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In honour of Prof. Ekhaguere at 70

Autoregressive time series modeling with asymmetric error innovations

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Abstract. Dynamic time series models were developed with the regular white noise assumption of the error terms. This has often time led to unreliable estimates of parameters and unrealistic forecast performance when the underlying data are not normally distributed. This paper therefore developed an Autoregressive model of order 2 [AR(2)] with Power-Exponential and Lognormal error innovations. The parameters of AR(2) with asymmetric error innovations were derived using Maximum Likelihood Estimation technique and its performance over the normal error innovations compared using the Akaike information Criterion (AIC) and forecasts performance criteria (the RMSE and MAE). Simulated data of various sample sizes and real data sets were used to validate the models. The results show that AR(2) with lognormal and exponential power error innovations are more appropriate and more efficient in modeling non normal time series data.

Keywords: maximum likelihood estimation, autoregressive model, innovations, exponential power, lognormal.

1. Introduction

Most of the robustness theories have concentrated on the development of robust statistical procedures for the case where the observations are independent [20]. He said the theory on robustness in time series setting has received less concentration, which is probably due to the increased technical problems imposed by serial dependence in the data.

Martin [15] gave an overview of robust methods for time series. Some of the important developments of robust statistical procedures in the time series setting may be found in [11, 15, 16]. Sanjoy [20] in his paper developed some robust methods for analyzing real time series data, where ordinary classical methods fail to give satisfactory results in the presence of influential observations.

Kuersteiner [12, 13] developed efficient instrument variable estimators for autoregressive and moving average (ARMA) models and autoregressive model of finite order AR(p). Goncalves and Kilian [4] used bootstrapping method to make robust inference in AR(p) and AR(1) model with unknown conditional heteroscedasticity. These methods and results rely on the assumption that conditional variance of error is constant over time. Unconditional homoscedasticity seems unrealistic in practice, especially in view of the recent emphasis in the empirical literature on structural change modelling for economic time series. To accommodate model when there are a finite number of step changes, [22] investigated the AR (1) model when there are a number of step changes at unknown time point in the error variance. These authors used iterative maximum likelihood method to locate the change points and then estimated the error variance in each block by averaging the squared least squares residuals. The resulting feasible weighted least square was shown to be efficient for the specific model considered. Alternative methods to detect step changes in the variances of time series model have been studied by [1, 21, 18, 14, 2, 3].

In practice, the pattern of variance changes over time, which may be discrete or continuous, is known to the econometrician and it seems desirable to use methods which can adapt for a wide range of possibilities. Accordingly, this research work attempts to develop a robust procedure to estimate parameters of autoregressive model AR(2) in the presence of assumption violation.

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2. Theory and methods

Robust statistics is concerned with the fact that many assumptions commonly made in statistics are not exactly true; they are mathematically convenient rationalizations of an often fuzzy knowledge or belief ([9] cited in [20]). The important pioneering work of [7] on the robust estimation of a location parameter is considered to be the basis for a theory of robust estimation. Some of the robust techniques are discussed in [9, 6]. Huber [10, 8] extended his results on robust estimation of a location parameter to the case of linear regression. Some of the recent research works in robust techniques are discussed as follows.

[22] investigated AR(1) model when there are a finite number of step changes at unknown time points in the error variance. He used iterative Maximum likelihood methods to locate the change point and then estimated the error variance in each block by averaging the squared least squared residual. His result showed to be efficient for the specific model considered.

Gudio [5] developed a new class of instrumental variables (IV) estimators for linear processes and in particular ARMA models. Previously, he used IV estimators based on lagged observations as instruments to account for un-modelled MA (q) errors in the estimation of the AR parameters. In his findings it was showed that these IV methods can be used to improve efficiency of linear time series estimators in the presence of un-modelled conditional heteroskedasticity. He said estimators based on a Gaussian likelihood are inefficient members of the class of IV estimators analysed when the innovations are conditionally heteroskedastic.

