\rho A}\right)\ddot{\gamma}_j(t)+\left(\frac{M\rho_i^2+K_L+MV^2D}{\rho A}\right)\gamma_j(t)=\frac{MgA_1}{\rho A}\qquad(29)$$

$$P=\frac{C+MB+2MVC}{\rho A}\qquad(30)$$

$$Q=\frac{M\rho_i^2+K_L+MV^2D}{\rho A}\qquad(31)$$

NUMERICAL SOLUTION

To solve the reduced second order differential equation (28), we apply finite difference method.
Equation (28) now becomes

$$\ddot{\gamma}_j(t) + p\dot{\gamma}_j(t) + Q\gamma_j(t) = \frac{MgA_1}{\rho A} \tag{32}$$

Using finite difference method

$$\dot{\gamma}_j(t) = \frac{\gamma_{j+1} - \gamma_{j-1}}{2h} \tag{33}$$

$$\ddot{\gamma}_j(t) = \frac{\gamma_{j+1} - 2\gamma_j + \gamma_{j-1}}{h^2} \tag{34}$$

Substituting (29), (30-31) into (32)

$$\frac{\gamma_{j+1} - 2\gamma_j + \gamma_{j-1}}{h^2} + p\frac{\gamma_{j+1} - \gamma_{j-1}}{2h} + Q\gamma_j = \frac{MgA_1}{\rho A} \tag{35}$$

Multiply through byh^2, we have

$$\gamma_{j+1} - 2\gamma_j + \gamma_{j-1} + \frac{ph}{2}(\gamma_{j+1} - \gamma_{j-1}) + Qh^2\gamma_j = -\frac{MgA_1h^2}{\rho A}$$

$$\gamma_{j+1} - 2\gamma_j + \gamma_{j-1} + \gamma_{j+1} - \frac{ph}{2}\gamma_{j-1} + Qh^2\gamma_j = -\frac{MgA_1h^2}{\rho A}$$

$$\left(1 + \frac{ph}{2}\right)\gamma_{j+1} + (Qh^2 - 2)\gamma_j + \left(1 - \frac{ph}{2}\right)\gamma_{j-1} = -\frac{MgA_1h^2}{\rho A} \tag{36}$$

Multiply equation (36) by 2 to obtain:

$$(2 + ph)\gamma_{j+1} + (2 - ph)\gamma_{j-1} +$$

$$(2Qh^2 - 4)\gamma_j = -\frac{2MgA_1h^2}{\rho A}$$

$$\gamma_j = \left[\frac{1}{2Qh^2 - 4}\right]\left[\begin{array}{l}\frac{-2MgA_1h^2}{\rho A} + \\ (ph-2)\gamma_{j+1} + (ph-2)\gamma_{j-1}\end{array}\right] \tag{37}$$

Substituting equation (37) into equation (1), we obtained:

$$W(x,t) = \sum_{j=1}^{\infty} X_j(x)\left[\frac{1}{2Qh^2 - 4}\right]\left[\begin{array}{l}\frac{-2MgA_1h^2}{\rho A} \\ +(ph-2)\gamma_{j+1} + (ph-2)\gamma_{j-1}\end{array}\right] \tag{38}$$

The above system of equation (38) which was obtained from the finite difference method was then simulated by MATLAB.

NUMERICAL RESULTS AND DISCUSSION

The above finite difference system of equation (38) is solved numerically. The package was used for the following set of data: M = 70kg, m = 7.04kg, h = 1.05m, L = 10m, V = 3.3m/s, K_D = 0, 0.2 MN/m^{3}and0.4 NM/m^3,E = 2x10^{-11}, g = 9.8m/s, I = 1.04 x 10^6, π = 3.142.

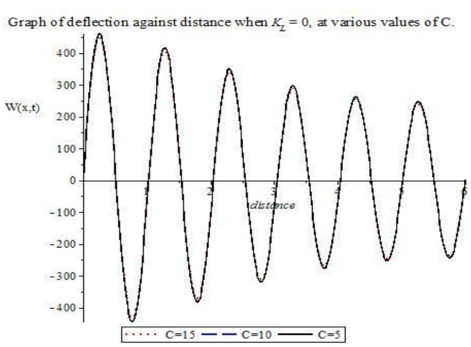

Fig. 1: Deflection of beam at various values of C when K_L = 0.

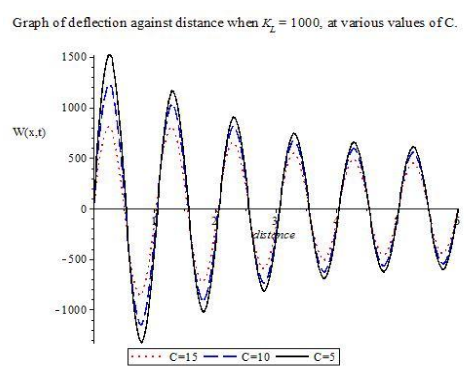

Fig. 2: Deflection of beam at various values of C when K_L=1000

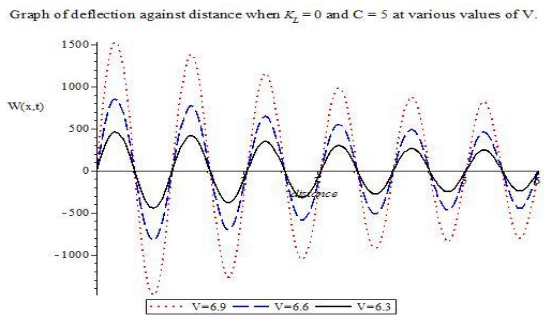

Fig. 3: Effect of Speed of the Moving load on the Beam when K_L = 0 and C = 5.

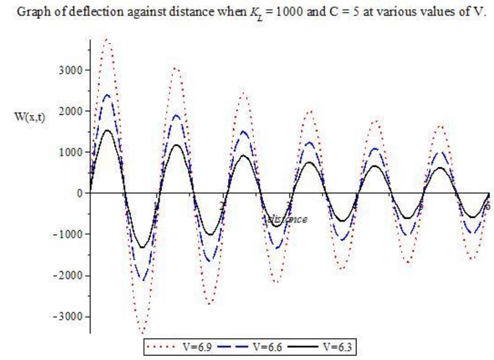

Fig. 4: Effect of Speed of the Moving load on the Beam when K_L = 1000 and C = 5.

CONCLUSION

From the analysis and figures 1-4, it was concluded that for an un-damped Euler-Bernoulli with moving load, the deflection of the beam kept on increasing, while for Euler-Bernoulli beam with damping coefficient, the deflection of the beam decreases as the speed of the moving load decreases with the various values of damping coefficient. Finally, the result in this work agrees with what was obtained in the Literature. Hence the method employed in this work is efficient and can be applied to more strongly nonlinear models that arise in physical sciences and engineering.

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