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DYNAMICALLY BROKEN SUPERSYMMETRY IN N = 1 SUPERGRAVITY AND HIGGS CONDENSATE

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Abstract

Gauge Theory is coupled to $N = 1$ supergravity. We showed how the D and F, terms are broken, and give a critical analysis mass splitting (Higgs effect), and the mass matrix for scalars and fermions of the O'Raifertaigh model. There is a strong evidence of a non-vanishing vacuum expectation of this model after supersymmetry is broken and there exists a massless Goldstone fermion oriented along the direction of supersymmetry breaking. The fermion mass matrix has one zero eigen value with eigen vector.

1.0 Introduction

In strongly interacting globally supersymmetry [1], there arise a condensate formation of gauge theories [2, 3]. This process leads to a broken chiral symmetry, but there are strong arguments that global supersymmetry remains unbroken [2-5]. The reason for this arguments are linked to the stability of global supersymmetry [4, 5]. The situation changes completely if these theories are coupled to gravity [6] where supersymmetry is realized locally. It is instructive to note that the coupling to supergravity give negative contributions [7,8] to the potential and the interpretation of the vacuum energy as an order parameter is lost in the process. It may be noted that gravity do not change qualitatively the dynamics of a strongly interacting gauge theory when the scale of these interaction is small compared to the planck mass $M = 10^{19} GeV$.Thus, condensation effect is expected.

Supersymmetry has to be broken in a realistic model, while preserving $E_{vacuum} = 0$ [7, 8]. It is thus highly suggestive in the proposed models [9] such as O'Raifertaigh and super-GUTs.

In this paper, we make an attempt in this direction, coupling a gauge theory to $N = 1$ supergravity. We showed how the D and Fterms are broken. While analyzing the mass matrix for scalars and fermions for the O'Raifertaigh model. We find a strong evidence that a non-vanishing vacuum expectation value of D, the auxillary field of V breaks supersymmetry. We find the spectrum of this model after supersymmetery is broken and discuss the mass splitting of the multiplet. As a result, the super-Higgs effect [10] occurs and the gravitino receives a mass by "eating" the Goldstino.

2.0 O'Raifertaigh Model

The O'Raifertaigh model involves a triplet of chiral superfieldsΦ_1, Φ_2, Φ_3 for which K … ahler and superpotential are given by

$$K = \Phi_i^{+}\phi_i, W = g\phi_1\left(\Phi_3^2 - m^2\right) + M\phi_2\phi_3, M \geq m \quad (2.1)$$

Now,

$$-F_1^* = \frac{\partial W}{\partial \varphi_1} = g(\varphi_3^2 - m^2) \quad (2.2)$$

$$-F_2^* = \frac{\partial W}{\partial \varphi_2} = g\varphi_3 \quad (2.3)$$

$$-F_3^* = \frac{\partial W}{\partial \varphi_3} = 2g\varphi_1\varphi_3 + M\varphi_2 \quad (2.4)$$

Remark: Equation (2.2) – (2.4) are the F equations of motion. If $F_i^* = 0$ for $i = 1, 2, 3$ simultaneously the form of W indeed breaks supersymmetry.

2.1 F-term Breaking

Theorem: The mass matrix for scalars and fermions for the O'Raifertaigh model is given by

$$ST_r\{M^2\} = \sum_j (-1)^{2j+1}(2j+1)\, m_j^2 = 0 \quad (2.5)$$

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where j represents the 'spin' of the particles.

Proof:

From [1], $N = 1$ supersymmetryLangrangian has theF-terms

$$F_i = -\frac{\partial W^*}{\partial \varphi_i^*}, F_i^* = -\frac{\partial W}{\partial \varphi_i} \tag{2.6}$$

and the scalar potential

$$V = \sum_j |F_j|^2 \tag{2.7}$$

$$\varphi_i = \varphi_{i,1} + i\varphi_{i,2} \tag{2.8}$$

$$M_{\alpha\beta}^2 = \frac{1}{2}\begin{pmatrix} \frac{\partial^2 V}{\partial\varphi_{1,1}\partial\varphi_{1,1}} & \frac{\partial^2 V}{\partial\varphi_{1,1}\partial\varphi_{1,2}} & \cdots & \frac{\partial^2 V}{\partial\varphi_{1,1}\partial\varphi_{n,2}} \\ \frac{\partial^2 V}{\partial\varphi_{1,2}\partial\varphi_{1,1}} & \frac{\partial^2 V}{\partial\varphi_{1,2}\partial\varphi_{1,2}} & \cdots & \frac{\partial^2 V}{\partial\varphi_{1,2}\partial\varphi_{n,2}} \\ \frac{\partial^2 V}{\partial\varphi_{1,n}\partial\varphi_{1,1}} & \frac{\partial^2 V}{\partial\varphi_{1,n}\partial\varphi_{1,2}} & \cdots & \frac{\partial^2 V}{\partial\varphi_{1,n}\partial\varphi_{n,2}} \end{pmatrix} \tag{2.9}$$

$$T_r\{M_{\alpha\beta}^2\} = \frac{1}{2}\sum_{j=1,2}\sum_i \frac{\partial^2 V}{\partial\varphi_{i,j}^2} \tag{2.10}$$