[4] used Standard residual-based bootstrap procedures for dynamic regression models and treat the regression error as independent and identically distributed. The procedures are invalid in the presence of conditional heteroskedasticity. Also they proposed three easy-to implement alternative bootstrap for stationary autoregressive processes with Martingale Difference Sequence (M.D.S.) errors subject to possible conditional heteroskedasticity of unknown form. The proposals are; the fixed-design wild bootstrap, the recursive-design wild bootstrap and the pairwise bootstrap. In a simulation study, it was found out that all the three procedures tend to be more accurate in small samples than the conventional large-sample approximation based on robust standard errors. In contrast, standard residual-based bootstrap methods for models with independent and identically distributed errors may be very inaccurate if the assumption is violated. They concluded that in many empirical applications the proposed robust bootstrap procedures should routinely replace conventional bootstrap procedures based on the independent and identically distributed error assumptions.

Phillips and Xu [19] considered stable autoregressive models of known finite order with martingale differences errors scaled by an unknown nonparametric time-varying function generating heterogeneity. He developed kernel-based estimators of the residual variances and associated Adaptive Least Squares (ALS) estimators of the autoregressive coefficients. These are shown to be asymptotically efficient, having the same limit distribution as the infeasible Generalized Least Squares (GLS). Comparisons of the efficient procedure and Ordinary Least Squares (OLS) reveal that least squares can be extremely inefficient in some cases while nearly optimal in others. Simulations show that, when least squares work well, the adaptive estimators perform comparably well, whereas when least squares work poorly, major efficiency gains are achieved by the new estimators.

Wang et. al. described an approach for a robust inference in parametric models that is attractive for time series models. Data from the postulated models were assumed to be measured with sporadic gross errors. It was shown that the tails of the error-contamination model kernel controlled the influence function properties (unbounded, bounded, re-descending), with heavier tails resulting in greater robustness. The method was studied first in location-scale models with independent and identically distributed data, allowing for greater theoretical development. In the application to time series data, they proposed a Bayesian approach and use Markov chain Monte Carlo methods to implement estimation and obtained outlier diagnostics. Simulation results showed that the new robust estimators are competitive with established robust location-scale estimators, and perform well for ARMA (p, q) models.

This research work attempts to use Maximum Likelihood Method in estimating the parameters of autoregressive model in the presence of assumption violation. Also estimate of parameters of AR(2) model with normal, exponential power, and lognormal error innovations will be derived. Simulated and real data were used to validate the models and comparative analysis based on their AIC and forecast performance was carried out.

3. Estimation of parameters

3.1 *Estimation of parameters of AR2 with Exponential power error innovations*

This is also known as generalized normal distribution. It allows β and σ to be any positive real numbers and μ to be any real number. If G is a random variable from a power exponential distribution, its probability density function is given by the following

$$f(g,\mu,\sigma,\beta) = \frac{1}{\sigma\Gamma\left(1+\frac{1}{2\beta}\right)2^{\left(1+\frac{1}{2\beta}\right)}} \exp\left\{-\frac{1}{2}\left|\frac{g-\mu}{\sigma}\right|2\beta\right\} \tag{3.1}$$

$$-\infty < \mu < \infty \ \ and\ \sigma > 0$$

Where σ^2 scale parameter, β is the shape parameter and μ is the location parameter. If X_t follows autoregressive model of order two AR (2), we have

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$$
$$\varepsilon_t = X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}$$

When error is no longer white noise, using AR (2) with power exponential error innovations, we have

$$f(\varepsilon_t) = \frac{1}{\sigma\Gamma\left(1+\frac{1}{2\beta}\right)2^{\left(1+\frac{1}{2\beta}\right)}} \exp\left\{\left|\frac{X_t-\phi_1X_{t-1}-\phi_2X_{t-2}}{\sigma}\right|2\beta\right\} \tag{3.2}$$

