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Interactions of 2-3 Prey-Predators in Diffusive Ecological System

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Abstract

A diagram illustrating the relationship between three (3) competing predators who also compete amongst themselves and two independent Preys is shown to see the nature of interdependence among the species. To this end, it is considered that the free population (without interaction) is the solution to the Logistic equation of individual specie. A semi analytical approach is developed for solving the dynamics of a 2-3 Prey-Predator system in a diffusive state. The resultant solution is then analyzed to see the impact of predation on the population of both the Prey and Predators. It was observed that the growth rate in the population of the Prey is inversely proportional to the growth rate of population of the Predators while the impact of population is directly proportional to time and distance. The equilibrium state of the system is also examined.

1. INTRODUTION

The question of two predator species competing for one prey was brought to the fore by ecological literature written by [1-4]. A mathematical model for the two predator species exploiting a single prey was proposed by [2]. He found out that when the interspecific interference coefficient is small, the winner competes with rivals successfully. [5]studied the permanent coexistence and global\newline stability of a simple Lotka-Volterra type mathematical model of a living resource supporting two predators. They showed that the permanent coexistence of the system depends on the threshold of the ratio between the coefficients of numerical responses of the two predators/consumers. [6]proposed a Guass-type model with diffusion of which is analyzed. For the research, they considered a system of two predators competing with interference for a limited prey. They showed that in the absence of intraspecific interaction of the predator series, the interior equilibrium is unstable.

For this research work, we propose a model of three interacting predators competing against two preys for which both preys are independent and are not competing against each other. To achieve this, a competitive Lotka-Volterra equation is employed. To ensure for proper analysis, some semi- analytic method of solution shall be employed.

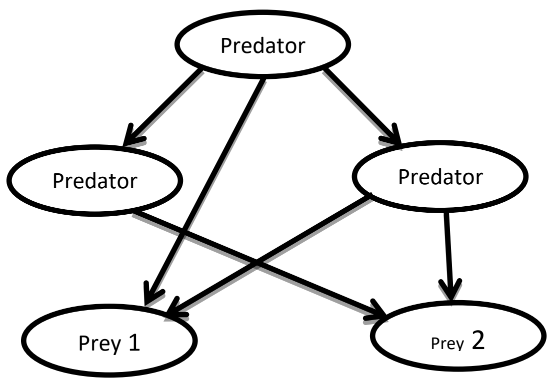

Fig 1: Diagram Showing the 2-3 Prey-Predator Interactions

The equations are modelled to obey the principles in [7].

$$\frac{\partial x_1}{\partial t} = x_1(r_1 - u_1 - v_1 - \alpha_1 y_1 - \alpha_2 y_2) + d_1.\frac{\partial^2 x_1}{\partial r^2} \quad (1)$$

$$\frac{\partial x_2}{\partial t} = x_2(r_2 - u_2 - v_2 - \beta_1 y_1 - \beta_2 y_2) + d_2.\frac{\partial^2 x_2}{\partial r^2} \quad (2)$$

$$\frac{\partial y_1}{\partial t} = -y_1(s_3 - \sigma_1 x_1 - \sigma_2 y_2 - \sigma_3 y_3) + D_1.\frac{\partial^2 y_1}{\partial r^2} \quad (3)$$

$$\frac{\partial y_2}{\partial t} = -y_2(s_4 - \delta_1 x_1 - \delta_2 x_2 + \delta_3 y_1) + D_2.\frac{\partial^2 y_2}{\partial r^2} \quad (4)$$

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$x_1(r,0) = f_1(r)$, $x_1(0,t) = 0$, $x_1(n,r) = K_1, \frac{\partial x_1}{\partial t}(r,0) = 0$

$x_2(r,0) = f_2(r)\ x_2(0,t) = 0,\ x_2(n,r) = K_2, \frac{\partial x_2}{\partial t}(r,0) = 0$

$y_1(r,0) = f_3(r)\ y_1(0,t) = 0\ y_1(n,r) = K_3, \frac{\partial y_1}{\partial t}(r,0) = 0$

