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Mathematical Model on the Control of HIV/AIDs Pandemic Using Condom, Vaccine, Therapeutic Doses and Public Health Campaign in Heterogeneous Population

[1]D. Omale, [2]R.O. Aja and [3]G.C.E. Mbah

[1]Mathematical Sciences Department, Kogi State University Anyigba,
[2]Department of Mathematics, Micheal Okpara University of Agriculture, Umudike
[3]Department of Mathematics, University of Nigeria, Nksuka

Abstract

A non linear mathematical model is proposed to study the effect of condom, vaccine therapeutic dose and public health campaign on the transmission dynamic of HIV/AIDS infection in a heterogeneous population. To achieve this, the population is divided into nine compartmental classes: susceptible class, susceptible that uses condom, susceptible that receive vaccination, the exposed or latent class; the infected class, the infected that receive the Highly Active Antiretroviral Therapy (HAART) treatment. The HIV/AIDS class, the AIDS class that receive treatment and the AIDS class that do not receive treatment. The HIV/AIDS class is divided into two classes of two full-blown AIDS stages. The positivity and invariant region of solutions were analysed for all the models. The numerical results suggest that effective use of public health campaign on the need for individual within the population to go for vaccination and on effective use of government approved and certified condom and for those who are screened and confirmed positive to continue their Highly Active Anti- retroviral Therapy (HAART) drugs faithfully to lower the HIV/AIDS viral load on the individual is a key factor in reducing the transmission rate; and eventual eradication of the disease from the population in no distant time and the educational programme of the need for abstinence for better behavioural changes should be given to all groups.

Keyword: *HIV/AID, Invariants, Condom, Vaccines, Therapeutic dose*

1. INTRODUTION

OVERVIEW OF HIV/AIDS

Human immunodeficiency virus infection/acquired immunodeficiency syndrome (HIV/AIDS) is a disease of the human immune system caused by infection with human immunodeficiency virus (HIV) [1]. During the initial infection, a person may experience a brief period of influenza-like illness. This is typically followed by a prolonged period without symptoms. As the illness progresses, it interferes more and more with the immune system, making the person much more likely to get infections, including opportunistic infections and tumors that do not usually affect people who have normal immune systems. HIV is transmitted primarily via unprotected sexual intercourse (including oral and anal sex), contaminated blood transfusions, infected hypodermic needles and from mother to child during pregnancy delivery or breast-feeding. Some bodily fluid like saliva and tears without do not transmit HIV [2]. Prevention of HIV infection programs is a key strategy to controlling of the spread of the disease. There is no cure or vaccine of effective use for it now. However antiretroviral treatment can slow the course of the disease and may lead to a near-normal life expectancy. These medications are expensive and may be associated with side effects on the patient receiving treatment. Genetic research indicates that HIV originated from West Africa during the early twentieth century [3]. Since the discovery of HIV/AIDS in 1981 by the centre for Disease control and prevention in the US [4], AIDS has caused nearly 30 million death world wide as of 2009. [5,6]. As at 2010 approximately 34 million people are living with HIV globally [7]. AIDS is considered a pandemic, a disease outbreak which is present over a large area and is actively spreading [8]. HIV/AIDS has had a great impact on society both as an illness and as a source of social and physical discrimination in the society and even in the work place. The disease has significant economic impacts. There are many misconceptions about HIV/AIDS such as the belief that it can be transmitted by casual non-sexual contact; this is a false belief [9].

The main goal of the research is to develop a mathematical model for the control of HIV/AIDS in the heterogeneous population and to solve the model system analytically using Homotopy perturbation method and to make good recommendation for the eradication of HIV/AIDS from our world following the results from our analysis. Develop a mathematical model using condom, vaccine, therapeutic dose and public health campaign for the control of HIV/AIDS.

This work will study HIV/AID dynamic and the control using the therapeutic and prophylactive vaccine, condom, HAART therapy and public health campaign to control the spread of HIV/AIDS. This is a theoretical investigation/work.

