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***In honour of Prof. Ekhaguere at 70***

# Two factors exponential type estimator for population mean under single phase sampling scheme

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**Abstract.** In this article, we have proposed two factors exponential type estimator for finite population mean under single phase sampling scheme. The bias and meansquare error (MSE) of the proposed estimator are obtained to the first degree of approximation in section 3. The efficiency of the proposed estimator over sample mean, ratio, product, Bahl and Tuteja [1], Singh and Espejo [3], Solankiet. al. [4] and Khan et. al. [2] estimators had been demonstrated in section 4. We carry out anumerical application showing that the proposed estimator outperforms the traditional estimators using different data sets.

**Keywords:** bias, efficiency, mean square error, parameters, precision.

## 1. Introduction

The uses of auxiliary information help in reducing the variability in the estimation of population parameters and thereby increase the precision of the estimators. Let us consider a finite population $U = U_1, U_{,2} ...U_N$ of size $N$ units and the value of the variables on the $i^{th}$ unit $U_i$, $i = 1, 2, ..., N$, be $(y_i, x_i)$. Let $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ be the population means of the study variable y and the auxiliary variable x, respectively.

The coefficients of variation of these variables are denoted by $C_x^2 = S_x^2/\bar{X}^2$ and $C_y^2 = S_y^2/\bar{Y}^2$ respectively. Further let $\mu_{20}\, or\, S_y^2$ and $\mu_{02}\, or\, S_x^2$ are corresponding variances. Also, $\rho_{xy}$ will represent population correlation coefficients between X and Y respectively.

For estimating the population mean, a simple random sample of size n is drawn without replacement from the population U. Then the sample mean, classical ratio and product estimators are respectively defined by

$$\hat{\bar{y}} = \bar{y}, \quad \hat{\bar{y}}_R = \bar{y}\frac{\bar{X}}{\bar{x}} \;\; if \;\; \bar{x} \neq 0 and \hat{\bar{y}}_P = \bar{y}\frac{\bar{x}}{\bar{X}} \tag{1}$$

Bahl and Tuteja [1] suggested ratio and product exponential type estimators for population mean as

$$\hat{\bar{y}}_{\text{Re}} = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{2}$$

$$\hat{\bar{y}}_{Pe} = \bar{y}\exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right) \tag{3}$$

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For estimating the population mean of the study variable $\bar{Y}$ , Singh and Espejo [3] considered an estimator of the ratio-product type given by

$$\hat{\bar{y}}_{RP} = \bar{y}\left(k\frac{\bar{X}}{\bar{x}} + (1-k)\frac{\bar{x}}{\bar{X}}\right) \tag{4}$$

Solankiet. al. [4] suggested a class of ratio type estimators for population mean $\bar{Y}$ as

$$\hat{\bar{y}}^{g}_{RHA} = \bar{y}\left(2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}\exp\left(\frac{\delta\left(\bar{x}-\bar{X}\right)}{\bar{x}+\bar{X}}\right)\right) \tag{5}$$

Khan et. al. [2] proposed exponential ratio and product type estimators for population mean as

$$\hat{\bar{y}}_{1\left(\sqrt{}\right)} = \bar{y}\exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right) \tag{6}$$

$$\hat{\bar{y}}_{2\left(\sqrt{}\right)} = \bar{y}\exp\left(\frac{\sqrt{\bar{x}}-\sqrt{\bar{X}}}{\sqrt{\bar{x}}+\sqrt{\bar{X}}}\right) \tag{7}$$

