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In honour of Prof. Ekhaguere at 70

Mixed optimal stopping and singular stochastic control model for investment in oil field project: identifying thresholds

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Abstract. Investing in projects involving huge financial risks demands great care in decision making and execution. Dealing with market uncertainty and taking effective decision for investment in oil field project therefore, requires a reliable guide - an optimal strategy. This strategy will emerge from addressing a problem involving an optimal stopping time with singular stochastic control for jump diffusions. The strategy therefore would identify two unique thresholds, one indicating when to apply the control and the other showing when to quit. In this paper, the optimal strategy for investment in oil field project is obtained. Two particular cases are also presented.

Keywords: stochastic control model, jump diffusion, optimal stopping time, financial risk.

1. Introduction

In managing risks involved in huge financial investments such as in oil field development projects, there is a need to develop viable strategies. Oil price is subject to fluctuations, this makes investment on energy projects uncertain. Technical uncertainties such as quantity of oil in the ground and geological structures also affect overall investment decisions. In order to meet the challenge of dealing with uncertainty, managers and investors have used some traditional capital investment tools such as net present value (NPV), discount factor analysis (DCF), expected monetary value (EMV). However, these calculations are done with fixed prices. (It is known that oil price is not fixed.) This grossly over estimates projected gains or severely undermines a project's viability leading to distorted decisions in each case.

When risk and uncertainty are involved, decisions cannot be taken with a "flip of the coin" strategy. To tackle the problem of "how and when" to invest, this work goes beyond calculations of expected return, and proposes an optimal strategy for investment in the project.

We attempt to answer the following questions: when should the investor invest and how should the investment be made. Ogbogbo (2016) has modelled the crude oil spot price as a Jump-diffusion process. The aim of the work is to obtain an optimal strategy for investment in an oil field project. The optimal strategy will involve a singular control and an optimal stopping time for the investment. Thus the work will identify two unique thresholds for the investor; One threshold points out when to apply the control and the other indicates when to quit. The rest of the paper is presented as follows: the second section gives a brief literature in optimal strategies, section three presents the dynamics and PDE of the system; strategy obtained and identified thresholds are also given here. Section four gives optimal strategy for particular cases for Brownian Motion and Geometric Brownian Motion. Fifth section gives the conclusion.

2. Brief literature on optimal strategies

Some models for optimal strategy and control have been obtained for gas storage, number of wells to drill and for oil discovery and extraction. The work by Bringedal (2003) was on gas storage

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valuation. He studied gas storage facility because of expanding gas market in Europe. Investing in a gas storage facility is similar to investing in an oil field development project. The objective of his work was to obtain a strategy which would identify a benchmark price level at which to refill the storage facility or sell off gas in it. The value of the storage facility was calculated with constant volatility and mean reversion parameters. Bringedal used a technique called *stochastic dual dynamic programming SDDP*.

The *optimal strategy* he obtained defined a bench mark price level x, at which one would sell if the spot price P_t is above it and buy if spot price is below it. i.e. sell if $P_t > x$ and Buy if $P_t < x$. Despite the effort at generating an optimal strategy, the assumption of constant volatility is considered a major simplification of the model. Though he used a mean reverting process in his model, the criticism of the Black- Schole model is based largely on assumption of constant volatility.

Benkherouf and Pitts (2005) obtained optimal strategy on the number of oil wells to drill. Their work developed an oil exploration model. They obtained their results analytically, the uncertainty element is in the fact that the wells n_1 and n_2 are unknown , but are represented by a two-dimensional distribution fixed apriori.(as Euler family of distributions). The objective of the work was to obtain optimal strategy for drilling, that maximizes the total expected return over an infinite horizon, based on the entire history and future prospects.

Maurer and Semmler (2010) worked on an Optimal control model of oil discovery and extraction. They obtained the optimal rate of extraction given the price trajectory, for an oil extraction and discovery problem. Using the Hamiltonian, and maximum principles they solved the finite horizon optimal control problem which they formulated. They solved the resulting non- linear programming problem numerically using NUDOCCCS. i.e. they used discretization technique to transcribe the optimal control problem into a non linear programming problem via the code NUDOCCCS.

Generally, the Maurer-Semmler model was a finite horizon optimal control model that used two state variables - known stock of resource and cumulated past extraction.

