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***In honour of Prof. Ekhaguere at 70***

# Estimators of finite population mean in two-phase sampling under transformation of sample means

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**Abstract.** Large sample provides adequate information for estimation but the cost and time involved are relatively high. In this paper, information on the large sample yet to be drawn has been used to improve Khan et al (2012) estimators for estimating population mean in two phase sampling. The MSE and PRE of proposed estimators were obtained. The efficiency of the proposed estimators was compare to some existing estimators of population mean theoretically and empirically and the results show that the proposed estimators are more efficient.

**Keywords:** bias, ratio estimator, mean square error, product estimator, precision.

## 1. Introduction

The role of auxiliary information is of prime importance in sampling theory. In survey sampling auxiliary variables are commonly used in order to obtain improved designs and to achieve higher precision in the estimates of some population parameters such as population total, population mean, population proportion, population ratio. In case when the relationship between auxiliary and study variable is positive then ratio estimation is suggested. However, product method of estimation is usually considered, when the correlation coefficient between study and auxiliary variable is negative. In most of the survey situations the auxiliary information is always available. It may either be promptly available or may be collected without much difficulty by averting a part of survey resources. In sample surveys it is also a usual practice to look for information on auxiliary variables which are either available from official records or can be collected inexpensively in the course of investigation. In the case of single auxiliary variable the ratio estimator and the regression estimator are two classical estimators making use of the auxiliary information to improve the efficiency of the finite population parameters such as population mean, total, variance, etc.

The work of Neyman (1938) may be referred to as the initial work where the use of auxiliary information has been established. The development is continue by using the ratio form of the ratio and product type estimators such as Bahl and Tuteja (1991), Noor ul amin and Hanif (2012) and many other researchers in sample survey.

Consider the finite population $U = U_1, U_{,2} ...U_n$ of size N. Let y and (x,z) be the variate of interest and auxiliary characteristics respectively related to y assume real non-negative $i^{th}$ value $(y_i, x_i, z_i)$ $i = 1, 2, 3..., N$ with population mean $\bar{Y}$, $\bar{X}$ $and$ $\bar{Z}$ respectively. Let a simple random sample without replacement (SRSWOR) is drawn in each phase, the two phases (or double) sampling scheme is as follows:
CASE-1: The first phase sample $S_1(S_1 \subset U)$ of size $n_1$ is drawn to measure x and z say $(x_1, z_1)$.
CASE-2 : The second phase sample $S_2(S_2 \subset S_1)$ of size $n_2$ is drawn from the first phase sample $S1$ to measure y say $y_2$.

Let $\bar{x}_1 = 1/n_1 \Sigma_{i \in s_1} x_i$, $\bar{x}_2 = 1/n_2 \Sigma_(i \in s_2) x_i$, $\bar{z}_1 = 1/n_1 \Sigma_(i \in s_1) z_i$, $\bar{z}_2 = 1/n_2 \Sigma_(i \in s_2) z_i$, $\bar{y}_2 = 1/n_2 \Sigma_(i \in s_2) y_i$. Singh and Espejo (2007) suggested a class of ratio-product estimators in two-phase

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sampling for population mean in the presence of two-auxiliary variables and also discussed their properties. The proposed estimator.

$$\bar{Y}_{SE} = \bar{y}^*\{b\frac{\bar{x}'}{\bar{x}^*} + (1-b)\frac{\bar{x}^*}{\bar{x}'}\} \tag{1}$$

$$Mse(\bar{Y}_S E) = \lambda' C_y^2 + (\lambda - \lambda')\{S_y^2 + \frac{D^*}{D}(\frac{D^*}{D - 2\beta_y x})S_x^2\} + \lambda^*\{S_{y2}^2 + \frac{D^*}{D}(\frac{D^*}{D - 2\beta_y x2})S_x^2 2\} \tag{2}$$