Taking into accounting [(2.8)]

and

$$\varphi_{i.1}^* = \varphi_{i,1} - i\varphi_{i,2} \tag{2.11}$$

$$\frac{\partial V}{\partial\varphi_{i,1}} = \frac{\partial V}{\partial\varphi_i} + \frac{\partial V}{\partial\varphi_i^*} \tag{2.12}$$

$$\frac{\partial V}{\partial\varphi_{i,2}} = i\frac{\partial V}{\partial\varphi_i} - i\frac{\partial V}{\partial\varphi_i^*} \tag{2.13}$$

$$\frac{\partial^2 V}{\partial\varphi_{i,1}^2} = \frac{\partial^2 V}{\partial\varphi_i^2} + \frac{\partial^2 V}{\partial(\varphi_i^*)^2} + 2\frac{\partial^2 V}{\partial\varphi_i\partial\varphi_i^*} \tag{2.14a}$$

$$\frac{\partial^2 V}{\partial\varphi_{i,2}^2} = -\frac{\partial^2 V}{\partial\varphi_i^2} - \frac{\partial^2 V}{\partial(\varphi_i^*)^2} + 2\frac{\partial^2 V}{\partial\varphi_i\partial\varphi_i^*} \tag{2.14b}$$

Hence,

$$T_\tau\{M_{\alpha\beta}^2\} = \frac{1}{2}\sum_{j=1,2}\sum_i \frac{\partial^2 V}{\partial\varphi_{i,j}^2}$$

$$= 2\sum_i \frac{\partial^2 V}{\partial\varphi_i\partial\varphi_i^*}$$

$$= 2\sum_i \frac{\partial^2 V}{\partial\varphi_i\partial\varphi_i^*}\left(\sum_j |F_j|^2\right)$$

$$= 2\sum_i \frac{\partial^2 V}{\partial\varphi_i\partial\varphi_i^*}\frac{\partial W^*}{\partial\varphi_j^*}\frac{\partial W}{\partial\varphi_j} \tag{2.15}$$

Now,

$$M_{ij} = \frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}\,(\text{Fermion mass matrix}) \tag{2.16}$$

$$M' = uMu^+ = \begin{pmatrix} m_1 e^{-i\varphi_1} & & & \\ & m_2 e^{-i\varphi_2} & & \\ & & \ddots & \\ & & & m_n e^{-i\varphi_n} \end{pmatrix} \tag{2.17}$$

$$M'(M'0^+) = \begin{pmatrix} m_1^2 & & & \\ & m_2^2 & & \\ & & \ddots & \\ & & & m_n^2 \end{pmatrix} \tag{2.18}$$

$$(M'(M')^+)_{i\tau} = (U_{ij}M_{jk}U_{kl}^+)(U_{lp}M_{pq}^+U_{q\tau}^+)$$

$$= (U_{ij}M_{jk}U_{kl}^+)(U_{lp}M_{pq}^*U_{q\tau}^+)$$

$$= U_{lj}M_{jk}M_{kl}^*U_{l\tau}^* \tag{2.19}$$

$T_\tau\{M'(M')^+\} = U_{lj}\, M_{jk}\, M^*_{kl}\, U^*_{l\tau}$
$= M_{jk}\, M^*_{kl}$
$$\sum_{j,k}\left(\frac{\partial^2 W}{\partial\varphi_j\,\partial\varphi_k}\right)\left(\frac{\partial^2 W^*}{\partial\varphi_j^*\partial\varphi_k^*}\right) \qquad (2.20)$$
Therefore,
$$\sum_j (-1)^{2.\frac{1}{2}+1}\left(2.\frac{1}{2}+1\right)K^2_{fermion} \;= 2\sum_{j,k}\left(\frac{\partial^2 W}{\partial\varphi_j\,\partial\varphi_k}\right)\left(\frac{\partial^2 W^*}{\partial\varphi_j^*\partial\varphi_k^*}\right) \qquad (2.21)$$
Now, consider table I

Table I: Mass Spectrum

O'Raifertaigh model	
Bosons	**Fermions**
$\varphi_1: 0.0$	$\psi_1: 0$
$\varphi_2: M_1 M$	$\psi: M$
$\varphi_3: \sqrt{M^2-2gm^2}, \sqrt{M^2+2gm^2}$	$\psi: M$

invoking tableI into formula for the supertrace leads to
$ST_r\{M\} = M^2 + M^2 + (M^2 - 2gm^2) + (M^2 + 2gm^2) - 2(M^2 + M^2) = 0$

Remark:

(i) We cannot have $F_i^* = 0$ for $i = 1, 2, 3$ simultaneously, consequently, the form W breaks supersymmetry.

(ii) The spectrum:
$V = \left(\frac{\partial W}{\partial\varphi_i}\right)\left(\frac{\partial W}{\partial\varphi_i}\right)^* = g^2|\varphi_3^2 - m^2|^2 + M^2|\varphi_3|^2 + M^2|2g\varphi_1\varphi_3 + M\varphi_2|^2$
If $m^2 < \frac{M^2}{2g^2}$, then the minimum is at $(\varphi_2) - (\varphi_3) = 0$.
(φ_1) arbitrary. This implies that $\langle V\rangle = g^2m^4 > 0$. This arbitrariness of φ_1 implies zero mass,$m_{\varphi_1} = 0$.