Inadequate information on the AIDS vaccine and non availability of mathematical laboratory for practical investigation is another very serious limitation and so secondary data from some authors as required were used for modal validation .The significance of this study can not be over emphasized as humanity will readily welcome any contribution for the cure or prevention of this world most severe infections in human history.

Correspondence Author: Omale D., E-mail: davidubahi@gmail.com, Tel: +2348165981963

The knowledge derived from this research will be used by policy makers on the need for HIV/AIDS vaccine and condom use with public health campaign for the control of AIDS. The knowledge will lead to formulation of vaccination strategies to curb the spread of HIV/AIDS. The study will eventually be useful in the medical communities.

Some existing literatures to assist us formulate our model and as a bases for subsequent analysis. A myriad of literatures exists on the mathematical dynamics of HIV/AIDS, and some on the vaccination, but we considered in this thesis, the impact of imperfect vaccine on HIV/AID dynamics. A mathematical model for the dynamics of an infectious disease in the presence of a preventive (prophylactic) vaccine and an effective therapeutic treatment as in [10]. The Study of the epidemiological impact of an HIV vaccine on the HIV/AIDS epidemic in Southern India. The author used a deterministic dynamic compartmental model to examine the impact of preventive HIV vaccine on the HIV epidemic in Southern India. Various assumption about the degree of protection offered by such a vaccine, the extent of immunological response in those vaccinated and the duration of protection afforded were explored as seen in [11]. A system of nonlinear ordinary differential equations model which were analysed to study the screening of unaware infectives on the spread of HIV/AIDS in a population with constant immigration of susceptible [12]. Non linear ordinary differential equation mathematical models were proposed to study the effect of vaccination on the speed of HIV/AID in the homogeneously mixing population [13]. Mathematical model for assessing therapeutic strategies for HIV infection, suggested that, to eradicate HIV from an infected individual requires the combination of highly active antiretroviral therapy (HAART) with immune activation interleuk was developed [14]. Imperfect vaccine aggregates the long-standing dilemma of voluntary vaccinations combined a simple but effective game theoretic model of vaccination behavior with an epidemiological process and in their analysis they showed that, in a population of self interested individuals, there exist an overshooting of vaccine uptake levels as the effectiveness of vaccination increases.in – 2 (al -2) [15]. Prophylactic vaccines,Risk Behaviour change and the probability of Eradicating HIV in San Francisco through the use of prophylactic vaccines was conducted [16]. A staged progression HIV model with imperfect vaccine which was formulated and analyzed and they used it to investigate the potential impact of an imperfect vaccine. In this study, the vaccine is assumed to have several desirable characteristics such as protecting against infection, causing bypass of the primary infection stage and offering a disease – altering therapeutic effect [17]. A dynamical model which allowed them to study the effects of an imperfect vaccine on a population prone to change in their risk behaviour [18]. Live attenuated HIV vaccines, predicting the trade off between efficacy and safety [19]. A mathematical model to simulate the impact of various partially effective preventive HIV vaccination scenarios and change in risk behavior within the population after was studied [20]. Potential public health impact of imperfect HIV type I, vaccines was considered and analyzed. The potential impact of a low-efficacy vaccine that induces a short duration of protection was also studied. Imperfect vaccines could act in various ways, including poor efficacy in protecting against infection [21]. Imperfect vaccine and herd immunity to HIV. Analytic and modelling techniques were used to investigate the effect of different levels of efficacy and coverage of a prophylactic vaccine on herd immunity and the possibility of eradicating the epidemic [22]. Low efficacy HIV vaccine potential for community based intervention programs as in [23]. The impact of a partially effective HIV vaccine on a population of intravenous drug users in Bangkok, Thailand. The paper focused on the effects of an HIV vaccine applied to the high prevalence setting of intravenous drug users (IDU) in Bangkok [24]. The epidemiological impact of an HIV/AIDs vaccine in developing country. The study used two different models applied to three different epidemic settings to examine the impact of vaccines with various characteristic on HIV incidence [25]. Stability of HIV/AIDS Treatment models with different stages. It proposed two stages of HIV infections; the HIV-positive in the asymptomatic stage and HIV-positive individual in the pre-AIDS stage [26]. The effect of screening and Treatment on transmission of HIV/AIDS infection in a population. The authors examined the effect of screening and treatment in the transmission dynamics of HIV/AIDS infection in the population [27]. The Homotopy perturbation method for solving model equation on the pollution of a system of Lakes. In this paper Homotopy perturbation method was used to give approximate and analytical solution to non linear differential equations [28], we hope to apply this method to solve our models later. Mathematical Analysis of the transmission dynamics of HIV/TB co infection in the prevention or treatment [29]. The author presented the synergistic interaction between HIV and Tuberculosis using a deterministic model, which incorporated many of the essential biological and epidemiological features of the two diseases.