Samiuddin and Hanif (2007) proposed estimator:

$$\bar{Y}_S H = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right]^1 \left[\frac{\bar{z}_1}{\bar{z}_2}\right]^2 \left[\frac{\bar{Z}}{\bar{z}_2}\right]^2 \tag{3}$$

$$Mse(\bar{Y}_S H) = \bar{Y}^2 {C_y}^2 \left[\theta_2(1 - \rho_(y.xz)^2) + \theta_1(1 - \rho_y z^2)\rho_(yx.z)^2\right] \tag{4}$$

Singh et al.(2004) generalised estimator:

$$\bar{Y}_S el = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right]^{\alpha_1} \left[\frac{a\bar{Z} + b}{a\bar{z}_1 + b}\right]^{\alpha_2} \left[\frac{a\bar{Z} + b}{a\bar{z}_2 + b}\right]^{\alpha_3} \tag{5}$$

$$Mse(\bar{Y}_S el) = \bar{Y}^2 {C_y}^2 \left[\theta_2 - \theta_1[\rho_y z]^2 - (\theta_2 - \theta_1)\right]\left[\rho_(y.xz)\right]^2 \tag{6}$$

Khan et al (2012) proposed estimators in two phase (double) sampling are as follows:

$$\bar{Y}_([Kel]_1) = \bar{y}_2 \left(\frac{\bar{x}_1}{\bar{x}_2}\right)^\alpha \left(\frac{a\bar{Z} + b}{a\bar{z}_1 + b}\right) \tag{7}$$

$$Bias(\bar{Y}_([Kel]_1)) = \bar{Y}\left[-\frac{\theta_3}{2}[C_y]^2[\rho_y x]^2 + \frac{\theta_3}{2}C_x(C_y\rho_x y - \frac{C_x}{4}) - \frac{\theta_1}{4}[C_y]^2[\rho_y z]^2\right] \tag{8}$$

where $\theta_3 = \theta_2 - \theta_1$

$$min.Mse(\bar{Y}_([Kel]_1)) = \bar{Y}^2[C_y]^2\left[\theta_2 - \theta_1[\rho_x y]^2 - \theta_3[\rho_y z]^2\right] \tag{9}$$

$$\bar{Y}_([Kel]_2) = \bar{y}_2\left[\frac{\bar{z}_1}{\bar{z}_2}\right]^\alpha\left[\delta\frac{(a\bar{X} + b)}{(a\bar{x}_1 + b)} + (1-\delta)\frac{(a\bar{x}_1 + b)}{(a\bar{X} + b)}\right] \tag{10}$$

$$Bias(\bar{Y}_([Kel]_2)) = \bar{Y}\left[\frac{\theta_3}{2}\left(C_y\rho_y z - \frac{C_z}{2}\right)^2\right] \tag{11}$$

$$min.Mse(\bar{Y}_([Kel]_2)) = \bar{Y}^2[C_y]^2\left[\theta_2 - \theta_1[\rho_x y]^2 - \theta_3[\rho_y z]^2\right] \tag{12}$$

## 2. Proposed estimators

Having studied some of the above existing estimators of population mean, we define the following transformations:

$$\bar{x}_1^* = \frac{\bar{X} - f_1\bar{x}_1}{1 - f_1},\ \bar{x}_2^* = \frac{\bar{X} - f_2\bar{x}_2}{1 - f_2},\ \bar{z}_1^* = \frac{a_z\bar{Z} - f_1\bar{z}_1}{1 - f_1} + b_z\ ,\ \bar{z}_2^* = \frac{\bar{Z} - f_2\bar{z}_2}{1 - f_2}$$

The following estimators based on the sample yet to be drawn $S_{N-n}$ of size $N-n$ are suggested.