(iii) $\langle\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}\rangle\,\varphi_i\varphi_j = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & M\\ 0 & M & 0\end{pmatrix}\varphi_i\varphi_j$ in the Lagrangian gives φ_i masses;
$m_{\varphi_1} = 0, m_{\varphi_2} = m_{\varphi_3} = M$

(iv) φ_1 turns out to be goldstino due to $\delta\varphi_1 \propto \langle F_1\rangle \neq 0$ zero mass.

(v) The quadratic terms in
$V: V_{quad} \;= -m^2g^2(\varphi_3^2 + \varphi_3^{*2}) + M^2|\varphi_3|^2 + +M^2|\varphi_2|^2$
implies $m_{\varphi_1} = 0, m_{\varphi_2} = M$ while φ_3 is regarded as a complex field
$\varphi_3 = a + ib$ where real and imaginary part have different masses. Thus
$m_a^2 = M^2 - 2g^2m^2, m_b^2 = M^2 - 2g^2m^2$
There arise heavier and lighter superpartners, the supertrace of M of the bosonic and fermionic parts vanishes, as seen in the generic for tree level of broken supersymmetry. It may be noted that W is not renormalized to all orders in perturbation theory.

(vi) If supersymmetry is unbroken at tree level, then it is also unbroken to all orders in perturbation theory. It means that in order to break supersymmetry we need to consider non-perturbative effects.

3.0 D-Term Breaking

Theorem: A chirasuperfieldϕ of charge q coupled to an Abelian vector superfield V yield the D-term part of the Lagrangian given by:
$$\mathcal{L}_D =_p D|\varphi|^2 + \frac{1}{2}D^2 + \frac{1}{2}D$$
Proof

When a chiral superfieldϕ of charge q coupled to an Abelian vector superfield V, the Lagrangian then reads
$$\mathcal{L} = \left(\phi^+ e^{q^V}\phi\right)_D + \frac{1}{4}(W^\alpha W_\alpha|_F + h.c) + \xi V_D \qquad (3.1)$$
Let
$$V_{W_z} = (\theta\sigma^\mu\bar\theta) + i(\theta\theta)(\bar\theta\bar\phi) - i(\bar\theta\bar\theta)(\theta\lambda) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D$$
$$Wess - Zumino\ gauge \qquad (3.2)$$
$$\Phi = \varphi + \sqrt{2}(\theta\psi) + (\theta\theta)F + i(\theta\sigma^\mu\bar\theta)\partial_\mu\varphi - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial_\mu\partial^\mu\varphi$$

$$-\frac{i}{\sqrt{2}}(\theta\theta)\left(\partial_\mu\partial^\mu\bar{\theta}\right) \quad (3.3)$$

$$\Phi^+ = \varphi^* + \sqrt{2}(\bar{\theta}\bar{\psi}) + (\bar{\theta}\bar{\theta})F^* + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi^* - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\varphi^*$$

$$+\frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\left(\theta\sigma^\mu\partial_\mu\bar{\psi}\right) \quad (3.4)$$

Φ and Φ^+ are the components of chiral superfield

NB:

$\partial^\mu\varphi^*\partial_\mu\varphi - i\bar{\psi}\partial^\mu\partial_\mu\psi + FF^*$ are dut to the D-term of $\Phi^+\Phi$ after integration by parts. But,

$$\Phi^+(x,\theta,\bar{\theta}) = \varphi^* + \sqrt{2}(\bar{\theta}\bar{\psi}) + (\bar{\theta}\bar{\theta})F^* + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi^* - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\varphi^*$$

$$+\frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\left(\theta\sigma^\mu\partial_\mu\bar{\psi}\right) \quad (3.5)$$

$$\Phi^+\Phi = \left(\varphi^* + \sqrt{2}(\bar{\theta}\bar{\psi}) + (\bar{\theta}\bar{\theta})F^* + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi^* - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\varphi^* + \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\left(\theta\sigma^\mu\partial_\mu\bar{\psi}\right)\right)$$

$$\left(\varphi + \sqrt{2}(\theta\psi) + (\theta\theta)F + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\varphi - \frac{i}{\sqrt{2}}(\theta\theta)\left(\partial_\mu\partial^\mu\bar{\theta}\right)\right)$$

$$\supset (\theta\theta)(\bar{\theta}\bar{\theta})\left[-\frac{1}{4}\varphi^*\partial_\mu\partial^\mu\varphi - \frac{1}{4}\varphi\partial_\mu\partial^\mu\varphi^* + |F|^2\right] + \left(\theta\sigma^\mu\partial_\mu\bar{\theta}\right)\left(\theta\sigma^v\partial_\mu\bar{\theta}\right)\partial_v\varphi\partial_\mu\varphi^*$$

$$-i\bar{\theta}\bar{\psi}(\theta\theta)\partial_\mu\varphi\sigma^\mu\bar{\theta} + i(\bar{\theta}\bar{\theta})\left(\theta\sigma^\mu\partial_\mu\bar{\psi}\right)(\theta\psi)$$

$$= (\theta\theta)(\bar{\theta}\bar{\theta})\left[-\frac{1}{4}\varphi^*\partial_\mu\partial^\mu\varphi - \frac{1}{4}\varphi\partial_\mu\partial^\mu\varphi^* + |F|^2\right] + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\varphi\partial_\mu\varphi^*$$

$$+i\bar{\theta}^\alpha\bar{\psi}_\alpha(\theta\theta)\partial_\mu\psi^\beta(\sigma^\mu)_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}} + i(\bar{\theta}\bar{\theta})\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}}\theta^\beta\varphi_\beta$$