Taking the likelihood of equation (3.2), we have

$$\prod_{i=1}^{n} f(\varepsilon_t) = \frac{1}{\sigma^n\Gamma\left(1+\frac{1}{2\beta}\right)^n 2^{n\left(1+\frac{1}{2\beta}\right)}} \exp\sum_{t=3}^{n}\left\{\left|\frac{X_t-\phi_1X_{t-1}-\phi_2X_{t-2}}{\sigma}\right|2\beta\right\}$$

The log likelihood is as follows

$$\log\prod_{i=1}^{n} f(\varepsilon_t) = \log\sigma^{-n}\Gamma\left(1+\frac{1}{2\beta}\right)^{-n} 2^{-n\left(1+\frac{1}{2\beta}\right)+\exp\sum_{t=3}^{n}\left\{\left|\frac{X_t-\phi_1X_{t-1}-\phi_2X_{t-2}}{\sigma}\right|2\beta\right\}}$$

$$= -n\log\sigma - n\log\Gamma\left(1+\frac{1}{2\beta}\right) - n\Gamma\left(1+\frac{1}{2\beta}\right)\log 2 - \frac{1}{2}\sum_{t=3}^{n}\left|\frac{X_t-\phi_1X_{t-1}-\phi_2X_{t-2}}{\sigma}\right|2\beta \tag{3.3}$$

Differentiate (3.3) with respect to ϕ_1, ϕ_2, σ and β we have

$$\frac{\partial\log\prod_{i=1}^{n} f(\varepsilon_t)}{\partial\phi_1} = \frac{\beta}{\sigma}X_{t-1}\sum_{t=3}^{n}\left|\frac{X_t-\phi_1X_{t-1}-\phi_2X_{t-2}}{\sigma}\right|2\beta - 1 = 0 \tag{3.4}$$

$$\frac{\partial\log\prod_{i=1}^{n} f(\varepsilon_t)}{\partial\phi_1} = \frac{\beta}{\sigma}X_{t-2}\sum_{t=3}^{n}\left|\frac{X_t-\phi_1X_{t-1}-\phi_2X_{t-2}}{\sigma}\right|2\beta - 1 = 0 \tag{3.5}$$

Equation (3.4) and (3.5) has no close form but if $\beta = 1$ the solution can be obtained. From equation (3.3)

$$\frac{\partial \log \prod_{i=1}^{n} f(\varepsilon_t)}{\partial \phi_1} = -\frac{n}{\sigma} + \frac{\beta}{\sigma^{2\beta-1}} = \frac{-n\Gamma\left(1+\frac{1}{2\beta}\right)}{\left(1+\frac{1}{2\beta}\right)} + \frac{n\log 2}{2\beta^2} - \sum_{t=3}^{n} \frac{\partial}{\partial \beta}\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| 2\beta \tag{3.6}$$

Multiply both sides by σ

$$\frac{n\sigma}{\sigma} = \frac{\sigma\beta}{\sigma^{2\beta+1}} \sum_{t=3}^{n} |X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}|^{2\beta-1} = 0$$

$$\sigma^{2\beta} = \beta^{\frac{\sum_{t=3}^{n}|X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}|^{2\beta}}{n}} \tag{3.7}$$

When $\beta = 1$, equation(3.7) becomes

$$\sigma^2 = \frac{\sum_{t=3}^{n} |X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}|^{2\beta}}{n}$$

From equation (3.3)

$$\frac{\partial \log \prod_{i=1}^{n} f(\varepsilon_t)}{\partial \beta} = \frac{-n\Gamma'\left(1+\frac{1}{2\beta}\right)}{\Gamma\left(1+\frac{1}{2\beta}\right)} + \frac{n\log 2}{1\beta^2} - \sum_{t=3}^{n} \frac{\partial}{\partial \beta}\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| 2\beta \tag{3.8}$$

From equ.(3.8), let us consider

$$\sum_{t=3}^{n} \frac{\partial}{\partial \beta}\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| 2\beta$$

$$Let\ \ p = \left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| 2\beta \tag{3.9}$$