$$\bar{y}_{A1} = \bar{y}_2\left(\frac{\left(\bar{X} - f_1\bar{x}_1\right)(1 - f_2)}{\left(\bar{X} - f_2\bar{x}_2\right)(1 - f_1)}\right)^{-\lambda_1}\left(\frac{\left(a_z\bar{Z} + b_z\right)(1 - f_1)}{a_z\left(\bar{Z} - f_1\bar{z}_1\right) + b_z(1 - f_1)}\right) \tag{13}$$

$$\begin{gathered}\bar{y}_{A2} = \bar{y}_2\left(\frac{\left(\bar{Z}-f_1\bar{z}_1\right)(1-f_2)}{\left(\bar{Z}-f_2\bar{z}_2\right)(1-f_1)}\right)^{-\lambda_2}\left(w\left(\frac{\left(a_x\bar{X}+b_x\right)(1-f_1)}{a_x\left(\bar{X}-f_1\bar{x}_1\right)+b_x(1-f_1)}\right)\right). \\ +(1-w)\left(\frac{a_x\left(\bar{X}-f_1\bar{x}_1\right)+b_x(1-f_1)}{\left(a_x\bar{X}+b_x\right)(1-f_1)}\right)\end{gathered} \tag{14}$$

where $\lambda_1$, $\lambda_2$ and $\omega$ are the unknown constants whose values are to be estimated. $a(\neq 0)$,and $b$ are assumed to be known as either real numbers or functions of some known parameters of auxilliary variable x.

## 3. Properties of proposed estimators (bias and mean square error)

In order to study the properties of the proposed estimator, we define the following error terms $e_0 = \left(\bar{y}_2 - \bar{Y}\right)/\bar{Y}$ , $e_1 = \left(\bar{x}_1 - \bar{X}\right)/\bar{X}$ , $e_2 = \left(\bar{x}_2 - \bar{X}\right)/\bar{X}$ , $e_3 = \left(\bar{z}_1 - \bar{Z}\right)/\bar{Z}$ such that $|e_0| < 1$ , $|e_1| < 1$ , $|e_2| < 1$ , $|e_3| < 1$

Under case I

$$\left.\begin{array}{l}\bar{x}_1 = \bar{X}(1 + e_1),\ \ \bar{z}_1 = \bar{Z}(1 + e_3),\ \bar{y}_2 = \bar{Y}(1 + e_0),\ \bar{x}_2 = \bar{X}(1 + e_2) \\ E(e_1) = E(e_3) = E(e_0) = E(e_2) = 0,\ E\left(e_0^2\right) = \theta_2 C_y^2,\quad E\left(e_1^2\right) = \theta_1 C_x^2, \\ E\left(e_3^2\right) = \theta_1 C_z^2,\ E\left(e_2^2\right) = \theta_2 C_x^2,\ E(e_0e_1) = \theta_1\rho_{xy}C_yC_x,\ E(e_1e_3) = \theta_1\rho_{yz}C_yC_z, \\ E(e_0e_2) = \theta_2\rho_{xy}C_yC_x,\ E(e_1e_3) = \theta_1\rho_{xz}C_xC_z,\ E(e_1e_2) = \theta_1 C_x^2,\ E(e_2e_3) = \theta_1\rho_{xz}C_xC_z \\ \theta_1 = \frac{1}{n_1} - \frac{1}{N},\ \theta_2 = \frac{1}{n_2} - \frac{1}{N},\ \theta_3 = \theta_2 - \theta_1\end{array}\right\} \tag{15}$$

Under case II

$$\left.\begin{array}{l}\bar{x}_1 = \bar{X}(1 + e_1),\ \ \bar{z}_1 = \bar{Z}(1 + e_3),\ \bar{y}_2 = \bar{Y}(1 + e_0),\ \bar{x}_2 = \bar{X}(1 + e_2) \\ E(e_1) = E(e_3) = E(e_0) = E(e_2) = 0,\ E\left(e_0^2\right) = \theta_2 C_y^2,\quad E\left(e_1^2\right) = \theta_1 C_x^2, \\ E\left(e_3^2\right) = \theta_1 C_z^2,\ E\left(e_2^2\right) = \theta_2 C_x^2,\ E(e_0e_1) = 0,\ E(e_1e_3) = \theta_1\rho_{yz}C_yC_z, \\ E(e_0e_2) = \theta_2\rho_{xy}C_yC_x,\ E(e_1e_3) = \theta_1\rho_{xz}C_xC_z,\ E(e_1e_2) = 0,\ E(e_2e_3) = 0 \\ \theta_1 = \frac{1}{n_1} - \frac{1}{N},\ \theta_2 = \frac{1}{n_2} - \frac{1}{N},\ \theta_3 = \theta_2 - \theta_1\end{array}\right\} \tag{16}$$