3. The system describing the crude oil price process

Ogbogbo (2017) has set the model for optimal invest strategy in oil field project . The assumptions of the model and background to model formulation including costs, concepts of singularity, optimal control are given by [7]. We therefore consider the stochastic system with jump component with singular control given in equation (1.1).

3.1 *Dynamics of the system*

The system of interest X_t, describing the oil price process, is a stochastic system with a jump component, also with singular control, and is of the form:

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + \int_{\mathbb{R}} h(t^-, X_{t^-}, \gamma(Z))\tilde{N}(dt, dZ) - d\Gamma_t \tag{1.1}$$

$$X(0) = x_0$$

With Lipschitz condition for existence of solution. Where $X = X^\Gamma = (X_1^\Gamma(t), \cdots, X_n^\Gamma(t)$. $W(t)$ is an n- dimensional Brownian motion independent of $\hat{N}$. $\hat{N}$ is a martingale measure of jumps. $\Gamma(t)$ is the singular control applied to the process $X(t)$. $X^\Gamma(s) = X = (X_1, \cdots, X_n) \in \mathbb{R}, s \leq t$. $\Gamma(t) = (\Gamma_1(t), \Gamma_2(t), \cdots, \Gamma_n(t)) \subset \mathbb{R}^n, t \geq s$. $\mu(X_t)$ is drift component, $\sigma(X_t)$ is diffusion component.

3.2 *The controller and the objective functional*

The controller or investor is observing a system that is evolving with time. There are costs involved. There is a cost paid over time for observing the system. Waiting before taking a decision is at a cost, anytime decision is taken, a cost is paid. Should she decide to stop, there is a terminal cost. The control is also applied at a cost. Giving rise to the following objective functional.

The objective functional or performance criterion

$J = J(s,x)$ of the form

$$J^{(\Gamma,\eta)}(s,x) = E^{(s,x)} \int_0^{\eta} g(t,x_t)dt + n(t,x_t)d\Gamma_t + m(\eta, x_\eta)|_{\{\eta<\infty\}} \tag{1.2}$$

$$J^{(\Gamma,\eta)}(s,x) = J^{(\Gamma,\eta)}(s,x) = E^{(s,x)} \left[\int_0^{\eta} g(t,x_t)dt + m(\eta,x_t)|_{\eta<\infty} \right] + \int_0^{\eta} n(t,x_t)d\Gamma_t. \tag{1.3}$$

Where m and g are continuous functions, $\eta_s = \eta_s(x) = \inf\{t > 0 : x = x^*\}$, $g(t,x_t)$ is a running cost or observation cost, $m(\eta, x_\eta)$ is the terminal cost, and $n(t,x_t)$ is the cost of applying the control. The running cost $g(t,x_t)$ is the cost of waiting to take decision. In the formulation for this particular problem, it is a constant which is not discounted, a sunk cost involved in production. The cost of applying the control, $n(t,x_t)$ is also a constant in this case.

The stopping cost or terminal cost $m(\eta, x)$, is actually the value of the project at the point the decision is taken. It is the revenue accruing. Interest is in $U(s,x) = \inf_{\Gamma,\eta} J^{(\Gamma,\eta)}_{(s,x)}$ i.e.

$$U(s,x) = \inf_{\Gamma,\eta} E^{(s,x)} \int_0^{\eta} g(t,x_t)dt + n(t,x_t)d\Gamma_t + m(\eta,x_\eta)|_{\{\eta<\infty\}} \Big\} \tag{1.4}$$

3.3 *Threshold and time*

Starting at some point in time and space the interest is in the first time the process hits the threshold. The idea of threshold and time raises the question of "how" and "when", with respect to the investment. "When" involves the threshold and time that the investor should call it quits and "how" is concerned with strategy. Since we have a mixed stochastic model, we desire two thresholds x^* and x_o. $x = x_o$ determines where to apply the control, $x = x^*$ gives the optimal stopping time.

The time (Optimal stopping time)
There is a non- empty time set. Therefore $\eta^* = \{\inf t > 0 : x_t \geq x^*\}$.

3.4 *Characterization of the process and domain of operation*

There is a PDE associated with this model. The PDE satisfies

$$LU(X,t) = -g(t,X) \tag{1.5}$$

$$U(X,t) = m(t,X_t) \tag{1.6}$$

$U(X,t)$ is the solution of the PDE.