To the second order approximation, the mean square errors of the estimators $\hat{\bar{y}}_R$, $\hat{\bar{y}}_P$, $\hat{\bar{y}}_{\text{Re}}$, $\hat{\bar{y}}_{Pe}$, $\hat{\bar{y}}_{RP}$, $\hat{\bar{y}}_{RHA}$, $\hat{\bar{y}}_{\sqrt{1}}$, and $\hat{\bar{y}}_{\sqrt{2}}$ respectively are given as

$$\text{var}\left(\hat{\bar{Y}}\right) = \frac{1-f}{n}S_y^2 \tag{8}$$

$$MSE\left(\hat{\bar{y}}_R\right) \simeq \frac{1-f}{n}\left(S_y^2 + R^2S_x^2 - 2RS_{xy}\right) \tag{9}$$

$$MSE\left(\hat{\bar{y}}_P\right) \simeq \frac{1-f}{n}\left(S_y^2 + R^2S_x^2 + 2RS_{xy}\right) \tag{10}$$

$$MSE\left(\hat{\bar{y}}_{\text{Re}}\right) \simeq \frac{1-f}{n}\left(S_y^2 + \frac{R^2}{4}S_x^2\left(1-4k\right)\right) \tag{11}$$

$$MSE\left(\hat{\bar{y}}_{Pe}\right) \simeq \frac{1-f}{n}\left(S_y^2 + \frac{R^2}{4}S_x^2\left(1+4k\right)\right) \tag{12}$$

$$MSE\left(\hat{\bar{y}}^{1}_{RHA}\right) \simeq \frac{1-f}{n}\left(S_y^2 + \frac{3R^2}{4}S_x^2\left(3-4k\right)\right) \tag{13}$$

$$MSE\left(\hat{\bar{y}}_{\sqrt{1}}\right) \simeq \frac{1-f}{n}\left(S_y^2 + \frac{R^2}{16}S_x^2 - \frac{R}{2}S_{xy}\right) \tag{14}$$

$$MSE\left(\hat{\bar{y}}_{\sqrt{2}}\right) \simeq \frac{1-f}{n}\left(S_y^2 + \frac{R^2}{16}S_x^2 + \frac{R}{2}S_{xy}\right) \tag{15}$$

Where $k = \frac{\rho\sqrt{\mu_{20}}}{R\sqrt{\mu_{02}}}$ *and* $R = \frac{\bar{Y}}{\bar{X}}$

## 2. Proposed estimator

After examining the related estimators in the studies mentioned in Section 1, for estimating the population mean of the main variable under study and motivated by Singh and Shukla [3] estimator of population mean, we proposed the following two factors exponential type estimator

$$\hat{\bar{y}}_f = \bar{y}\exp\left(\frac{(d-1)\left(\bar{x}-\bar{X}\right)+(d-2)\left(\bar{X}-\bar{x}\right)}{(d-1)\left(\bar{x}+\bar{X}\right)+(d-2)\left(\bar{X}+\bar{x}\right)}\right) \tag{16}$$

where $0 < d < \infty$. If $d = 1$, $\hat{\bar{y}}_f = \hat{\bar{y}}_{\mathrm{Re}}$ in eqn(2) and if $d = 2$, $\hat{\bar{y}}_f = \hat{\bar{y}}_{Pe}$ in eqn(3). Our main goal in this paper is to determine the value of $d$ such that the bias and/or the mean square error (MSE) of $\hat{\bar{y}}_f$ is minimal.

## 3. Bias and Mean Square Error (MSE) of $\hat{\bar{y}}_f$

To obtain the bias and MSE of $\hat{\bar{y}}_f$ , we write $e_0 = \left(\bar{y}-\bar{Y}\right)/\bar{Y}$ *and* $e_1 = \left(\bar{x}-\bar{X}\right)/\bar{X}$ . So that $\bar{y} = \bar{Y}\left(1+e_0\right)$ and $\bar{x} = \bar{X}\left(1+e_1\right)$ where $|e_0| < 1$ $|e_1| < 1$ are error terms. Hence $E\left(e_0\right) = E\left(e_1\right) = 0,\ E\left(e_0\right) = \frac{1-f}{n}C_y^2,\ E\left(e_1\right) = \frac{1-f}{n}C_x^2,\ E\left(e_0e_1\right) = \frac{1-f}{n}\rho C_yC_x$ and $f = \frac{n}{N}$

$$\hat{\bar{y}}_f = \bar{Y}\left(1+e_0\right)\exp\left(\frac{(B-A)\,e_1}{(B+A)\left(2+e_1\right)}\right) \tag{17}$$