Expressing equations (13) and (14) in terms of es to second order approximation as
$\bar{y}_{A_1} = \bar{Y} +\ barY\left[e_0 + \phi e_3 + \phi^2{e_3}^2 - \lambda_1 he_2 - \lambda_1\phi he_2e_3 - \lambda_1 h^2[e_2]^2 + \lambda_1 he_1 + \lambda_1\phi he_1e_3 +\right.$

$$\lambda_1 h^2 e_1 e_2 + \tfrac{1}{2}[\lambda_1]^2 h^2 [e_1]^2 + \tfrac{1}{2}[\lambda_1]^2 h^2 [e_2]^2 - [\lambda_1]^2 h^2 e_1 e_2 \tfrac{1}{2}\lambda_1 h^2 [e_1]^2 + \tfrac{1}{2}\lambda_1 h^2 [e_2]^2 - \lambda_1 h^2 e_1 e_2 + \phi e_0 e_3 - \lambda_1 h e_0 e_3 + \lambda_1 h e_0 e_1$$

Where $\varphi = \frac{(abarZ)}{(abarZ+b)}$

$$\bar{y}_{A_2} = \bar{Y} + \ barY \left[e_0 - \lambda_2 h e_2 - \lambda_2 h^2 {e_2}^2 - \lambda_2 h e_3 + \lambda_2 h^2 e_2 e_3 + \tfrac{1}{2}{\lambda_2}^2 h^2 {e_2}^2 + \tfrac{1}{2}{\lambda_2}^2 h^2 {e_3}^2 - {\lambda_2}^2 h^2 e_2 e_3 + \tfrac{1}{2}\lambda_2 h^2 {e_2}^2 + \tfrac{1}{2}\lambda_2 h^2 {e_3}^2 - \lambda_2 h^2 e_2 e_3 - \lambda_2 h e_0 e_2 + \lambda_2 h e_0 e_3 \phi e_1 + \lambda_2 \phi h e_1 e_2 - \lambda_2 \phi h e_1 e_3 - \phi e_0 e_1 + \omega \phi^2 {e_1}^2 + 2\omega\phi e_1 - 2\lambda_2 \omega h e_1 e_2 + 2\lambda_2 h \phi e_1 e_3 + 2\omega\phi e_0 e_1\right.$$

Where $\varphi = \frac{(a\bar{X})}{(b+a\bar{X})}$. After applying expectation to equations (15) and (16) above and using the definitions above, the biases of the proposed estimators are obtain:

CASE1:

$$Bias(\bar{Y}_{A_1}) = \bar{Y}\left[\theta_1 \emptyset \rho_y z C_y C_z + \theta_1 \emptyset^2 [C_z]^2 + \frac{1}{2}\theta_3 [\lambda_1]^2 h^2 [C_x]^2 + \frac{1}{2}\theta_3 \lambda_1 h^2 [C_x]^2 - \theta_3 \lambda_1 h^2 [C_x]^2 - \theta_3 \lambda_1 h \rho_x y C_x C_y\right. \quad (17)$$

$$Bias(\bar{Y}_{A_2}) = \bar{Y}\left[\tfrac{1}{2}\theta_3 [\lambda_2]^2 h^2 {C_z}^2 - \tfrac{1}{2}\theta_3 \lambda_2 h^2 {C_z}^2 - \theta_3 \lambda_2 h \rho_y z C_y C_z - \theta_1 \phi \rho_x y C_x C_y + \theta_1 \omega \phi^2 [C_x]^2 - 2\theta_1 \lambda_2 \omega h \rho_x y C_x C_y + 2\theta_1 \lambda_2 h \phi \rho_x z C_x C_z + 2\theta_1 \omega \phi \rho_x y C_x C_y\right]$$