Domain of operation

The threshold separates the system into two Domains. The non-intervention region is D, connoting “Wait” and B region is “below the threshold”. Above the threshold, the process is described by the PDE, below the threshold we have $U(X,t) = m(t,X_t)$. Thus we are interested in the solution U(X,t), that defines the threshold. This is illustrated in Fig 1 below.

Fig 1

$$U(X,t) = \begin{cases} U^* & x \geq x^* \quad D \\ \text{------------------------------} & (threshold) \\ m(t,X) & 0 < x \leq x^* \quad B \end{cases}$$

For example, for investor in stock, D region connotes "wait" and the threshold is "invest".

Remark 1 An important condition in the formulation of the model is that the process must not jump at the threshold, otherwise the threshold which is being tracked can be missed. Hence at the threshold, the process must be *continuous and differentiable.*

3.5 *Generation of PDE*

The price process X_t satisfies (1.1) and is a Jump-diffusion. Let

$$Y_t = V(t, X_t) \tag{1.7}$$

V is $C^{1,2}$, hence by Itô's lemma

$$dY_t = V_t(t, X_t)dt + V_x(t, X_t)\mu(X_t, t)dt + \frac{1}{2}V_{xx}(t, X_t)\sigma^2(X_t, t)dt + V_x(t, X_t)\sigma(X_t, t)dW_t$$

$$= V_t dt + V_x \left[\mu(X_t)dt + \sigma(X_t)dW_t)\right] + \frac{1}{2}V_{xx}\sigma^2(X_t)dt \tag{1.8}$$

With the singular control, we have

$$dY_t = V_t dt + V_x\{(X_t)dt + \sigma(x_t)dW_t - d\Gamma_t\} + \frac{1}{2}V_{xx}\sigma^2(X_t)dt$$

$$= \left[V_t + \mu(X_t)V_x + \tfrac{1}{2}V_{xx}\sigma^2(X_t)\right]dt + V_x\sigma(X_t)dW_t - V_x d\Gamma_t \tag{1.9}$$

Equation (1.9) describes the dynamics of the process including the control.

To have a complete description of the control problem, the costs are added (through the $J(t,x)$ functional) to dY_t process. The performance criterion is given in equation (1.3) $J = J(t,x)$

$$J(t,x) = E^x\left[\int_0^\eta g(t, x_t)dt + m(\eta, x_t)|_{\{\eta<\infty\}}\right] + \int_0^\eta n(t, x_t)d\Gamma_t \tag{1.10}$$

Let $\mathcal{Z}_t$ denote $J(t,x)$, since $J(t,x)$ is a process. Then the $\mathcal{Z}_t$ process is added to dY_t. From (1.3/ 1.10)

$$d\mathcal{Z}_t = g(t, x_t)dt + n(t, x_t)d\Gamma_t$$

Then

$$d\mathcal{Z}_t + dY_t = \left[V_t + \mu(X_t)V_x + \frac{1}{2}V_{xx}\sigma^2(x,t) + g(t, x_t)\right]dt + V_x\sigma(X_t)dW_t + (n(t, x_t) - V_x)d\Gamma_t \tag{1.11}$$

To solve for $U(x,t)$, $g(t,x)$ is chosen to be of an exponential form $g(t,x) = e^{-\rho s} f(x)$. Since $U(s,x)$ is the function that solves the PDE, by the principle of optimal control.

$$L_x U(s,x) + g(s,x) = 0 \tag{1.12}$$

$U(s,x) = m(s,x)$

$$L_x U(s,x) = \frac{1}{2}\sigma^2(x)\frac{\partial^2 u}{\partial x^2} + \mu(x)\frac{\partial u}{\partial x} + \frac{\partial u}{\partial s} + \int_{\mathbb{R}} U(s,x,\gamma) - \gamma(Z)\frac{\partial u}{\partial x} - U(s,x)\}\Big]\pi dZ \tag{1.13}$$

(1.13) includes the jumps. From (1.12), and excluding the jumps in (1.13), we have

$$\frac{1}{2}\sigma^2(x)\frac{\partial^2 u}{\partial x^2} + \mu(x)\frac{\partial u}{\partial x} + \frac{\partial u}{\partial s} + g(s,x) = 0 \tag{1.14}$$