Where $A = d-1$ and $B = d-2$. Let $\theta_1 = A+B$, $\theta_2 = B-A$ *and* $\theta = \frac{\theta_2}{\theta_1}$, then, equation (17) can be written as

$$\hat{\bar{y}}_f = \bar{Y}\left(1+e_0\right)\exp\left(\frac{\theta}{2}e_1\left(1+\frac{e_1}{2}\right)^{-1}\right) \tag{18}$$

Expressing $\hat{\bar{y}}_f$ in terms of es to second degree terms of order $n^{-1}$ , we have

$$\hat{\bar{y}}_f - \bar{Y} = \bar{Y}\left(e_0 + \frac{\theta}{2}e_1 - \frac{\theta}{4}e_1^2 + \frac{\theta^2}{8}e_1^2 + \frac{\theta}{2}e_0e_1\right) \tag{19}$$

Taking the expectation of equation (19), we obtain the bias of the proposed estimator as

$$Bias\left(\hat{\bar{y}}_f\right) \simeq \bar{Y}\frac{1-f}{n}\left(\frac{\theta}{4}C_x^2\left(\frac{\theta}{2}-1\right) + \frac{\theta}{2}\rho C_yC_x\right) \tag{20}$$

Also, taking square and expectation of equation (19), we obtain the MSE of the proposed estimator up first order approximation as

$$MSE\left(\hat{\bar{y}}_f\right) \simeq \bar{Y}^2\frac{1-f}{n}\left(C_y^2+\frac{\theta^2}{4}C_x^2+\theta\rho C_yC_x\right) \tag{21}$$

The minimum MSE of the proposed estimator can be obtained by differentiate the equation (21) partially with respect to $\theta$ and equate to zero to obtain the optimum value of $\theta$ , say $\theta_{opt}$ .

$$\frac{\partial MSE\left(\hat{\bar{y}}_f\right)}{\partial\theta} = \bar{Y}^2\frac{1-f}{n}\left(\frac{\theta}{2}C_x^2+\rho C_yC_x\right) \tag{22}$$

Equate equation (22) to zero; we obtain the optimum value of $\theta$ as

$$\theta_{opt} = \frac{-2\rho C_y}{C_x} \tag{23}$$

Substituting the result of equation (23) into equation (21), the minimum MSE of $\hat{\bar{y}}_f$ is given as

$$MSE\left(\hat{\bar{y}}_f\right)_{\min} = \frac{1-f}{n}S_y^2\left(1-\rho^2\right) \tag{24}$$

*Remark* 1 It worth noting that the minimum MSE of $\hat{\bar{y}}_f$ is the same with that of regression estimator when the slope $\beta_1 = \frac{S_{xy}}{S_x^2}$

To obtain the optimum value of $d$, we have that $\theta = -v, \quad \Rightarrow \quad \frac{\theta_2}{\theta_1} = \frac{B-A}{B+A} = -v$ where $v = \frac{2\rho C_y}{C_x}$ and therefore

$$\frac{(d-2)-(d-1)}{(d-2)+(d-1)} = -v. \tag{25}$$

By simplifying equation (25), the optimum value of $d$ is obtain as

$$d = \frac{3}{2}+\frac{C_x}{4\rho C_y} \tag{26}$$

## 4. Efficiency comparisons

From (24) and (8), we have

$$\text{var}\left(\hat{\bar{Y}}\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\rho^2S_y^2 > 0.$$

From (24) and (9), we have

$$MSE\left(\hat{\bar{y}}_R\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y - RS_x\right)^2 > 0.$$