$y_2(r,0) = f_4(r)\ y_2(0,t) = 0\ y_2(n,r) = K_4, \frac{\partial y_2}{\partial t}(r,0) = 0$

$y_3(r,0) = f_5(r)\ y_3(0,t) = 0\ y_3(n,r) = K_5, \frac{\partial y_3}{\partial t}(r,0) = 0$

y_1 is the 1st Predator

y_2 is the 2nd Predator

y_3 is the 3rd Predator

x_1 is the 1st Prey

x_2 is the 2nd Prey

D_1, D_2, D_3, d_1 and d_2 are diffusion coefficient of the is the 1st Predator, 2nd Predator, 3rd Predator, 1st Prey and 2nd Prey respectively.

K_1, K_2, K_3, K_4 and K_5 are carrying capacities of 1st Prey, 2nd Prey, 1st Predator, 2nd Predator and 3rd Predator respectively.

$\alpha_1, \alpha_2, \beta_1, \beta_2, \sigma_1, \sigma_2, \sigma_3, \delta_1, \delta_2, \delta_3, \phi_1$ and ϕ_2 are interspecies interaction coefficients.

r_1 and r_2 are birth ratesof 1st Prey and 2nd Prey respectively.

u_1 and u_2 are death rates by accident of 1st Prey and 2nd Prey respectively.

v_1 and v_2 are death rates by old age of 1st Prey and 2nd Prey respectively.

s_3, s_4 and s_5 are death rates of 1st Predator, 2nd Predator and 3rd Predator respectively.

t is the time.

r is the radius or the area under consideration

The term $\frac{\partial^2}{\partial r^2}$ denotes the population density of area under consideration.

3.0 Solutions to Resulting Models

The Standard Garlekin method is to be applied to solve the above problem. The assumed basis function for Prey 1 (X_1) is;

$$U^{x_1} = \varphi_0 + c_1(t).r + c_2(t)r^2 \qquad (6)$$

Applying boundary conditions $x_1(0,t) = 0$,

$$U^{x_1}(0,t) = \varphi_0 + 0 + 0 = 0$$

$$\varphi_0 = 0$$

Applying boundary conditions $x_1(n,t) = K_1$,

$$U^{x_1}(n,t) = \varphi_0 + c_1(t).n + c_2(t)n^2$$

Let $c_1(t) = c_1$ and $c_2(t) = c_2$

$$c_2 n^2 = K_1 - c_1 n$$

$$c_2 = \frac{K_1 - c_1 n}{n^2}$$

Hence, (6) becomes;

$$U^{x_1} = c_1.r + \left(\frac{K_1 - c_1 n}{n^2}\right) r^2$$

$$U^{x_1} = c_1.r + \frac{K_1 r^2}{n^2} - \frac{c_1 n r^2}{n^2}$$

$$U^{x_1} = \frac{K_1 r^2}{n^2} + \left(\frac{rn - r^2}{n}\right) c_1$$

$$U^{x_1} = \frac{K_1 r^2}{n^2} + \frac{r}{n}(n-r)c_1$$

Applying boundary conditions $\frac{\partial x_1}{\partial t}(r,0) = 0$:

$$\frac{\partial U^{x_1}}{\partial t}(r,0) = \frac{r}{n}(n-r)c_1(0)' = 0$$

$$\Rightarrow c_1(0)' = 0$$

$$c_1(0) = a_1$$

where a_1 is a constant.

Applying boundary conditions $x_1(r,0) = f_1(r)$,

$$U^{x_1}(r,0) = \frac{K_1 r^2}{n^2} + \frac{r}{n}(n-r)c_1(0) = f_1(r)$$

$$f_1(r) = \frac{K_1 r^2}{n^2} + \frac{r}{n}(n-r)a_1$$

$$f_1(r) = \frac{(K_1 - a_1 n)r^2 + a_1 n^2 r}{n^2}$$

$$U^{x_1} = \frac{K_1 r^2}{n^2} + \frac{r}{n}(n-r)c_1(t) \tag{7}$$

$$U^{x_2} = \frac{K_2 r^2}{n^2} + \frac{r}{n}(n-r)c_2(t) \tag{8}$$

$$U^{y_1} = \frac{K_3 r^2}{n^2} + \frac{r}{n}(n-r)c_3(t) \tag{9}$$

$$U^{y_3} = \frac{K_4 r^2}{n^2} + \frac{r}{n}(n-r)c_4(t) \tag{10}$$

$$U^{y_3} = \frac{K_5 r^2}{n^2} + \frac{r}{n}(n-r)c_5(t) \tag{11}$$

Solving the resultant equations using the basis functions we have the following equations:
For **Prey1**:

Neglecting the terms c_3 and c_4 we have the equation below;

$$k_1 c_1' - k_2 c_1 + k_3 c_1^2 + k_4 c_1 c_3 + k_5 c_1 c_4 + k_6 = 0 \tag{12}$$

Divide (12) by k_1, we have;

$$c_1' - \frac{k_2}{k_1} c_1 \left(1 - \frac{k_3}{k_2} c_1\right) + \frac{k_4}{k_1} c_1 c_3 + \frac{k_5}{k_1} c_1 c_4 + \frac{k_6}{k_1} = 0 \tag{13}$$

For **Prey2:**

Neglecting the terms c_4 and c_5 we have the equation below;

$$l_1 c_2' - l_2 c_2 + l_3 c_2^2 + l_4 c_2 c_4 + l_5 c_2 c_5 + l_6 = 0 \tag{14}$$

Divide (14) by l_1, we have;

$$c_1' - \frac{l_2}{l_1} c_1 \left(1 - \frac{l_3}{l_2} c_1\right) + \frac{l_4}{l_1} c_1 c_3 + \frac{l_5}{l_1} c_1 c_4 + \frac{l_6}{l_1} = 0 \tag{15}$$

For **Predator 1**:

Neglecting the terms c_1, c_4 and c_5 we have the equation below;

$$m_1 c_3' - m_2 c_3 + m_3 c_3^2 + m_4 c_3 c_1 + m_5 c_3 c_4 + m_6 c_3 c_5 + m_7 = 0 \tag{16}$$

Divide (16) by m_1, we have;

$$c_3' - \frac{m_2}{m_1} c_3 \left(1 - \frac{m_3}{m_2} c_3\right) + \frac{m_4}{m_1} c_3 c_1 + \frac{m_5}{m_1} c_3 c_4 + \frac{m_6}{m_1} c_3 c_5 + \frac{m_7}{m_1} = 0 \tag{17}$$

For **Predator 2:**

Neglecting the terms , c_1 c_2 and c_3 we have the equation below;

$$n_1 c_4{}' - n_2 c_4 + n_3 c_4{}^2 + n_4 c_4 c_1 + n_5 c_4 c_2 + n_6 c_4 c_3 + n_7 = 0 \qquad (18)$$

Divide (18) by n_1, we have;

$$c_4{}' - \frac{n_2}{n_1} c_4\left(1 - \frac{n_3}{n_2} c_4\right) + \frac{n_4}{n_1} c_4 c_1 + \frac{n_5}{n_1} c_4 c_2 + \frac{n_6}{n_1} c_4 c_3 + \frac{n_7}{n_1} = 0 \qquad (19)$$

For **Predator 3**

Neglecting the terms c_2 and c_3 we have the equation below;

$$p_1 c_5{}' - p_2 c_5 + p_3 c_5{}^2 + p_4 c_5 c_2 + p_5 c_5 c_3 + p_6 = 0 \qquad (20)$$

Divide (20) by p_1, we have;

$$c_5{}' - \frac{p_2}{p_1} c_5\left(1 - \frac{p_3}{p_2} c_5\right) + \frac{p_4}{p_1} c_5 c_2 + \frac{p_5}{p_1} c_5 c_3 + \frac{p_6}{p_1} = 0 \qquad (21)$$

We assume that our equation is of the form

$$\frac{dp}{dt} - ap(1 - \alpha p) + F_s = 0$$

With solution

$$p_n = \frac{p_0 e^{at}}{\left(1 - \alpha . p_0 (1 - e^{at})\right)} - \int_0^t \frac{e^{a(t-s)}\left(1 - \alpha . p_0 (1 - e^{as})\right)}{\left(1 - \alpha . p_0 (1 - e^{at})\right)} . F ds$$