3.6 *Solution of PDE: identification of strategy*

We desire $U(s,x)$ that solves this time dependent PDE. We consider

$$g(s,x) = e^{-\rho s}$$

$$U(s,x) = m(s,x) = e^{-\rho s} x^{\alpha} \qquad \alpha > 0$$

$g(s,x) = e^{-\rho s})$ means that a constant cost is paid for observing the system, which is discounted in time. By the theory of PDE for optimal control we let

$$U(s,x) = e^{-\rho s}\Psi(x)$$

We have

$$L_x U(s,x) = e^{-\rho s} L\Psi(x)$$

From (1.14). Since $U(s,x) = e^{-\rho s}\Psi(x)$, then

$$\frac{1}{2}\sigma^2(x)U''_x + \mu(x)U'_x + U_s + e^{-\rho s} = \left[\frac{1}{2}e^{-\rho s}\sigma^2(x)\Psi''(x) + \mu(x)e^{-\rho s}\Psi'(x) - \rho e^{-\rho s}\Psi(x) + e^{-\rho s}\right]$$

$$= e^{-\rho s}\left[\tfrac{1}{2}\sigma^2(x)\Psi''(x) + \mu(x)\Psi'(x) - \rho\Psi(x) + 1\right] = 0$$

Dividing through by $e^{-\rho s}$ we have

$$\frac{1}{2}\sigma^2(x)\Psi''(x) + \mu(x)\Psi'(x) - \rho\Psi(x) = -1 \tag{1.15}$$

$\Psi(x) = x^{\alpha}$, $\quad \alpha \geq 0$ is a fixed constant. In particular we consider **case when $\boldsymbol{\rho = 0}$**. Then the controller is paying a constant running cost which is not discounted, then we have

$$\frac{1}{2}\sigma^2(x)\Psi''(x) + \mu(x)\Psi'(x) = -1 \tag{1.16}$$

$\Psi(x) = x^{\alpha}$. By change of variable argument, we let

$$\Psi'(x) = f(x) \tag{1.17}$$

Then (1.16) reduces to a first order ODE which is solved explicitly for $f(x)$, and $\Psi(x)$ is recovered by integration.

$$\frac{1}{2}\sigma^2(x)f'(x) + \mu(x)f(x) = -1 \tag{1.18}$$

Hence

$$U(s,x) = \begin{Bmatrix} x^\alpha & 0 < x \leq x^* \\ \Psi(x) & x^* < x \leq x_0 \\ & x_0 \leq x < x^* \\ \Psi(x) + (x - x^*) & x \geq x^* \end{Bmatrix} \tag{1.19}$$

Remark 2 The solution set is a piecewise continuous solution.

- $\Psi(x)$ is solution of the PDE when x lies within the interval $x^* < x \leq x_0$ (resp $x_0 \leq x \leq x^*$, depending on which threshold is above the other)
- we have x^α (which is the terminal cost), for $0 < x \leq x^*$. This happens, if the controller decides to stop abruptly.
- $\Psi(x)+(x-x^*)$ is solution of the PDE in the last interval $x \geq x^*$. This describes points slightly above the threshold. (What happens a little after the solution point is usually observed after solving a PDE).

From equation (1.18)

$$f'(x) + 2\frac{\mu(x)}{\sigma^2(x)}f(x) = \frac{-2}{\sigma^2(x)} \qquad \sigma(x) > 0.$$

For Integrating factor, $I.F = e^{2\int_0^x \frac{\mu(s)}{\sigma^2(s)}ds}$

$$\frac{d}{dx}\left[f(x)e^{2\int_0^x \frac{\mu(s)}{\sigma^2(s)}ds}\right] = \frac{-2}{\sigma^2(x)}e^{2\int_0^x \frac{\mu(s)}{\sigma^2(s)}ds}$$

$$f(x) = \frac{\int \frac{-2}{\sigma^2(x)}e^{2\int_0^x \frac{\mu(s)}{\sigma^2(s)}ds}dx}{e^{2\int_0^x \frac{\mu(s)}{\sigma^2(s)}ds}} \tag{1.20}$$