If log is introduced to both sides of equ (3.9) we have

$$\log p = 2\beta \log\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right|$$

$$\frac{\partial p}{\partial \beta} \times \frac{1}{p} = 2\beta \log\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right|$$

$$\frac{\partial p}{\partial \beta} = 2\beta \log\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| \times p$$

$$\frac{\partial p}{\partial \beta} = 2\beta\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| 2\beta \times \log\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| \tag{3.10}$$

By putting (3.10) into (3.8) we have:

$$\frac{\partial \log \prod_{i=1}^{n} F(\varepsilon_t)}{\partial \beta} = \frac{-n\Gamma'\left(1+\frac{1}{2\beta}\right)}{\left(1+\frac{1}{2\beta}\right)} + \frac{n\log 2}{2\beta^2} - 2\beta\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right|^{2\beta} \times \log\left|\frac{X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}}{\sigma}\right| \tag{3.11}$$

Equation (3.3), (3.4) and (3.11) are solved iteratively using numerical method to obtain maximum likelihood estimates of β, ϕ_1 and ϕ_2 because there is no close form solution for the parameters.

Equation (3.7) has been solved analytically to obtain σ^2. When $\beta = 1$, it becomes $\sigma^2 = \frac{\sum_{t=3}^{n}|X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2}|^2}{n}$ i.e. the variance of AR(2) with normal error term.

3.2 *Estimation of parameters of AR2 with lognormal error innovations*

In probability theory, a lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable X is log-normally distributed, then $Y = ln(X)$ has a normal distribution.

If x is a random variable from a generalized normal distribution, its probability density function is given by the following

$$f(x;\mu,\sigma) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{\left|\frac{lnX_t - \mu}{\sigma}\right|\right\} \quad -\infty < \mu < \infty \;\; and \;\; \sigma > 0 \tag{3.12}$$

Where σ^2 is the scale parameter and is the location parameter, x is a random variable. If X_t follows autoregressive model of order 2 AR(2), we have

$$\begin{aligned} X_t &= \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t, \\ lnX_t &= \phi_1 lnX_{t_1} + \phi_2 lnX_{t-2} + ln\, ln\varepsilon_t \end{aligned} \tag{3.13}$$

Let $lnX_t = X_t^*$

$$\begin{aligned} X_t^* &= \phi_1 X_{t-1}^* + \phi_2 X_{t-2}^* + \varepsilon_t,^* \\ \varepsilon_t^* &= X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^* \end{aligned} \tag{3.14}$$

$$f(\varepsilon_t^*) = \frac{1}{X_t\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left|\frac{\varepsilon_t^*}{\sigma}\right|^2\right\} \tag{3.15}$$

Put (3.14) into (3.15), we have

$$f(\varepsilon_t^*) = \frac{1}{X_t\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left|\frac{X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*}{\sigma}\right|^2\right\} \tag{3.16}$$

By taking the likelihood function of equation (3.16), we have

$$L(\varepsilon_t) = \frac{(2\pi\sigma^2)^{-\frac{n}{2}}}{\prod_{t=1}^{n}} - e^{-\frac{1}{2\sigma^2}\sum_{t=3}^{n}(X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2})^2} \tag{3.17}$$

By taking the Log likelihood function of (3.17), we have

$$\log L(\varepsilon_t) = -\frac{n}{2}\log(2\pi\sigma^2) - \sum(\log X_t) - \frac{1}{2}\sum_{t=3}^{n}\left|\frac{X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*}{\sigma}\right|^2 \tag{3.18}$$

By differentiating (3.18) with respect to σ^2, ϕ_1 and ϕ_2 we have

$$\frac{\partial L(\varepsilon_t)}{\partial\sigma^2} = -\frac{n}{2}\left(\frac{2\pi}{2\pi\sigma^2}\right) + \frac{1}{2(\sigma^2)^2}\sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|^2$$

$$-\frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|^2 = 0 \tag{3.19}$$

$$\frac{n}{2\sigma^2} = \frac{1}{2(\sigma^2)^2}\sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|^2$$