To solve (13), (15), (17), (19) and (21) giving the solution below;

$$c_1 = \frac{a_1 e^{\frac{k_2}{k_1}t}}{\left(1 - \frac{k_3}{k_2} . a_1 (1 - e^{\frac{k_2}{k_1}t})\right)} \left(1 - \left(\begin{array}{l} \frac{k_4 m_1}{k_1 m_3} \ln\left(1 - \frac{m_3}{m_2} . a_3 (1 - e^{\frac{m_2}{m_1}t})\right) \\ + \frac{k_5 n_1}{k_1 n_3} \ln\left(1 - \frac{n_3}{n_2} . a_4 (1 - e^{\frac{n_2}{n_1}t})\right) \\ - \frac{k_6 k_2}{a_1 k_1{}^2}\left(1 - \frac{k_3}{k_2} . a_1\right) e^{\frac{-k_2}{k_1}t} + \frac{k_6 k_3}{k_1 k_2} t \end{array}\right)\right) \qquad (22)$$

$$c_2 = \frac{a_2 e^{\frac{l_2}{l_1}t}}{\left(1 - \frac{l_3}{l_2} . a_2 (1 - e^{\frac{l_2}{l_1}t})\right)} \left(1 - \left(\begin{array}{l} \frac{l_4 n_1}{l_1 n_3} \ln\left(1 - \frac{n_3}{n_2} . a_4 (1 - e^{\frac{n_2}{n_1}t})\right) \\ + \frac{l_5 p_1}{l_1 p_3} \ln\left(1 - \frac{p_3}{p_2} . a_5 (1 - e^{\frac{p_2}{p_1}t})\right) \\ - \frac{l_6}{a_2 l_1}\left(1 - \frac{l_3}{l_2} . a_2\right) e^{\frac{-l_2}{l_1}t} + \frac{l_6 l_3}{l_1 l_2} t \end{array}\right)\right) \qquad (23)$$

$$c_3 = \frac{a_3 e^{\frac{m_2}{m_1}t}}{\left(1 - \frac{m_3}{m_2} . a_3 (1 - e^{\frac{m_2}{m_1}t})\right)} \left(1 - \left(\begin{array}{l} \frac{m_4 k_1}{m_1 k_3} \ln\left(1 - \frac{k_3}{k_2} . a_1 (1 - e^{\frac{k_2}{k_1}t})\right) \\ + \frac{m_5 n_1}{m_1 n_3} \ln\left(1 - \frac{n_3}{n_2} . a_4 (1 - e^{\frac{n_2}{n_1}t})\right) \\ + \frac{m_6 p_1}{m_1 p_3} \ln\left(1 - \frac{p_3}{p_2} . a_5 (1 - e^{\frac{p_2}{p_1}t})\right) \\ - \frac{m_7}{a_3 m_1}\left(1 - \frac{m_3}{m_2} . a_3\right) e^{\frac{-m_2}{m_1}t} + \frac{m_7 m_3}{m_1 m_2} t \end{array}\right)\right) \qquad (24)$$

$$c_4 = \frac{a_4 e^{\frac{n_2}{n_1}t}}{\left(1-\frac{n_3}{n_2}.a_4(1-e^{\frac{n_2}{n_1}t})\right)}\left(1-\left(\begin{array}{l}\frac{n_4k_1}{n_1k_3}\ln\left(1-\frac{k_3}{k_2}.a_1(1-e^{\frac{k_2}{k_1}t})\right)\\ +\frac{n_5l_1}{n_1l_3}\ln\left(1-\frac{l_3}{l_2}.a_2(1-e^{\frac{l_2}{l_1}t})\right)\\ +\frac{n_6m_1}{n_1m_3}\ln\left(1-\frac{m_3}{m_2}.a_3(1-e^{\frac{m_2}{m_1}t})\right)\\ -\frac{n_7}{a_4n_1}\left(1-\frac{n_3}{n_2}.a_4\right)e^{\frac{-n_2}{m_n}t}+\frac{n_6n_3}{n_1n_2}t\end{array}\right)\right) \qquad (25)$$