From (1.17), $\Psi'(x) = f(x)$

$$\Psi(x) = \int_0^x f(s)ds + c \tag{1.21}$$

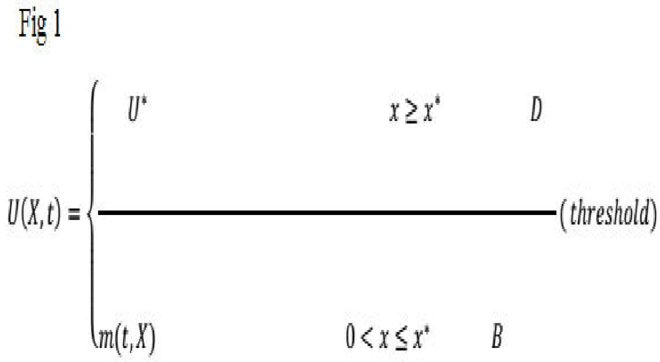
For $x \in (0, \infty)$. Hence

$$U(s,x) = \begin{Bmatrix} x^\alpha & 0 < x \leq x^* \\ \int_0^x f(s)ds + c & x^* < x \leq x_0 \\ \int_0^x f(s)ds + c + (x - x^*) & x \geq x^* \end{Bmatrix} \tag{1.22}$$

$x \neq \infty, x \in (0, \infty)$. Equation (1.22) presents the general solution or general case, subsequently we examine particular cases for Brownian motion and Geometric Brownian motion. $f(s)$ can be given explicitly when the system is a Brownian motion and Geometric Brownian motion. $\Psi(x)$ must converge. We have equation (1.19) because we have a singular control problem, the function is not absolutely continuous over the interval.

Two thresholds x^* and x_o are involved in this model. Which threshold is above or below is determined by conditions on $\Psi(x)$. If the thresholds coincide i.e. $x^* = x_0$, then we have strictly an optimal control problem or strictly an optimal stopping time problem. The Control is flat (not applied) while in the D domain (See fig. 1). It is applied at the threshold to ensure the system does not fall out of order e.g. A financial institution does not go bankrupt by paying dividend, fish population does not become extinct by over harvesting, an investor investing in an oil field project does not invest at a loss.

Figures 2 and 3 below illustrate position of the thresholds



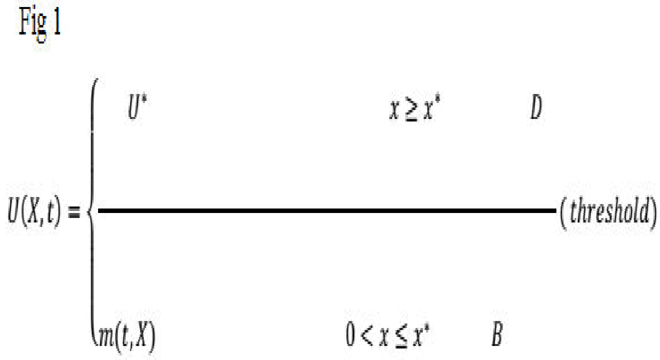


Fig 2: Threshold(price) that determines stopping time, attained before threshold (price) that determines when to apply control

Fig 3: Threshold(price) that determines when to apply control, attained before threshold (price) that determines stopping time

3.7 *Continuity and differentiability of $\Psi(x)$ at x_0 and x^**

Continuity at $\mathbf{x} = \mathbf{x}^*$ and differentiability at $\mathbf{x} = \mathbf{x}^*$ are established but not given here.

From equation (1.19)

$$\Psi(x) = \Psi(x^*). \tag{1.23}$$

$$\Psi(x^*) = x^{*\alpha}. \tag{1.24}$$

Then equation (1.24)

$$\Psi'(x^*) = \alpha x^{*\alpha-1}. \tag{1.25}$$

Dividing Equation (1.24) by Equation (1.25), we obtain

$$x^* = \alpha\frac{\Psi(x^*)}{\Psi'(x^*)}. \tag{1.26}$$

Similarly,

$$x_0 = \alpha\frac{\Psi(x_o)}{\Psi'(x_o)}. \tag{1.27}$$

The expressions $\Psi(x^*)$ and $\Psi(x_0)$ are obtained from differentiability and continuity at $x = x^*$ and $x = x_0$ respectively. From (1.26) and (1.27) x^* and x_0 can be obtained explicitly for particular cases of Brownian motion, BM and Geometric Brownian motion, GBM. The emerging thresholds must be unique.