$$= (\theta\theta)(\bar{\theta}\bar{\theta})\left[-\frac{1}{4}\varphi^*\partial_\mu\partial^\mu\varphi - \frac{1}{4}\varphi\partial_\mu\partial^\mu\varphi^* + |F|^2\right] + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\varphi\partial_\mu\varphi^*$$

$$+i\frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}(\bar{\theta}\bar{\theta})\bar{\psi}_{\dot{\alpha}}(\theta\theta)\partial_\mu\psi^\beta(\sigma^\mu)_{\beta\dot{\beta}} + i\frac{1}{2}(\bar{\theta}\bar{\theta})(\theta\theta)\epsilon^{\alpha\beta}(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}}\varphi_\beta$$

$$= (\theta\theta)(\bar{\theta}\bar{\theta})\left[-\frac{1}{4}\varphi^*\partial_\mu\partial^\mu\varphi - \frac{1}{4}\varphi\partial_\mu\partial^\mu\varphi^* + |F|^2 + i\frac{1}{2}\partial_\mu\psi\partial_\mu\varphi^* + i\frac{1}{2}\partial_\mu\psi(\sigma^\mu)\bar{\psi} - i\frac{1}{2}\psi(\sigma^\mu)\partial_\mu\bar{\psi}\right]$$

$$= (\theta\theta)(\bar{\theta}\bar{\theta})\left[|F|^2 + \partial^\mu\varphi\partial_\mu\varphi^* - i\frac{1}{2}\psi(\sigma^\mu)\partial_\mu\bar{\psi}\right] + total\ derivatives \quad (3.6)$$

hence,

$$\Phi^+V_\Phi\big|_D = -i\left(\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi\right) + \partial_\mu\varphi^* + |F|^2 \quad (3.7)$$

Now,

$$\Phi^+V\phi|_D = \varphi^*(\theta\sigma^\mu\bar{\theta})V_\mu i(\theta\sigma^V\bar{\theta})\partial_v\varphi - i\varphi^*(\bar{\theta}\bar{\theta})(\theta\,\lambda)\left(\sqrt{2}\theta\psi\right) + \frac{|\varphi|^2}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D$$

$$+\sqrt{2}(\bar{\theta}\bar{\psi})(\theta\sigma^\mu\bar{\theta})V_\mu\sqrt{2}(\theta\psi) + i\sqrt{2}(\bar{\theta}\bar{\psi})(\theta\theta)(\bar{\theta}\,\bar{\lambda})\varphi - i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi^*{}_\mu(\theta\sigma^V\bar{\theta})V_v\varphi|_D$$

$$= \frac{1}{2}|\varphi|^2D + \frac{i}{2}\left(\varphi^*V^\mu\partial_\mu\varphi - \varphi V^\mu\partial_\mu\varphi^*\right) + \frac{i}{\sqrt{2}}\left((\bar{\lambda}\,\bar{\psi})\varphi - (\lambda\,\psi)\varphi^*\right) - \frac{1}{2}(\bar{\psi}\bar{\sigma}^\mu\psi)V_\mu \quad (3.8)$$

Using Fierz identities,

$$\Phi^\dagger\frac{V^2}{2}\phi|_D = \frac{1}{2}\varphi^*(\theta\sigma^\mu\theta)V_\mu(\theta\sigma^\mu\bar{\theta})V_v|_D$$

$$= \frac{1}{4}|\varphi|^2V_\mu V^\mu \quad (3.9)$$

$$\Phi^\dagger e^{2\ or\ v}\phi|_D = \partial_\mu\varphi\partial^\mu\varphi^* + |F|^2 - i\left(\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi\right) + qV^\mu\left[-\left(\bar{\psi}\bar{\sigma}_\mu\right) + i\left(\varphi^*\partial_\mu\varphi - \varphi\partial_\mu\varphi^*\right)\right]$$

$$+\sqrt{2}iq[\varphi(\bar{\lambda}\,\bar{\psi}) - \varphi^*(\bar{\lambda}\,\bar{\psi})] + a\left(D + qV_\mu V^\mu\right)|\varphi|^2 \quad (3.10)$$

Hence the D-term part of the Lagrangian is

$$\mathcal{L}_D = qD|\varphi|^2 + \frac{1}{2}D^2 + \frac{1}{2}\xi D \quad (3.11)$$

where $\frac{1}{2}D^2$ term comes from $\frac{1}{4}W_\alpha W^\alpha + hc$. Solving (3.11) for D yields

$$D = -q|\varphi|^2 - \frac{\xi}{2} \quad (3.12)$$

Put *3.12) into (3.11) yields the D-term potential

$$V_D = \frac{1}{8}(\xi + 2a|\varphi|^2)^2 \quad (3.13)$$

Remark:

(i) Supersymmetry is broken when $\left(q|\varphi|^2 + \frac{\xi}{2}\right) = -D \neq 0$. This is the condition that the Fayet-Iliopoulos term and the charge q have to satisfy for supersymmetry to be broken.