2.0 MATHEMATICAL MODEL FORMULATION.

The development of our model is based on the following assumptions that,

i. The diseases HIV/AIDS is killing continuously
ii. Individual who contact this diseases will definitely die of the disease if untreated or on control drug.
iii. There is no medicine right now for total cure of this particular disease, therefore infected individual will live with the disease in his/her life time. Individual on HIV drug will remain on the drug forever.
iv. Individual who is faithful to the drug will not die of HIV/AIDS
v. There is no vaccine with 100% efficacy to prevent HIV/AIDS
vi. The available vaccines are imperfect; and so the vaccine will wane with time.
vii. That not all the people within the sexually active population are willing to use condom whenever they have sex..
viii. There are no vertical transmissions of the diseases .
ix. That campaign reduces the rate of transmission; because those who are properly informed will reduce their exposure to infection whenever they meet any infectious opportunity.

We develop and analyze a mathematical model for HIV/AIDS transmission dynamics and control improving on the existing models as discoursed in our literature review. This is done by incorporating vaccination coverage, condom usage, campaign and therapeutic doses. The model is defined as a set of ordinary differential equations based on our assumptions about the dynamics of HIV/AIDS, and some biological interventions. The interaction between the classes is describe as follows: The susceptible is divided into three groups: (S) represent the number of individuals not yet infected with the virus (HIV/AIDS) virus but are susceptible to the disease and its recruitment is not vaccinated, denoted by π, the other susceptible group is the vaccinated susceptible population denoted by (V), when the susceptible population, as a result of public enlightment campaign get vaccinated at the rate δ_1 and its recruitment is vaccinated at a proportion P, the vaccine has the ability to reduce the infection rate by a factor $(1-\theta_1)k$ where θ_1 is the vaccine efficacy. When the efficacy is low, the infection may occur at the rate $(1-\theta_1)k$, θ_1, measure the efficacy of the vaccine such that $0 \le \theta_1 \le 1$ If $\theta_1 = 1$,vaccine is completely effective in preventing the population from infections, if the θ_1 is equal to 0 the vaccine is useless, as the whole population will be infected if they interact with infected population. The third susceptible class are those who use condom at the δ_2 and its recruitment is denoted by ω, the failure rate in protecting an individual is denoted by ε, in that case the condom users will be susceptible again. The effectiveness of the condom is denoted by φ, such that $0 \le \varphi \le 1$,If $\varphi = 1$, the condom is very effective and it can prevent the population from the infections, but if the condom efficacy is equal to zero (0), the condom is useless. The waning rate of the vaccine is denoted by θ and the individual become susceptible again. Exposed class (E) is made of individuals who have contracted the infection at the early stage, but are not capable of infecting others in the population yet, the exposed individual will become