Since $\Psi(x)$ is given as an integral, Continuity and differentiability of $\Psi(x)$ has been established using Riemann integration, fundamental theorems of Calculus, Order Preserving property of integrals, Leibnitz Integral Rule (for differentiation under the integral sign). Proof is lengthy and not given here

3.8 *Uniqueness and position of the thresholds*

Uniqueness of x* and x_o have been established but not given here.

Position of the thresholds

The position of the threshold is determined by the following inequalities, $\Psi(x^*) < \Psi(x_0)$ or $\Psi(x^*) > \Psi(x_0)$. This determines which threshold is above or below the other. If $\Psi(x^*) - \Psi(x_0) < 0$, then $\Psi(x^*) < \Psi(x_0)$. Conversely, if $\Psi(x^*) - \Psi(x_0) > 0$ then $\Psi(x^*) > \Psi(x_0)$.

3.9 *Existence of the integral; existence of solution*

From solution of the PDE, equation (1.21)

$$\Psi(x) = \int_0^x f(s)ds \qquad x \neq \infty \tag{1.28}$$

Existence of (1.28) above implies existence of the solution. Then we can obtain $\Psi(x_o)$ and $\Psi(x^*)$ and by implication x_o and x^* for this general case. (with appropriate boundary and initial conditions.). Other conditions that guarantee existence of solution are given as follows. (i) $f(x)$ must be continuous on the bounds of the integral. (ii) Solution must converge. (iii) It is given that there are no jumps at the initial process, we expect jumps at some point in time $X(0^-) = x$. Then we may give the optimal strategy for investment for the general case as follows:

- Stop immediately if $0 \leq x \leq x^*; x = x^*$ or $x = 0$. (This includes stopping abruptly).
- Do nothing if $x \geq x^*$ (ought to have stopped investment already)
- Start investing at x_0, if $x_0 \leq x < x^*$

4. Particular cases

4.1 *Optimal strategy for Brownian motion*

For the Brownian motion, σ, μ are constants

$$\frac{1}{2}\sigma^2\Psi''(x) + \mu\Psi'(x) = -1 \tag{1.29}$$

σ, μ are constants. $x \in (0, \infty)$. Let $\Psi'(x) = P(x)$. Then (1.29) becomes

$$P'(x) + \frac{2}{\sigma^2}\mu P(x) = \frac{-2}{\sigma^2} \tag{1.30}$$

with $I.F = e^{2\int \frac{\mu}{\sigma^2}dx} = e^{\frac{2\mu}{\sigma^2}x}$. Then we obtain the solution for $\Psi(x)$ as

$$\Psi(x) = \frac{-x}{\mu} + c.\frac{\sigma^2}{-2\mu}e^{\frac{-2\mu x}{\sigma^2}} + k_2.$$

Since C, σ and μ are constants, let $c\dfrac{\sigma^2}{-2\mu}$ be k_1. Then

$$\Psi(x) = \frac{-x}{\mu} + k_1 e^{\frac{-2\mu x}{\sigma^2}} + k_2 \tag{1.31}$$

This explicit solution, (1.31) is the same result obtained, when σ, μ are substituted as constants in equation (1.20). Hence for the Brownian Motion case

$$U(s,x) = \begin{cases} x^\alpha & 0 < x \leq x^* \\ \dfrac{-x}{\mu} + k_1 e^{\frac{-2\mu x}{\sigma^2}} + k_2 & x^* < x \leq x_0, x_0 \leq x < x^* \\ \dfrac{-x}{\mu} + k_1 e^{\frac{-2\mu x}{\sigma^2}} + k_2 + (x - x^*) & x \geq x^* \end{cases} \tag{1.32}$$

Optimal strategy is specified as follows:

- **Stop immediately if** $0 < x \leq x^*$ i.e. $x = x^*$ or $x = 0$
- **Do nothing if** $x > x^*$
- **Start producing and selling copiously at** $x = x_0$, if $x_0 \leq x < x^*$

Recall equation (1.26) $x^* = \alpha\dfrac{\Psi(x^*)}{\Psi'(x^*)}$ and equation (1.27) $x_0 = \alpha\dfrac{\Psi(x_0)}{\Psi'(x_0)}$.