(ii) If ξ and q are given the same sign, then V_D is minimized at $\varphi = 0$, with a positive potential and supersymmetry is broken. In this case

$$V_D = \frac{1}{8}\xi^2 + \frac{q\xi}{2}|\varphi|^2 + \frac{q^2}{2}|\varphi|^4$$

(iii) If $\langle\varphi\rangle = 0$, the mass of φ is $m_\varphi^2 = q\xi$, as the kinetic energy terms are $\partial_\mu \psi \partial^\mu \varphi^*$

Since now other fields obtain $V\theta Vs$, no mass is generated for the fermions. Therefore the mass splitting in the multiplet is $m_\varphi = \sqrt{q\xi}$ and $m_\varphi = 0$.

Theorem:

A renormalisable$N = 1$ supersymmetry theory with chiral superfields $\Phi_1 = (\varphi_1, \psi_1, F_1)$ and vector superfields $V_a\left(\lambda_a, D_a^\mu, D_a\right)$ with both D and F term supersymmetry breaking ($F_1 \neq 0$ and $D_a \neq 0$) in the vacuum is given by:

$$\frac{\partial V}{\partial \varphi_i} = F^j \frac{\partial^2 W}{\partial \varphi_i \varphi_j} + g^a D^a \varphi_j^\dagger (T^a)_i^j = 0$$

where $g^a T^a$ and W are the gauge coupling condition, generators of the gauge group and superpotential respectively.

Proof

By definition

$$V = \sum |F_i|^2 + \frac{1}{2}\sum_a D^a D^a$$

$$= \sum_i \left(\frac{\partial W}{\partial \varphi_i}\right)\left(\frac{\partial W^*}{\partial \varphi_j^*}\right) + \frac{1}{2}\sum_a \left(\sum_j \varphi_j^\dagger T^a \varphi_j\right)\left(\sum_k \varphi_k^\dagger T^a \varphi_k\right) \quad (3.14)$$

Now,

$$\frac{\partial V}{\partial \varphi_i} = \sum_i \left(\frac{\partial^2 W}{\partial \varphi_i \varphi_j}\right) F_i + \sum_i D^a \left(\sum_k \varphi_k^\dagger (T^a)_{kj}\right)$$

$$= \frac{\partial^2 W}{\partial \varphi_i \varphi_j} \sqrt{2} \sum_k \varphi_k^\dagger (T^a)_{kj} \begin{pmatrix} F_i \\ \frac{D^a}{\sqrt{2}} \end{pmatrix} = 0 \quad (3.15)$$

Lemma 1:

The gauge variation of W is given by

$$\delta_{gauge}^{(a)} W = \frac{\partial W}{\partial \varphi_i} \delta_{gauge}^{(a)} \varphi^j$$

$$= -F_i^\dagger (T^a)_j^i \varphi_j \quad (3.17)$$

multiply (3.15) with a non-vanishing complex number c, we obtained the matrix

$$= \begin{pmatrix} c\sum_k \varphi_k^\dagger (T^a)_{kj} & 0 \end{pmatrix} \begin{pmatrix} F_i \\ \frac{D^a}{\sqrt{2}} \end{pmatrix} \quad (3.18)$$

Combining (3.15) and (3.18), lead to the matrix

$$0 = \begin{pmatrix} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} & \sqrt{2}\sum_k \varphi_k^\dagger (T^a)_{kj} \\ \sqrt{2}\sum_k \varphi_k^\dagger (T^a)_{kj} & 0 \end{pmatrix} \begin{pmatrix} F_i \\ \frac{D^a}{\sqrt{2}} \end{pmatrix} \quad (3.19)$$

Lemma 2:
The matrix (3.19) is the same as that of the fermion mass matrix
$\begin{pmatrix}\psi_i\\ \lambda_a\end{pmatrix}^T (M_{ia})\begin{pmatrix}\psi_i\\ \lambda_a\end{pmatrix}$

Proof
To find the entries of the mass matrix, we need to know the standard contribution for the fermion mass matrix given by $\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}$. The off-diagonal terms can be obtained from the structure of the kinetic terms

$$\int d^2\theta d^2\bar\theta\,\bar\Phi^\dagger e^{V^a T^a}\Phi$$

Now,

$$\left.\begin{aligned}\Phi &\sim\psi+\sqrt{2}\theta\psi+\theta\theta Ff\\ \Phi &\sim\psi^\dagger+\sqrt{2}\bar\theta\bar\psi+\bar\theta\bar\theta F^*\\ V^a &\sim(\theta\sigma^\mu\bar\theta)V_\mu^a+i\theta\theta theta\lambda^a\\ &-i(\bar\theta\bar\theta\theta\lambda^a)+\frac{1}{2}\theta\theta\bar\theta\bar\theta D^a\end{aligned}\right\} \qquad (3.20)$$

As deduced for the abelian case (3.1) – (3.10), the term $\psi\lambda$ arises from $\Phi_i^\dagger e^{V^a T^a}\Phi_i$. We identify the following cross term by looking at the superfield expansion (3.1) – (3.10).
$\sqrt{2}\psi_i^\dagger T_{ij}^a\psi_j\lambda_a$
From the superfield expansion, no gaugino mass term $(\lambda\lambda)$ is generated.

Hence,

$$M_{ia}\begin{pmatrix}F_i\\ D^a\\ \hline \sqrt{2}\end{pmatrix}=0 \qquad (3.21)$$

where M_{ia} is the fermion mass matrix.
Remark:
Eqn. (3.21) implies that there is at least one zero eigenvalue with eigenvector
$\begin{pmatrix}F_i\\ \frac{D^a}{\sqrt{2}}\end{pmatrix}$
This means that there exists a massless Goldstone fermion, oriented along the direction of supersymmetry breaking.