CASE 2:

$$Bias(\bar{Y}_{A_1}) = \bar{Y}\left[\theta_1 \lambda_1 \phi h \rho_x z C_x C_z + \phi^2 \theta_1 {C_z}^2 + \tfrac{1}{2}\theta_1 {\lambda_1}^2 h^2 {C_x}^2 + \tfrac{1}{2}\theta_1 \lambda_1 h^2 [C_x]^2 - \theta_2 \lambda_1 h^2 {C_x}^2 - \theta_2 \lambda_1 h \rho_x y C_x C_y + \tfrac{1}{2}\theta_2 \lambda_1 h^2 {C_x}^2 + \tfrac{1}{2}\theta_2 {\lambda_1}^2 h^2 {C_x}^2\right.$$

$$Bias(\bar{Y}_{A_2}) = \bar{Y}\left[\tfrac{1}{2}\theta_2 {\lambda_2}^2 h^2 {C_z}^2 + \tfrac{1}{2}\theta_1 {\lambda_2}^2 h^2 {C_z}^2 - \theta_2 \lambda_2 h^2 {C_z}^2 + \tfrac{1}{2}\theta_1 \lambda_2 h^2 {C_z}^2 + \tfrac{1}{2}\theta_2 \lambda_2 h^2 {C_z}^2 - \theta_2 \lambda_2 h \rho_y z C_y C_z + \theta_1 \lambda_2 h \phi \rho_x z C_x C_z + \theta_1 \omega \phi^2 {C_x}^2\right.$$

Taking square and applying expectation, given in (15) and (16), and ignoring terms of degree greater than two the mean square error (MSE) of the proposed estimators are obtained as:

CASE 1:

$$Mse(\bar{Y}_(A_1)) = \bar{Y}^2[\theta_2 {C_y}^2 + \varphi^2 \theta_1 {C_z}^2 + 2\theta_1 \varphi \rho_y z C_y C_z - \theta_1 {\lambda_1}^2 h^2 {C_x}^2 - 2\theta_3 \lambda_1 h \rho_x y C_x C_y + \theta_2 {\lambda_1}^2 h^2 {C_x}^2] \quad (18)$$

Differentiating(23) with respect to $\lambda_1$ and setting equal zero. we have:

$$\lambda_1 = \frac{(-\rho_x y C_y)}{(hC_x)} \quad (19)$$

$$Mse(\bar{Y}_(A_2)) = \bar{Y}^2\left[\theta_2 {C_y}^2 - 2\theta_3 \lambda_2 h \rho_y z C_y C_z - 2\theta_1 \phi \rho_x y C_x C_y + 4\theta_1 \omega \phi \rho_x y C_x C_y + \theta_3 {\lambda_2}^2 h^2 {C_z}^2 + 4\theta_1 \lambda_2 h \omega \phi \rho_x z C_x C_z + \theta_1 \phi^2 {C_x}^2 - 4\theta_1 \omega \phi^2 {C_x}^2 + 4\theta_1 \omega^2 \phi^2 {C_x}^2 - 2\theta_1 \lambda_2 h \phi \rho_x z C_x C_z\right.$$

Differenting (25) with respect to $\lambda_2$ and setting equal zero. We have:

$$\lambda_2 = \frac{(\phi^2 {C_x}^2 - \phi \rho_x y C_x C_y - 2\omega \phi^2 {C_x}^2)}{(h \phi \rho_x z C_x C_z)} \quad (20)$$

Also differentiating (25) with respect to $\omega$ and making $\lambda_2$ the subject, then equating the result with the value of $\lambda_2$, we have:

$$\omega = \frac{(\theta_3 h^2 \varphi^2 {C_x}^2 {C_z}^2 - \theta_3 h^2 \varphi \rho_x y C_x C_y {C_z}^2 + \theta_3 h^2 \varphi \rho_x z_y z C_x C_y {C_z}^2 + \theta_1 h^2 \varphi^2 \rho_x z {C_x}^2 {C_z}^2)}{(2h^2 \varphi^2 {C_x}^2 (\theta_1 \rho_x z + \theta_3))} \quad (21)$$