From (24) and (10), we have

$$MSE\left(\hat{\bar{y}}_P\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y + RS_x\right)^2 > 0.$$

From (24) and (11), we have

$$MSE\left(\hat{\bar{y}}_{\mathrm{Re}}\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y - \frac{1}{2}RS_x\right)^2 > 0.$$

From (24) and (12), we have

$$MSE\left(\hat{\bar{y}}_{Pe}\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y + \frac{1}{2}RS_x\right)^2 > 0.$$

From (24) and (13), we have

$$MSE\left(\hat{\bar{y}}^1_{\mathrm{R}HA}\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y - \frac{3}{2}RS_x\right)^2 > 0.$$

From (24) and (14), we have

$$MSE\left(\hat{\bar{y}}_{\sqrt{1}}\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y - \frac{1}{4}RS_x\right)^2 > 0.$$

From (24) and (15), we have

$$MSE\left(\hat{\bar{y}}_{\sqrt{2}}\right) - MSE\left(\hat{\bar{y}}_f\right) = \frac{1-f}{n}\left(\rho S_y + \frac{1}{4}RS_x\right)^2 > 0.$$

Table 1. Parameters of Populations (Subramani and Kumarapandiyan [6])

| Parameters | Population1 | Population2 | Population3 |
|---|---|---|---|
| $N$ | 103 | 80 | 49 |
| $n$ | 40 | 20 | 20 |
| $\bar{Y}$ | 62.6212 | 51.8264 | 116.1633 |
| $\bar{X}$ | 556.5541 | 11.2646 | 98.6765 |
| $\rho$ | 0.7298 | 0.9413 | 0.6904 |
| $S_y$ | 91.3549 | 18.3569 | 98.8286 |
| $C_y$ | 1.4588 | 0.3542 | 0.8508 |
| $S_x$ | 610.1643 | 8.4563 | 102.9709 |
| $C_x$ | 1.0963 | 0.7507 | 1.0435 |

Table 2. Optimum values of $d$

| | Population1 | Population2 | Population3 |
|---|---|---|---|
| Value of $d$ | 1.76 | 2.06 | 1.94 |

## 5. Conclusion

From efficiency comparisons in Section 4 and the empirical results of the percentage relative efficiency of the proposed estimator $\hat{\bar{y}}_f$ with respect to the existing estimators $\hat{\bar{y}}$, $\hat{\bar{y}}_R$, $\hat{\bar{y}}_P$, $\hat{\bar{y}}_R$, $\hat{\bar{y}}_{Pe}$, $\hat{\bar{y}}_{RHA}$, $\hat{\bar{y}}_{\sqrt{1}}$, $and\,\hat{\bar{y}}_{\sqrt{2}}$ and in Table 2, we infer that the proposed estimator is more efficient and can produce better estimate than the mentioned existing estimators in the sense that it has least mean squared error..

Table 3. PRE of Proposed Estimator over Existing Estimators

| Estimators | PRE of Proposed Estimator $\hat{\hat{y}}_f$ | | |
|---|---|---|---|
| | Population-1 | Population-2 | Population3 |
| $\hat{\hat{y}}$ | 213.95 | 877.54 | 191.08 |
| $\hat{\hat{y}}_R$ | 100.10 | 1317.99 | 154.93 |
| $\hat{\hat{y}}_P$ | 569.48 | 8320.83 | 802.16 |
| $\hat{\hat{y}}_{Re}$ | 126.82 | 112.30 | 101.14 |
| $\hat{\hat{y}}_{Pe}$ | 361.50 | 3613.72 | 424.75 |
| $\hat{\hat{y}}_{RHA}$ | 133.79 | 4494.62 | 352.45 |
| $\hat{\hat{y}}_{\sqrt{1}}$ | 162.84 | 248.56 | 128.14 |
| $\hat{\hat{y}}_{\sqrt{2}}$ | 280.17 | 1999.27 | 289.95 |

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