4.0 Yang-Mills coupled to $N = 1$ supergravity

In this section, we consider pure $SU(N)$ supersymmetry Yang-Mills coupled to $N = 1$ supergravity and analyzed the effective Lagrangian formulation noted in [3].
Thus, the Langrangian is expressed as

$$\mathcal{L}=\int d^4\theta\left[(SS^*)^{\frac{1}{3}}+\left(\frac{SS^*}{\mu^4}\right)\right]+\int d^2\theta\left[Slog\left(\frac{S}{\mu^3}\right)-S+h.c\right] \qquad (4.1)$$

where $S=W^\alpha W_\alpha$ and $W_\alpha=\bar D^2 e^V D_\alpha e^V$. S is a chiral multiplet with components.

$$S=\left(\lambda\lambda.\lambda\sigma_{\mu\nu}F^{\mu\nu}+\cdots+F_{\mu\nu}F^{\mu\nu}+\cdots\right) \qquad (4.2)$$

λ denotes the gauge fermion of $SU(N)$ and $F^{\mu\nu}$ is the field strength tensor $\left(F_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu p\lambda}F^{\mu\nu}\right)$.
We observed that $\lambda\lambda$ condensation occurs [3] without breaking global supersymmetry. The description is in good faith because of its success when applied to QCD [3]. It may be noted that

$$W^\alpha W_\alpha|_F=D^2-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}-2i\lambda\sigma^\mu\partial_\mu\bar\lambda+\frac{i}{2}F_{\mu\nu}F^{\mu\nu} \qquad (4.3)$$

and F term of $W^\alpha W_\alpha$ is verified using

$$T_\tau\{\sigma^{\mu\nu}\sigma^{\mu\tau}\}-\frac{1}{2}(\eta^{\mu k}\eta^{\nu k}-\eta^{\mu\nu}\eta^{\nu k}+i\epsilon^{\mu\nu k\tau}) \qquad (4.4)$$

In the QED choice, we write $f = \frac{1}{4}$ with kinetic terms of the vector superfield given by

$$\mathcal{L}_{kin} = \frac{1}{4}W^{\alpha}W_{\alpha}|_{F} + h.c = \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} \qquad (4.5)$$

Theorem:

The F-components of $\frac{1}{4}W^{\alpha}W_{\alpha}$ is given by:

$$\frac{1}{4}W^{\alpha}W_{\alpha}|_{F} = -\frac{1}{2}\lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} + \frac{1}{4}D^2 - \frac{1}{8}F_{\mu\nu}F^{\mu\nu} + \frac{i}{8}\overline{F_{\mu\nu}}|F^{\mu\nu}$$

Proof

The F-components of $\frac{1}{4}W^{\alpha}W_{\alpha}$ is given by:

$$\frac{1}{4}W^{\alpha}W_{\alpha}|_{F} = -\frac{1}{2}\lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} + \frac{1}{4}D^2 - \frac{1}{8}F_{\mu\nu}F^{\mu\nu} + \frac{i}{8}\overline{F_{\mu\nu}}|F^{\mu\nu}$$

Proof $\frac{1}{4}W^{\alpha}W_{\alpha}|_{F} = \frac{1}{4}(\theta\theta)\left(-2i \leftthreetimes^{\alpha}\ \sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\ \overline{\leftthreetimes}^{\dot{\alpha}} + D^2\right) - \frac{1}{16}(\sigma^{\mu}\bar{\sigma}^{\mu}\theta)^{\alpha}(\sigma^{\mu}\bar{\sigma}^{\leftthreetimes}\theta)_{\alpha}F_{\mu\nu}F^{\mu\nu}$

$$+\frac{i}{4}D\theta^{\alpha}(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)_{\alpha}F_{\mu\nu}$$

$$= \frac{1}{4}(\theta\theta)\left(-2i \leftthreetimes^{\alpha}\ \sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\ \overline{\leftthreetimes}^{\dot{\alpha}} + D^2\right) - \frac{1}{31}(\theta\theta)T_{r}\{\sigma^{\mu}\bar{\sigma}^{\mu}\sigma^{\leftthreetimes}\bar{\sigma}^{p}\}F_{\mu\nu}F^{\mu\nu} \qquad (4.6)$$

From (4.6)

$$\frac{i}{4}D\theta^{\alpha}(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)_{\alpha}F_{\mu\nu} = \frac{i}{4}DF_{\mu\nu}\theta^{\alpha}\theta^{\gamma}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\sigma}^{\nu})^{\dot{\alpha}\beta}\epsilon\beta\gamma F_{\mu\nu}$$

$$= -\frac{i}{8}DF_{\mu\nu}(\theta\theta)\epsilon^{\alpha\gamma}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\sigma}^{\nu})^{\dot{\alpha}\beta}\epsilon\beta\gamma$$

$$= \frac{i}{8}DF_{\mu\nu}(\theta\theta)(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\sigma}^{\nu})^{\dot{\alpha}\beta}\delta^{\alpha}_{\beta}$$

$$= \frac{i}{8}DF_{\mu\nu}(\theta\theta)\left(\sigma^{\mu}_{\alpha\dot{\alpha}}\right)(\bar{\sigma}^{\nu})^{\dot{\alpha}\alpha}$$