Multiply through by $\frac{2\sigma^4}{n}$

$$\sigma^2 = \sum_{t=3}^{n} \frac{\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|^2}{n} \tag{3.20}$$

$$\begin{aligned}
\frac{\partial L(\varepsilon_t)}{\partial \phi_1} &= -\frac{2}{2\sigma^2}\left[-\sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right| X_{t-1}^*\right] = 0 \qquad (3.21)\\
&= \sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right| X_{t-1}^*\\
&= \left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right| X_{t-1}^* = 0\\
\phi_1 \sum_{t=3}^{n} (X_{t-1}{}^*)^2 &= \sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|\\
\phi_1 &= \frac{\sum\limits_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|}{\sum\limits_{t=3}^{n}\left(X_{t-1}^*\right)^2} \qquad (3.22)
\end{aligned}$$

Similarly,

$$\frac{\partial L(\varepsilon_t)}{\partial \phi_2} = -\frac{2}{2\sigma^2}\left[-\sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right| X_{t-2}^*\right] = 0 \tag{3.23}$$

$$\phi_2 = \frac{\sum_{t=3}^{n}\left|X_t^* - \phi_1 X_{t-1}^* - \phi_2 X_{t-2}^*\right|}{\sum\limits_{t=3}^{n}\left(X_{t-2}^*\right)^2} \tag{3.24}$$

Substituting equation (3.24) into (3.23) to obtain estimate of ϕ_1 we have

$$\phi_1 \sum_{t=3}^{n}(X_{t-1}{}^*)^2 = \left\{\sum_{t=3}^{n} X_t{}^* X_{t-1}{}^* - \left[\left(\frac{\sum_{t=3}^{n}\left|X_t^* X_{t-2}{}^* - \phi_1 X_{t-2}^* X_{t-1}^*\right|}{\sum\limits_{t=3}^{n}\left(X_{t-2}^*\right)^2}\right) X_{t-2}{}^* X_{t-1}{}^*\right]\right\}$$

$$\phi_1 \sum_{t=3}^{n}(X_{t-1}{}^*)^2 \sum_{t=3}^{n}(X_{t-2}{}^*)^2 = \left\{\sum_{t=3}^{n} X_t{}^* X_{t-1}{}^* \sum_{t=3}^{n}(X_{t-2}{}^*)^2 \left[\sum_{t=3}^{n}\left|X_t^* X_{t-2}^* X_{t-1}^* + \phi_1 (X_{t-2}^* X_{t-1}^*)^2\right|\right]\right\}$$

$$\phi_1 \sum_{t=3}^{n}(X_{t-1}{}^*)^2 \sum_{t=3}^{n}(X_{t-2}{}^*)^2 - \phi_1 (\sum_{t=3}^{n}(X_{t-2}{}^*)^2 \sum_{t=3}^{n}(X_{t-1}{}^*)^2) =$$

$$\left\{\sum_{t=3}^{n} X_t^* X_{t-1}^* \sum_{t=3}^{n}(X_{t-2}{}^*)^2 - \left[\sum_{t=3}^{n}\left|X_t^* X_{t-2}^* X_{t-2}^* X_{t-1}^*\right|\right]\right\}$$

$$\phi_1 = \frac{\sum\limits_{t=3}^{n} X_t^* X_{t-1}^* \sum\limits_{t=3}^{n}(X_{t-2}^*)^2 - \sum\limits_{t=3}^{n}\left|X_t^* X_{t-2}^* X_{t-2}^* X_{t-1}^*\right|}{\sum\limits_{t=3}^{n}(X_{t-1}^*)^2 \sum\limits_{t=3}^{n}(X_{t-2}^*)^2 - \phi_1 (X_{t-2}^* X_{t-1}^*)^2} \tag{3.25}$$

The above derivation has a closed form solution therefore the estimates can be obtained using the equations (3.20), (3.24) and (3.25).