CASE 2: $Mse(\bar{Y}_(A_1)) = \bar{Y}^2 \left[\varphi^2 \theta_1 {C_z}^2 + 2\theta_1 \lambda_1 \varphi h \rho_x z C_x C_z - 2\theta_2 \lambda_1 h \rho_x y C_x C_y + \theta_2 {\lambda_1}^2 h^2 [C_x]^2 + \right.$

$$\theta_1 [\lambda_1]^2 h^2 [C_x]^2 + \theta_2 [C_y]^2 \quad (22)$$

Differentiating (28) with respect to $\lambda_1$ and setting equal to zero. We have:

$$\lambda_1 = \frac{(\theta_2 \rho_x y C_y - \theta_1 \varphi \rho_x z C_z)}{((\theta_2 + \theta_1) h C_x)} \quad (22)$$

$$Mse(\bar{Y}_(A_2)) = \bar{Y}^2 \left[\theta_2 {C_y}^2 - \theta_3 \lambda_2 h \rho_y z C_y C_z + \theta_2 [\lambda_2]^2 h^2 [C_z]^2 - 2\theta_1 \lambda_2 h \phi \rho_x z C_x C_z + \right.$$

$$4\theta_1 \lambda_2 h \omega \phi \rho_x z C_x C_z + \theta_1 \phi^2 [C_x]^2 - 4\theta_1 \omega \phi^2 [C_x]^2 + 4\theta_1 \omega^2 \phi^2 [C_x]^2$$

Differentiating (30) with respect to $\lambda_2$ and setting equal zero. We have:

$$\lambda_2 = \frac{(\theta_3 \rho_y z C_y + 2\theta_1 \phi \rho_x z C_x - 4\theta_1 \omega \phi \rho_x z C_x)}{(2(\theta_2 + \theta_1) h C_z)} \quad (23)$$

Also differentiating (30) with respect to $\omega$ and setting equal zero. making $\lambda_2$ the subject of the formula and equating the result with the value of $\lambda_2$ above. We have:

$$\omega = \frac{(\theta_3 h \rho_x z \rho_y z C_y C_z + 2\theta_1 h \phi \rho_x z^2 C_x C_z - 2(\theta_2 + \theta_1) h \phi C_x C_z)}{(4\theta_1 h \phi \rho_x z^2 C_x C_z - 4\theta_1 (\theta_2 + \theta_1) \phi h C_x C_z)} \quad (24)$$

## 4. Empirical analysis

This section particularly deals with numerical analysis based on the data use in this research work in order to see the performances for the proposed estimators in comparisons to some other existing estimators.

### *Population 1: Data used by Anderson (1958)*

Y: Head length of second son, X: Head length of first son, Z: Head breadth of first son
$\mathrm{N} = 25, \bar{\mathrm{Y}} = 183.84, \bar{\mathrm{X}} = 185.72, \bar{\mathrm{Z}} = 151.12, \mathrm{C}_y = 0.0546, \mathrm{C}_x = 0.0526,$
$C_z = 0.0488, \rho_{xy} = 0.7108, \rho_{yz} = 0.6932, \rho_{zx} = 0.7346, \mathrm{n}_1 = 10, \mathrm{n}_2 = 7$

### *Population 2: ( Population census of Okara district(1998),Pakistan)*

Y: population matric and above, X: primary but below matric, Z : Population both sexes
$\mathrm{N} = 300, \bar{\mathrm{Y}} = 41.5233, \bar{\mathrm{X}} = 141.58, \bar{\mathrm{Z}} = 1518.767, \mathrm{C}_y = 1.2185, \mathrm{C}_x = 1.088,$
$C_z = 0.9757, \rho_{xy} = 0.894, \rho_{yz} = 0.94, \rho_{zx} = 0.7346, \mathrm{n}_1 = 60, \mathrm{n}_2 = 12$

### *Population 3: (Population census report of Gujrat district(1998),Pakistan)*

Y: Population Matric and above, X: Primary but below Matric, Z: Population both sexes

