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DYNAMICAL BEHAVIOR OF THE CLASSICALLY DAMPED DRIVEN MORSE OSCILLATOR, WITH A VARYING FORCING AMPLITUDE AND A CONSTANT DRIVING FREQUENCY.

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Abstract

The dynamical behavior of the damped driven Morse oscillator with a fixed forcing frequency is presented; the values of the damping, the natural and the forcing frequencies are kept at constant, while the forcing amplitude is varied. The trajectories, the phase portraits the Poincare sections of some specially selected points suggested from the bifurcation structure and the top Lyapunov exponents were presented. The results show that as the forcing amplitude is varied in some particular range the Morse oscillator under consideration show a very rich dynamical behavior, chaotic and periodic orbits of different periods are found in co-existence, implying that a proper selection of the forcing amplitude can be used in suppressing chaotic behavior and enhancing chaos where it is required.

1.0 Introduction

The Morse oscillator is frequently used in the description of the motion of diatomic molecules in an external electromagnetic field. A lot of investigation on the Morse oscillator has been done with the classical, semi classical and quantum mechanical methods for the description of the diatomic molecule [1-11]. Some areas where Morse oscillator is used include the modeling of multi photon excitation of diatomic molecule in a dense medium or in a gaseous cell under a high pressure and the modeling of the pumping of a local mode of a poly atomic molecule by an infra red laser where the energy flow out of the molecule decay with a constant rate. Not much has been done on the Morse oscillator in the area of nonlinear dynamics. The dynamics of the damped driven Morse oscillator has not received much attention in comparison with the Duffing and the van-der Pol oscillators with has been looked into by many. The bifurcation structure of the classically damped driven oscillator has been looked into where the driving frequency was considered as the bifurcation parameter [12, 13]. Where the authors considered the driving frequency in the range of 0.0 to 3.0 and presented their results, a very rich dynamical behavior was presented; in particular they showed from a bifurcation diagram that a chaotic orbit is found for a driving frequency value of 0.5. In this paper the driving frequency value 0.5 was closely looked into by keeping it fixed, and varied the other parameters, in particular the forcing amplitude is considered the varying parameter and presented the results.

2.0 System description

The motion of the classically damped, driven Morse oscillator can be described by the equation of motion

$$\ddot{x}+\alpha\dot{x}+\tfrac{dV}{dx}= f_0 cos(\omega t) \qquad (1)$$

Where α, v, f_0 and ω are the damping, the Morse potential function, the forcing amplitude and the driving frequency respectively for Morse oscillator given by (1).

The Morse potential is given by

$$V(x)=(1-e^{-\beta x})^2 \qquad (2)$$

This can be represented as in figure 1, where β sets the curvature of the potential function.

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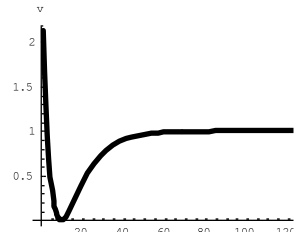
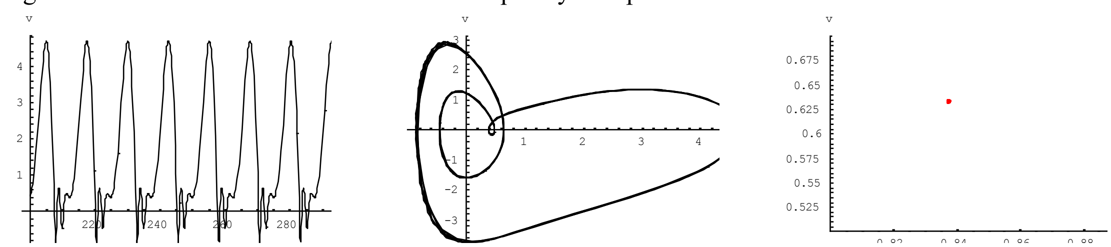


Figure 1. The Morse potential function versus position

Showing a potential function of a single minimum, which can be overcomed with a proper choice of energy from the initial velocity and position.

3.0 Results and discussion

The dynamics of the Morse oscillator is governed by equation (1) which is a nonlinear differential equation whose analytic solution can be difficult to obtain in a closed form. As a result the fourth order Runge-Kutta method is used to obtain the numerical solutions. The results obtained include the trajectories, the phase portraits the Poincare sections at some specially chosen points. The bifurcation diagram and the top Lyapunov exponents for the forcing parameter range where the various dynamical behavior is captured is presented. The driving frequency is held fixed at the value $\omega = 0.5$, the value of the damping constant held at $\alpha = 0.8$ and the natural frequency is kept at the value of 8.0.