4. Results and discussions

The summary statistics of the 180 data points were calculated and plotted in charts and diagrams as a form of data cleaning exercise given in table (3.1) below. The data used in validating these

Table 1. Descriptive Statistic

MIN	1st QU	MEDIAN	MEAN	3rd QU	MAX
96.96	107.8	118.5	128	142.6	216.6

models is import commodity price index obtained from Central Bank of Nigeria Statistical Bulletin. AIC/BIC criterion was used to determine the suitable order for the model as seen in Table (2) below. Table (3) with p-value $2.365e-12$,

Table 2. Order Determination Criterion

AR	**AIC**	**BIC**
1	985.5	991.9
2	947.6	957.2
3	949.6	962.4
4	950.4	966.4
5	952.1	971.2
6	952.7	975
7	952.9	978.5

Source: Source: R statistics software

Table 3. Shapiro-Wilk Normality Test

Name of the test:	**Shapiro-Wilk Normality Test**
Data:	180
Test statistic:	0.84935
P-value:	2.37E-12

Figure 1 and Figure 2 show that some values stand out in the data set which indicates that there are outliers in the data set.

Table 4. Estimation of parameter of AR (2) with normal error innovations

Coefficient	**Estimate**	**Standard error**	**t-value**	$Pr(>t)$
ϕ_1	0.4662	0.0666	7.0004	2.552e-12 ***
ϕ_2	0.4606	0.0671	6.7175	1.849e-11 ***

log likelihood = -726.19 : AIC = 1460.38

Signif. codes: 0 '***'0.001 '**'0.01 '*'0.05 '.'0.1 ''1

Source: R Statistics software

Table 5. Estimation of parameter of AR(2) with power exponential error innovations

Coefficient	**Estimate**	**Standard error**	**t-value**	$Pr(>t)$
ϕ_1	0.4994	0.2288	2.182	0.0291
ϕ_2	0.5006	0.2288	2.188	0.0289

log likelihood = -145.8869 : AIC = 301.7738

Signif. codes: 0 '***'0.001 '**'0.01 '*'0.05 '.'0.1 ''1

Source: R Statistics software

Tables (4) to (6) presented the estimate of parameters of Auto-Regressive model of order two and AIC of each model. It was shown that Log normal with AIC = -674.384 appear to be the best

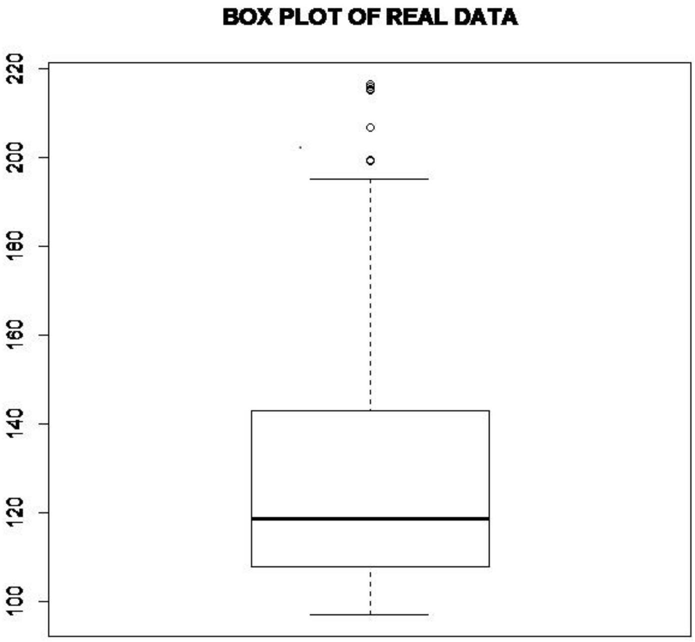

Figure 1. Box plot of price index of import commodity in Nigeria between 2000 and