For a Brownian motion x^* and x_0 are obtained explicitly from equations (1.26) and (1.27), since α is a fixed constant and $\alpha > 0$ given, k_2 is to be obtained by extra initial conditions.

4.2 *Optimal strategy for geometric Brownian motion*

For the Geometric Brownian motion, $\mu = x, \sigma = x^2$

$$\frac{1}{2}x^2\Psi''(x) + x\Psi'(x) = -1$$

Let $m = \Psi' = \frac{d\Psi}{dx}$Then. $\Psi'' = \frac{dm}{dx} = \frac{d^2\Psi}{dx^2} \Rightarrow \frac{dm}{dx} + \frac{2}{x}m = -2x^{-2}$. With $IF = e^{\int \frac{2}{x}dx} = e^{2lnx} = x^2$ we obtain the solution for $\Psi(x)$ as

$$\Psi = -2lnx - \frac{C}{x} + D.$$

C and D are constants to be determined using initial conditions. For given initial conditions, the constants C and D can be obtained leading to explicit solution for $\Psi(x), x^*$ and x_0. These initial conditions could be project-specific information, in addition to crude oil price data for particular oil fields during a given period. We now have the optimal strategy for investment for the GBM case.

$$U(s,x) = \begin{cases} x^{\alpha} & 0 < x \leq x^* \\ -2lnx - \dfrac{C}{x} + D & x^* < x \leq x_0, x_0 \leq x < x^* \\ \\ -2lnx - \dfrac{C}{x} + D + (x - x^*) & x \geq x^* \end{cases} \tag{1.33}$$

To obtain optimal strategy for the Jump-diffusion case, equation (1.13) will be used, jumps will be included in equation (1.14). Thus the analysis and method of solution will include jumps in obtaining $\Psi(x), x^*$ and x_0. Work continues to obtain precise optimal strategy in this case.

5. Conclusion

In this work, an optimal strategy for investment in oil field project was obtained. The model used in the work considers oil price as a jump-diffusion process. The strategy involves two important thresholds, one that determines the stopping time and the other which determines when to apply the control. Running cost is given as a constant cost which is not discounted in time. The results obtained so far and some particular cases have been presented. Conditions for existence of solution were given. The Brownian motion process and Geometric Brownian Motion process were used as particular cases, for which the work obtained explicit solutions, and distinct optimal strategy. BM and GBM cases provide strategy for investment which is good enough, however, it is expected that the best strategy to invest will be attained when jumps are included in the result. Work therefore continues to obtain possibly explicit optimal strategy for the Jump-diffusion case, and to validate the optimal strategy using empirical data and project information from fields in the Niger-Delta. Also running cost may be considered as a constant cost which is discounted in time

References

[1] Bard Bringedal (2003) "Valuation Of Gas Storage: A real Options Approach": Master Thesis: Department Of Industrial Economics And Technology Management. NTNU

[2] Benkherouf Lakedere and Pitts Susan (2005) "On a multidimensional oil exploration problem" Journal of Applied Mathematics and Stochastic Analysis 2005 Vol 2 p 97 - p 118.

[3] Benkherouf, L. (1990) "Optimal Stopping in Oil Exploration with Small And Large Oilfields". Probability in the Engineering and Informational Sciences, Vol. 4, P. 399 - 411.

[4] Benkherouf, L., Bather, J.A. (1998): "Oil Exploration: Sequential Decisions Decisions in the Face of Uncertainty", Journal of Applied Probability, V, 25, P. 529 - 543.

[5] Helmurt Maurer and Willi Semmler (2010) "An Optional Control model of oil discovery and extraction" Applied Mathematics and Computation Vol 217 p 1163- p 1169.

[6] Ogbogbo C. P. (2016), "A stochastic Model of crude oil spot Price as Jump- diffusion process". A Ph.D thesis in the Department of Mathematics, Faculty of Science, University of Ibadan. Nigeria.

[7] Oksendal Bernt and Sulem Agnes (2009), "Applied Stochastic Control of Jump Diffusions". Springer. Third Ed.

[8] Yong J; and Zhou X.Y. (1999) Dynamic Programming and the Hamilton -Jacobi- Bellman equation" In: stochastic controls, SpringerLink. link.springer.com/../10.1007% 2F9 78-1- chapter.