infectious at the rate ϕ the public health campaign is denoted by c, the rate at which the infectious individual through effective public health campaign go for treatment is τ, the non effectiveness of therapy is denoted by σ_1, σ_2 such that $1 \le \sigma_1, \sigma_2 \le 1$, the infectious individual progress to full blown AIDS at the rate η, the delay rate in developing symptom is e, (A) is the population of individual with clinical AIDs, it is a function of (I), (I_2) and (A_2) developing disease symptoms. The susceptible may become infectious at the rate of infection k, the force of infection is given by $k = \frac{n_1\beta_1 I_2 + n_2\beta_2 A_2 + n_3\beta_3 A_T}{N}$

where

n = number of sex partners

β_1 = transmission rate from infectious individual not receiving treatment

β_2 = transmission rate from infectious individual receiving treatment

β_3 = transmission rate of AIDS individual who is undergoing therapy, (HAARTS)

In the force of infection $\beta_1 > \beta_2 > \beta_3$. This show that β_1 contribute much on the transmission of the infection due to the fact that they are not receiving treatment, so they are not protected, β_2 contribute much less on the transmission of the infection due to their HIV status, they have acquired HIV/AIDS but receiving treatment so their viral load will be significantly reduced, unless if they desist from taking their daily pills. β_2 Is expected to contribute least to the infection, since they just acquired the full virus and are aware of the AIDS status and they are receiving the daily therapy. There is natural death rate (μ) in the whole compartments, but there is an HIV/AIDS induced death rate in the (A) and (A_2) classes. (A) And A_2 are the same if proportion of (A) class stop receiving treatment.

The total population at any time t is given by

$$N(t) = S(t) + V(t) + H(t) + E(t) + I(t) + I_2(t) + A_2(t) + A(t) + A_T(t)$$

The population is homogeneously mixed and each susceptible individual has equal chance of acquiring HIV infection when the individual come in contact with an infectious individuals.

The full description of the variables and parameters to be used in the model are as follows in the table below;

Table 1 State variable of the HIV/AIDS with control strategies;

$S(t)$	Number of susceptible at time *t*.
$V(t)$	Number of preventive vaccinated individual at time *t*.
$H(t)$	Number of susceptible that are condom users at time *t*.
$E(t)$	Latent/exposed individuals at time *t*.
$I(t)$	Infectious individuals at time t not receiving any treatment
$I_2(t)$	Number of infectious individuals who are undergoing treatment
$A(t)$	Number of individuals with full blown AIDS.
$A_T(t)$	Number or proportion of full blown AIDS who are undergoing therapy.
$A_2(t)$	Proportion of full blown AIDS who are not receiving the therapy.

Table 2 Parameter descriptions

π	Population recruited into the susceptible class.
P	Proportion of susceptible recruited individual with lost preventive vaccination
ω	Proportion of susceptible recruited individual that uses condom
μ	Per capita death rate (Nature death)
α_i	Disease induced death rate
δ_1	Preventive Vaccination rate in the population
δ_2	Rate of condom usage in the population
θ	Waning rate of the vaccine
ε	Improper condom usage
φ	Condom efficacy or effectiveness
θ_1	Vaccination efficacy rate
ϕ	Progression rate of latent individual to infectious class.
c	Public health campaign rate
σ_1, τ_C	Rate of non effectiveness of the drug.
τ	Treatment rate of infectious individual
η	Rate of progression to full blown AIDS
e	Reduction in developing symptom.
r_1	Rate at which those in the AID class receive treatment due to effectiveness of public health campaign
k	Effective contact rate of the susceptible with the infectious classes and called force of infection.
δ_2	The rate at which the susceptible individual uses condom effectively
r	Rate at which unvaccinated and those who voluntarily refused to use condom become exposed to the infections.
r_2	Rate at which proportion of those in *A* class refused to receive the therapy and remain with AIDS.
α_2	Disease induced death rate of those who refused therapy as AIDS individuals.