$\mathrm{N} = 300, \bar{\mathrm{Y}} = 131.5133, \bar{\mathrm{X}} = 356.8433, \bar{\mathrm{Z}} = 1407.407, \mathrm{C}_y = 1.2532, \mathrm{C}_x = 0.991,$
$C_z = 0.9545, \rho_{xy} = 0.927, \rho_{yz} = 0.893, \rho_{zx} = 0.972, \mathrm{n}_1 = 60, \mathrm{n}_2 = 12$

Table 1. Efficiency comparisons of some estimators CASE 1

| *Estimators* | *Populations* | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Sample mean $\bar{y}$ | 29.55 | 844.79 | 8963.84 |
| Khan et al (2012) $\bar{y}_(Kel_1)$ | 21.89 | 804.65 | 7809.98 |
| Khan et al (2012) $\bar{y}_(Kel_2)$ | 20.44 | 743.78 | 7632.45 |
| Proposed $\bar{y}_{A1}$ | 19.70 | 718.40 | 6725.48 |
| Proposed $\bar{y}_{A2}$ | 11.60 | 635.22 | 6001.16 |

Table 2. Efficiency comparisons of some estimators CASE 2

| *Estimators* | *Populations* | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Sample mean $\bar{y}$ | 29.55 | 844.79 | 8963.84 |
| Khan et al (2012) $\bar{y}_(Kel_1)$ | 21.89 | 804.65 | 7809.98 |
| Khan et al (2012) $\bar{y}_(Kel_2)$ | 20.44 | 743.78 | 7932.45 |
| Proposed $\bar{y}_{A1}$ | 19.29 | 204.55 | 2170.43 |
| Proposed $\bar{y}_{A2}$ | 10.47 | 204.80 | 2173.08 |

Table 3. PRE comparisons of some estimators under CASE 1

| *Estimators* | *Populations* | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Sample mean $\bar{y}$ | 100.00 | 100.00 | 100.00 |
| Khan et al (2012) $\bar{y}_(Kel_1)$ | 135.00 | 105.00 | 114.77 |
| Khan et al (2012) $\bar{y}_(Kel_2)$ | 144.57 | 113.58 | 117.44 |
| Proposed $\bar{y}_{A1}$ | 150.00 | 117.60 | 133.28 |
| Proposed $\bar{y}_{A2}$ | 254.74 | 132.99 | 149.39 |

Table 4. PRE comparisons of some estimators under CASE 2

| *Estimators* | *Populations* | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| Sample mean $\bar{y}$ | 100.00 | 100.00 | 100.00 |
| Khan et al (2012) $\bar{y}_(Kel_1)$ | 135.00 | 105.00 | 114.77 |
| Khan et al (2012) $\bar{y}_(Kel_2)$ | 144.57 | 113.58 | 117.44 |
| Proposed $\bar{y}_{A1}$ | 153.19 | 413.01 | 413.00 |
| Proposed $\bar{y}_{A2}$ | 282.23 | 412.50 | 412.48 |

Tables 1 and 2 above show the relative efficiency of the transformed ratio-type $\bar{y}_(Kel_1)$ and $\bar{y}_([Kel]_2)$ estimators and estimators $\bar{y}$ , $[\bar{y}_A]_1$ and $[\bar{y}_A]_2$ for finite population mean in two-phase sampling under cases 1 and 2 respectively. This numerical comparison shows that the proposed ratio-type estimators have minimum MSEs.

Tables 3 and 4 above show the percentage relative efficiency of the transformed ratio-type $\bar{y}_(Kel_1)$ and $\bar{y}_([Kel]_2)$ estimators and estimators $\bar{y}$ , $\bar{y}_{A1}$ and $\bar{y}_{A2}$ for finite population mean in two-phase sampling under cases 1 and 2 respectively. This numerical comparison shows that the proposed ratio-type estimators are more precise and accurate

## 5. Recommendation

Since the proposed estimators has the largest gain in efficiency, over unbiased sample mean , ratio estimator by khan *et al* (2012). Hence the proposed estimators is recommended for practical survey.

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