$$= \frac{i}{4}DF_{\mu\nu}(\theta\theta)\eta^{\mu\nu}$$

$$= 0 \qquad (4.7)$$

$$-\frac{1}{16}(\sigma^{\mu}\sigma^{\mu}\theta)^{\alpha}(\sigma^{\rho}\bar{\sigma}^{\leftthreetimes}\theta)_{\alpha}F_{\mu\nu}F_{\rho\leftthreetimes} = -\frac{1}{6}\epsilon_{\alpha\beta}(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)^{\alpha}(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)^{\beta}F_{\mu\nu}F_{\rho\leftthreetimes}$$

$$= -\frac{1}{16}\epsilon^{\alpha\beta}(\sigma^{\mu})_{\alpha\dot{\alpha}}(\bar{\sigma}^{\nu})^{\dot{\alpha}\gamma}\theta_{\gamma}(\sigma^{\rho})_{\beta\dot{\beta}}(\bar{\sigma}^{\leftthreetimes})^{\dot{\beta}\delta}\theta_{\delta}F_{\mu\nu}F_{\rho\leftthreetimes}$$

$$= -\frac{1}{32}(\theta\theta)T_{r}\{\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\leftthreetimes}\bar{\sigma}^{\rho}\}F_{\mu\nu}F_{\rho\leftthreetimes}$$

$$= -\frac{i}{16}(\theta\theta)\epsilon^{\mu\leftthreetimes\tau\rho}F_{\mu\leftthreetimes}F_{\rho\tau} - \frac{1}{8}(\theta\theta)F_{\mu\nu}F^{\mu\nu} \qquad (4.8)$$

and

$$T_{\tau}\{\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\leftthreetimes}\bar{\sigma}^{\rho}\} = 2i\epsilon^{\mu\nu\leftthreetimes\rho} + 2\eta^{\mu\nu}\eta^{\leftthreetimes\rho} - 2\eta^{\mu\leftthreetimes}\eta^{\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\leftthreetimes} \qquad (4.9)$$

If we write $\tilde{F}_{\mu} = \frac{1}{2}\epsilon_{\mu\nu\rho\leftthreetimes}F^{\rho\leftthreetimes}$ noted in (4.2), we have

$$\frac{1}{4}W^{\alpha}W_{\alpha}|_{F} = \frac{i}{2}\leftthreetimes\sigma^{\mu}\partial_{\mu}\ \overline{\leftthreetimes} + \frac{1}{4}D^2 - \frac{1}{8}F_{\mu\nu}F^{\mu\nu} + \frac{i}{8}F_{\mu\nu}F^{\mu\nu} \qquad (4.10)$$

Hence the prove.

Lemma 2:

A new ingredient of supersymmetry theories is that an extra term can be added to the Lagrangian $\mathcal{L}$. It is also invariant for $u(1)$ gauge theories know as Fayet Iliopoulos (FI) term:

$$\mathcal{L}_{F1} = \xi V|_{D} = \frac{1}{2}\xi D \qquad (4.11)$$

Now,

$$\mathcal{L}_{D} = qD|\varphi|^2 + \frac{1}{2}D^2 + \frac{1}{2}\xi D \qquad (4.12)$$

Imposing the condition $\left(\frac{\delta S_{(D)}}{\delta D} = 0\right)$ yields

$$D = \frac{\xi}{2} - q|\varphi|^2 \qquad (4.13)$$

Putting (4.13) into (4.12) yields

$$\mathcal{L}_D = \frac{1}{8}(\xi + 2q|\varphi|^2)^2$$

$$=: -V_{(D)}(\varphi) \quad (4.14)$$

Lemma 2:

The Lagriangian depending on the auxiliary field F takes the simple form:

$$\mathcal{L}_{(F)} = FF^* + \frac{\partial W}{\partial \varphi}F + \frac{\partial W^*}{\partial \varphi^*}F^* \quad (4.15)$$

We eliminate F using the field equations

$$\frac{\delta S_{(F)}}{\delta F} = 0 \Rightarrow F^* + \frac{\partial W}{\partial \varphi} = 0 \quad (4.16)$$

$$\frac{\delta S_{(F)}}{\delta F^*} = 0 \Rightarrow F + \frac{\partial W^*}{\partial \varphi^*} = 0 \quad (4.17)$$

Put (4.16) and (4.17) into (4.15) yields

$$\mathcal{L}_{(F)} \rightarrow -\left|\frac{\partial W}{\partial \varphi}\right|^2 =: -V_{(F)}(\varphi) \quad (4.18)$$

Hence, the positive definite scalar potential $V_{(F)}(\varphi), V \geq 0$.

Now, combining (4.14) together with (4.18) yields

$$V(\varphi) = V_{(F)}(\varphi) + V_{(D)}(\varphi)$$

$$= \left|\frac{\partial W}{\partial \varphi}\right|^2 + \frac{1}{8}(\xi + 2q|\varphi|^2)^2 \quad (4.19)$$

Hence the total potential.

Remarks: When supersymmetry is broken Dynamically in the F-term simultaneously. There exists a semidefinite scalar potential $V_{(D)}(\varphi)$ and $V_{(F)}(\varphi)$ in the D and F term breaking respectively. Hence, the total potential is the contribution for both $V_{(F)}(\varphi)$ and $V_{(D)}(\varphi)$.