(a) (b) (c)

Figure 2(a) Show the trajectory (b) Show the phase portrait (c) Show the Poincare section for the forcing amplitude of $f_0 = 2.85$

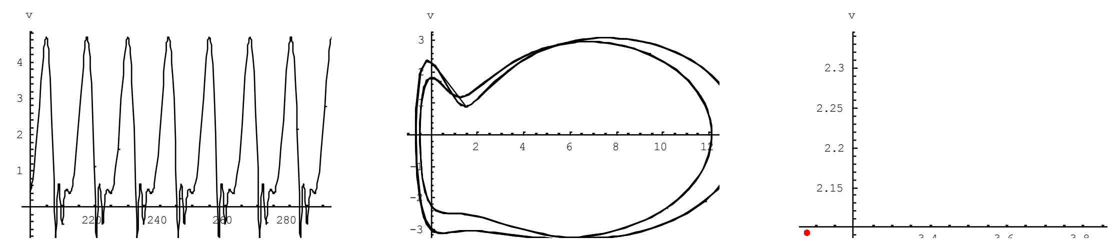


Figure 3(a) Show the trajectory (b) Show the phase portrait (c) Show the Poincare section for the forcing amplitude of $f_0 = 3.1$

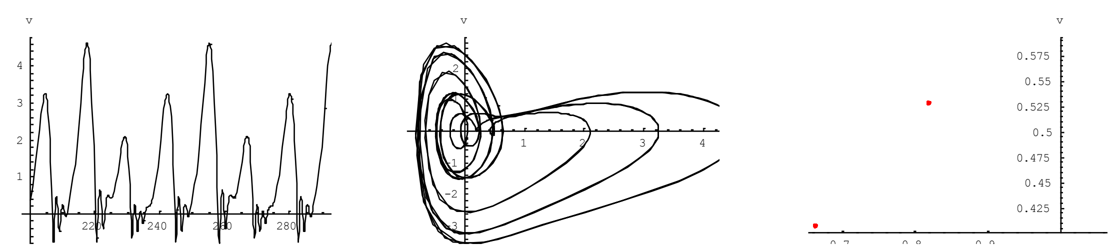


Figure 4(a) Show the trajectory (b) Show the phase portrait (c) Show the Poincare section for the forcing amplitude of $f_0 = 2.57309$

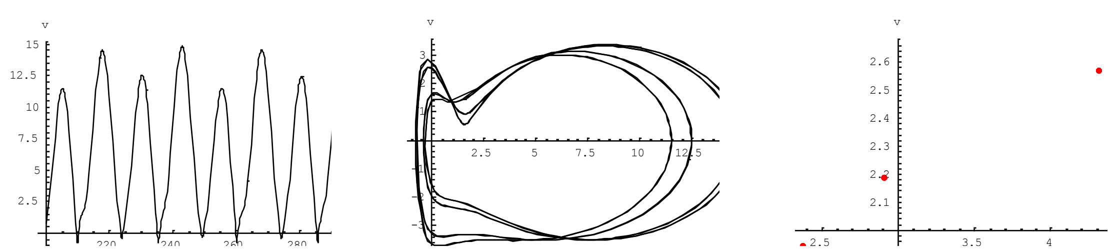


Figure 5(a) Show the trajectory (b) Show the phase portrait (c) Show the Poincare section for the forcing amplitude of $f_0 = 3.3051$

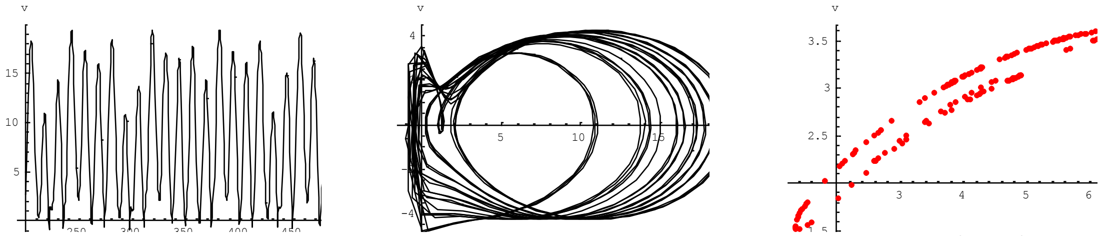


Figure 6(a) Show the trajectory (b) Show the phase portrait (c) Show the Poincare section for the forcing amplitude of $f_0 = 4.0$

It is clear from figures 2-6 the dynamical behavior show many types of periodic and chaotic orbits as the forcing amplitude is varied. This behavior is suggested by the bifurcation structure shown in figure 7, where the forcing amplitude versus the velocity was plotted, for the bifurcation parameter f_0 ranging from 0.0 to 7.0., where periodic orbit are seen up to $f_0 \leq 2.5$, a chaotic orbit found for $f_0 \geq 3.8$, and alternating periodic orbits of different periods and chaotic orbits are found in between.