Table 6. Estimation of parameter of AR(2) with log normal error innovations

Coefficient	**Estimate**	**Standard error**	**t-value**	$Pr(>t)$
ϕ_1	0.4687469	0.0082090	57.10	$< 2e - 16$
ϕ_2	-0.4310135	0.0082090	-52.51	$< 2e - 16$

log likelihood = 890.6804: AIC = -674.384

Signif. codes: 0 '***'0.001 '**'0.01 '*'0.05 '.'0.1 ''1

Source: R Statistics software

model with minimum AIC, this is followed by AR2 with power exponential error innovation with AIC = 1446.328, normal error innovation has the least AIC = 1460.38. It could be deduced that Log normal and power exponential distribution is superior to normal distributions in terms of dynamic model fitting.

Table 7. Summary of Results

Distribution	**AIC**	**Log lik**
Normal	1460.38	-726.19
Power exponential	1446.33	-719.16
Log normal	-674.38	340.192

From Table (7), it was observed that Log normal error innovation performed best followed by power exponential error innovations and normal error innovation perform least with non-normal data judging from their AIC.

5. Forecast performance

From Table (8), the error measures of each indicated that power exponential error innovations is the most efficient model for forecasting with the least RMSE and MAE followed by Log-normal error innovation while normal error innovation is the least efficient judging from the error measures.

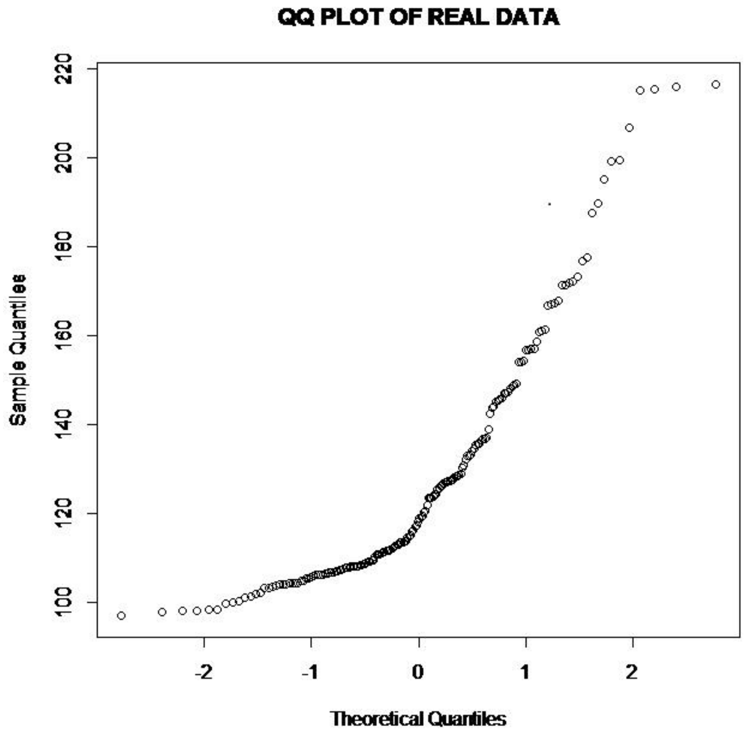

Figure 2. QQ plot of price index of import commodity in Nigeria between 2000 and

Table 8. Error Measures

INNOVATIONS	RMSE	MAE
NORMAL	7.6795	5.1828
POWER EXPONENTIAL	7.0418	4.7574
LOG NORMAL	7.2688	4.767

RMSE → Root Mean Square Error
MAE→ Mean Absolute Error

6. Conclusion

The focus of this paper was to develop an Autoregressive model of order 2 [AR(2)] with Power-Exponential and lognormal error innovations. The parameters of AR(2) model with asymmetric error innovations were derived using Maximum Likelihood Estimation technique, and the performance of the model over the normal error innovations was compared using the Akaike Information Criterion (AIC) and forecasts performance criteria (the RMSE and MAE). Based on these criteria, the results showed that AR(2) models with lognormal and exponential power error innovations are more appropriate and efficient in modelling non normal time series processes.

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