5.0 Strongly Coupled Chiral Superfield(S)

In this section, we coupled the chiral superfield(S) with the action (4.1) to supergravity, following the pioneering work of ref. [8]. We adopt the same notation chosen in [8]. Now, we deduced from (4.1)

$f: g(z) \longrightarrow G(z, z^*)$, where $z = (\curlywedge\curlywedge)$(5.1)

$$G(z, z^*) = 3log\left(-\frac{\emptyset}{3}\right) - log * \left(\frac{|g|^2}{M^6}\right) \quad (5.2)$$

$$\emptyset = -1 + \left(\frac{\alpha}{M^2\mu^4}\right)(zz^*) + \left(\frac{b}{M^2}\right)(zz^*)^{\frac{1}{3}} \quad (5.3)$$

$$g(z) = c\left\{zlog\left(\frac{z}{\mu^3}\right) - z\right\} \quad (5.4)$$

where $g(z)$ is the super-potential, μ and M are the mass scales. Correcting the coefficients a, b and c to order one, we have:

$$V = -\exp(-G)\left(3 + \frac{|G_{1z}|^2}{G_{1zz^*}}\right) \quad (5.5)$$

This potential (V) is given by [8].

Results

1. If we imposed the condition $V(z = 0) = \infty$, and $\langle z \rangle \neq 0$, and then $\curlywedge\curlywedge$ condensation occurs.

Now

$$V(z, z^*) = f(z, z^*)B(z, z^*) \quad (5.6)$$

$$f = exp(-G)/G_{1zz^*}\emptyset|g|^2 \quad (5.7)$$

$$B = 3\emptyset_{1z}g(g_{1z})^* + 3(\emptyset_{1z})^*g_{1z}g^* - 0\emptyset_{1zz^*}|g|^2 - |g_{1z}|^2\emptyset \quad (5.8)$$

2. If f is strickly positive, $\emptyset < 0, G_{1zz^*} < 0$,

$$B_{1z} = 0 \text{ and } z_0 = \mu^3 \quad (5.9)$$

3. If we allow $b = -9a$ i.e. parameter adjustment.

$$B(z_0) = 0 \text{ whenever } f > 0 \quad (5.10)$$

Remarks:

1. These results (5.6) – (5.10) shows that for $\langle \lambda\lambda \rangle = \mu^3$, the potential is minimum with zero energy, where μ is the scale.
2. If the gauge interaction is strong, then the scale governing the scale of condensation is not disturbed by the gravitational effect.
3. The absolute minimum of the potential is situated at $G_{01z} = 0$ with negative energy.
   This result however does not implying vacuum at $z_0 = \mu^3$, thus $E = 0$ is stable [11]. The stable vacuum with $E = 0$ was chosen in the history of the early universe [12]. Again $E = 0$ does not imply exact local supersymmetry.
4. The absence of the cosmological term and the appearance of Poincare supersymmetry in the local limit is noted in [13], thus,

$$\langle u \rangle = -\frac{9}{2}\left(\frac{g_0^*}{\emptyset_0}\right)\left(1 + \frac{3|\emptyset_{01z}|^2}{\emptyset_0^2 G_{01zz^*}}\right) = 0 \qquad (5.11)$$

6.0 Higgs Condensate

In this section, we discuss the mass splitting based on super-Higgs effect noted in our previous work [10]. Now, the gravitino mass term is

$$M_\Psi^2 = M^2 exp(-G_0) \qquad (6.1)$$

The super-Higgs effect occurs when $\lambda\, \sigma^{\mu v} F_{\mu v}$ of $\lambda\lambda$ is absorbed by gravitino. One can observe these in [8].

$$X = \lambda\, \sigma^{\mu v} F_{\mu v} \qquad (6.2)$$

$$\delta X = 2exp(-G_0/2)\left[G_{01zz^*}/(-2G_{01zz^*})^{\frac{1}{2}}\right]\epsilon + \cdots \qquad (6.3)$$

In $SU(3) \times SU(2) \times U(1)$ gauge bosons. The mass splitting of $10TeV$ between gauge bosons and the ganginos will induced a mass for the scalar partners of quark and lepton at the two loop level of order $(\alpha/\pi)m_g \approx (100 - 1000)EgV$, where α is one of the $SU(3) \times SU(2) \times U(1)$ couplings. However, Higgs boons receive a negative mass-square due to the Yukawa interactions [10] and $SU(2) \times U(1)$ is broken $U(1)$ at a scale of $100GeV$.

Finally, continuously broken gauge theories realize the Higgs mechanism in which the corresponding Goldstone is “eaten” by the corresponding gauge field to get a mass. In supersymmetry, the Goldstino field join the originally massless gravitino field and give it a mass. In this case, the gravitino receives its mass by “eating” the Goldstino.

7.0 Conclusion

In this paper, we have coupled gauge theory to $\mathcal{N} = 1$ supergravity and have analyzed super-Higgs effect. We find a strong evidence of a non-vanishing vacuum expectation of O’refitaigh model after supersymmetry is broken. We have shown how Yang-mills theory is coupled to $\mathcal{N} = 1$ supergravity and establish a creteria for Lagrangians. The condition for strongly coupled chiral superfields were fully analyzed. For future work, one can establish the effects of strongly coupled superfield within the frame work of Higgs mechanics.

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