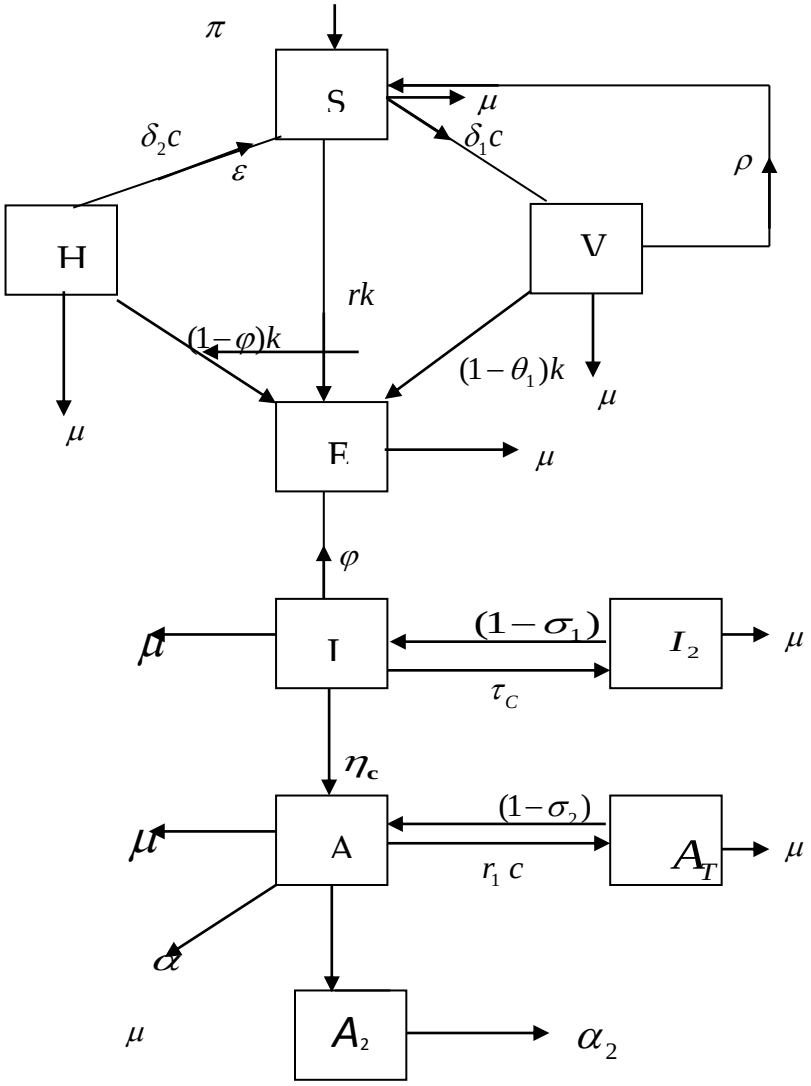

FIGURE. 1 FLOW DIAGRAM ILLUSTRATING THE INTERACTIONS OF THE DIFFERENT COMPARTMENTS

From our assumptions and the flow chart we obtain the system of ordinary differential equations.

$$\frac{dS}{dt} = \pi - \delta_1 cS - rkS - \delta_2 cS + \varepsilon H - \mu S + pV$$

$$\frac{dV}{dt} = \delta_1 cS - (1-\theta_1)kV - pV - \mu V$$

$$\frac{dH}{dt} = -(1-\varphi)kH - \varepsilon H + \delta_2 cS - \mu H$$

$$\frac{dE}{dt} = (1-\varphi)kH + rkS + (1-\theta_1)kV - \phi E - \mu E$$

$$\frac{dI}{dt} = \phi E + (1-\sigma_1)I_2 - \tau cI - \eta cI - \mu I$$

$$\frac{dI_2}{dt} = \tau cI - (1-\sigma_1)I_2 - \mu I_2 \qquad ..(1)$$

$$\frac{dA_T}{dt} = r_1 cA - (1-\sigma_2)A_T - \mu A_T$$

$$\frac{dA}{dt} = \eta cI + (1-\sigma_2)A_T - r_1 cA - \alpha A - \mu A - r_2 A$$