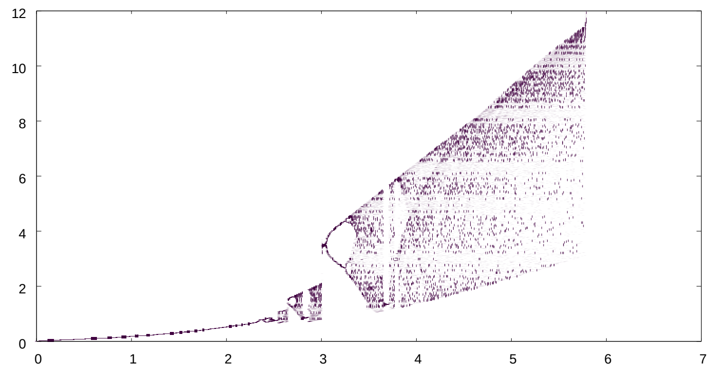


Figure7: Bifurcation diagram for the bifurcation parameter f_0 value in the range [0.0 - 7.0] versus the velocity.

Which show agreement with the results obtained from the trajectories, the phase portraits and the Poincare sections.

To further support the obtained results the top Lyaponov exponent is computed. The top Lyapunov exponent is among one of the important chaotic indicators, which is actually a measure of the average rate of expansion (contraction) of nearby trajectories. A positive measure indicating chaos, zero indicating quasi-periodicity while a negative exponent indicating a stable periodic orbit [13]. Computation of the top Lyapunov exponent for this system is done by converting equation (1) to three first order differential equations given by

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -\alpha v - \beta e^{-x}(1-e^{-x}) + f_0 \cos(\omega z) \\ \dot{z} &= t \end{aligned} \tag{3}$$

Leading to the variational equations

$$\begin{pmatrix} \delta\dot{u}_1(t) \\ \delta\dot{u}_2(t) \\ \delta\dot{u}_3(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \beta e^{-u_1}(2e^{-u_1}-1) & -\alpha & -f_0\omega\sin(\omega u_3) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta u_1(t) \\ \delta u_2(t) \\ \delta u_3(t) \end{pmatrix} \tag{4}$$

Where $u_1 = x$, $u_2 = v$ and $u_3 = z$

Represented in a compact form by

$$\delta\dot{U} = J(U(t))\delta U \tag{5}$$

Where J is the Jacobian matrix. From (5) the top Lyapunov exponent is defined as

$$\lambda = \lim_{t\to\infty} \frac{\|\delta U(t)\|}{\|\delta U(t)\|} \tag{6}$$

Results computed are given in figure 8.

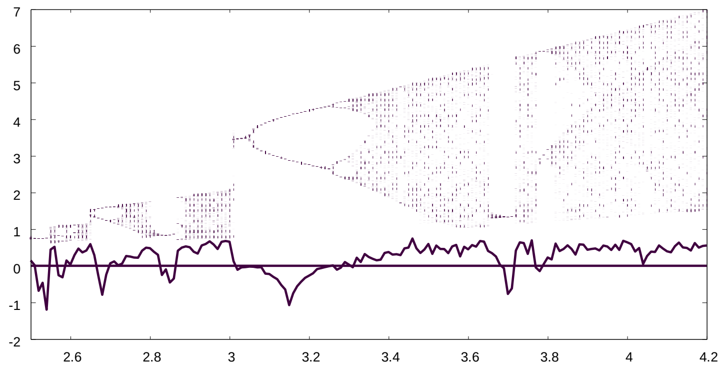


Figure8: Bifurcation diagram for the bifurcation parameter versus velocity ".", for f_0 value in the range [2.5 - 4.2] , the top Lyapunov exponent λ versus f_0"-" for the same f_0 range and the "zero line" on the same frame to show the level of agreement. Which also show agreement with the results obtained from the trajectories, the phase portraits and the Poincare sections.

4.0 Concluding Remarks:

This work considered the dynamical behavior of the classically damped driven Morse oscillator for a fixed value of the forcing frequency. The bifurcation diagram and the top Lyapunov exponents was used to carefully select some special values for forcing amplitudes for which the trajectories the phase portraits and the Poincare sections were presented, the results show (1) As the forcing amplitude is varied a co-existence of chaotic and periodic orbits of different periods are found in an alternating order, similar to the results for driving frequency considered the bifurcation parameter found in literature. (2) The range of the forcing function for which a rich dynamical behavior is found is obtained from the bifurcation diagram to be [2.5 – 4.2] agreeing in structure with driving frequency considered as bifurcation parameter found in literature. (3) The range of the forcing amplitude below 2.5 and above 4.1 where the bifurcation structure needed to be treated with caution can be seen from the potential function for the Morse oscillator (1) given in fig. 1 where it is seen that for low values of r the potential becomes infinitely high and for high values of r the potential reaches a saturation level were not much is expected to occur (the dissociation limit). Leaving only the range presented, where a proper choice of the initial conditions, position and velocity enough to overcome the dissociation limit produces the rich dynamical behavior found.

5.0 Acknowledgment

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