$$c_5 = \frac{a_5 e^{\frac{p_2}{p_1}t}}{\left(1-\frac{p_3}{p_2}.a_5(1-e^{\frac{p_2}{p_1}t})\right)}\left(1-\left(\begin{array}{l}\frac{p_4l_1}{p_1l_3}\ln\left(1-\frac{l_3}{l_2}.a_2(1-e^{\frac{l_2}{l_1}t})\right)\\ +\frac{p_5m_1}{p_1m_3}\ln\left(1-\frac{m_3}{m_2}.a_3(1-e^{\frac{m_2}{m_1}t})\right)\\ -\frac{p_6}{a_5p_1}\left(1-\frac{p_3}{p_2}.a_5\right)e^{\frac{-p_2}{p_1}t}+\frac{p_6p_3}{p_1p_2}t\end{array}\right)\right) \qquad (26)$$

4.0 Numerical Applications

A set of feasible values were assigned to the solutions (7)-(11) to arrive at the following relations;

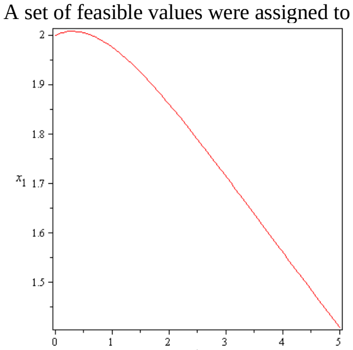

Fig 1: Population of Prey 1 against time

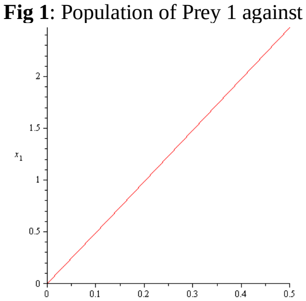

Fig 2: Population of Prey 1 against distance r

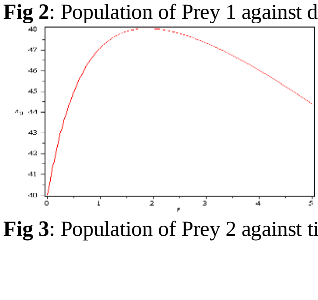

Fig 3: Population of Prey 2 against time t

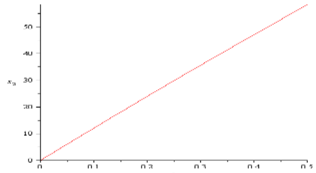

Fig 4: Population of Prey 2 against distance r

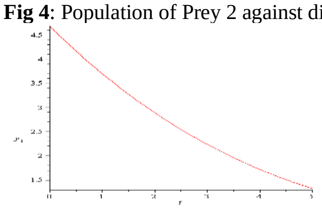

Fig 5: Population of Predator 1 against time

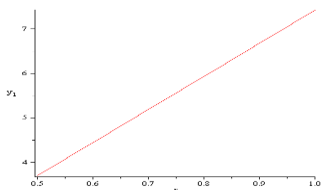

Fig 6: Population of Predator 1 against distance r

The effect of predation is noticeable in Prey 1 (Fig 1& 2) far more than Prey 2 (Fig 3 & 4) hence, fewer predators attack Prey 2 compared to Prey 1. Despite being the dominant Predator, the population of Predator 1 decreases because its death rate is substantially higher than its birth rate. Both Predator 2 and 3 depends on the population of Predator 1.

All the species (both Prey and Predator) increase with increase in distance covered hence the larger the area covered the more the population.

5.0 Conclusion

In Prey-Predator System, a Prey tends to move from Predator dominated areas to less dominated areas. Multiple Prey-Predator interactions often exhibit similar growth pattern among the prey and also among the predators. At equilibria points often the absence of one or two specie leads the stagnation of the population of other species. Hence, we note that the method of solution aided in solving the resulting modelling equation of the Prey-Predator system by giving us a solution for each of the species.

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