$$\frac{dA_2}{dt} = r_2 A - (\alpha_2 + \mu)A_2$$

where k is the effective contact rate given as

$$k = \frac{n_1\beta_1 I_2 + n_2\beta_2 A_2 + n_3\beta_3 A_T}{N}$$

With the following initial conditions

$S(0)>0, V(0)>0, H(0)>0, E(0)>0, I(0)>0, I_2(0)>0, A_2(0)>0, A_T(0)>0, A(0)>0$

With the effective contact rate

$\beta_1 > \beta_2 > \beta_3$

And $$N = S + V + H + E + I + I_2 + A + A_T + A_2$$

3.0 MODEL ANALYSIS

The normalized model system of equation will be analyzed qualitatively to get insight into the dynamical features of the systems which will allow us to understand the effect of condom, vaccines, public health campaign and therapeutic doses on the treatment and prevention of HIV/AID infections in the population. The threshold quantity which governs the persistence or the elimination of HIV/AIDS will be determined and studied. We begin by finding the invariant region and show that all the solution of system are positive for all time *t.*

3.1 BASIC PROPERTIES OF THE MODEL: INVARIANT REGION

Since the model system of the normalized equation is studying HIV/AIDS in the population, the model is dealing with human population. We assume that all the state variables and parameters are positive all the time $t \geq 0.$

We will obtain the region by considering the following theorem.

Theorem (1): The solutions of the system are feasible for all $t \geq 0.$ if they enter the invariant region $\Omega \in R_+^9.$

***Proof*:**

We first show that all the feasible solutions are uniformly bounded in a proper subset of the region $\Omega \in R_+^9.$

Let $\{s(t), h(t), v(t), e(t), i(t), i_2(t), a(t), a_1(t), a_2(t)\} \in R_+^9$

be any solution of the system (4.0.7) given by $\{s(t), h(t), v(t), e(t), i(t), i_2(t), a(t), a_1(t), a_2(t)\} \in R_+^9$

where $s>0,\ h>0,\ v>0,\ e>0,\ i>0,\ i_2>0,\ a>0,\ a_1>0,\ a_2>0$ with non-negative initial conditions

From the normalized model system we have

$N = s + h + v + e + i + i_2 + a + a_1 + a_2$

$$\frac{dN}{dt} = \pi + \varepsilon H - \delta_1 cS - \delta_2 cS - \omega S + pV - \mu S - rkS + \delta_1 cS$$
$$-(1-\theta_1)kV - pV - \mu V + \delta_2 cS + \omega S - \varepsilon H - (1-\varphi)kH - \mu H$$
$$+rkS + (1-\varphi)kH + (1-\theta_1)kV - \phi E - \mu E + \phi E + (1-\sigma_1)I_2 - \tau_c I - \eta c_3 I - \mu I$$
$$+\eta c_3 I + (1-\sigma_2)A_T - r_1 cA - r_2 A - \alpha A - \mu A + \tau_c I - (1-\sigma_1)I_2 - \mu I_2$$
$$+r_1 cA - (1-\sigma_2)A_T - \mu A_T + r_2 A - \alpha_2 A_2 - \mu A_2$$

$$\frac{dN}{dt} = \pi - \alpha A - \alpha_2 A_2 - \mu S - \mu V - \mu H - \mu E - \mu I - \mu A - \mu I_2 - \mu A_T - \mu A_2$$

$$\frac{dN}{dt} = \pi - \alpha A - \alpha_2 A_2 - \mu N$$

at equilibrium, $\frac{dN}{dt} = 0$

$\Rightarrow 0 = \pi - \alpha A - \alpha_2 A_2 - \mu N.$

if it is disease free equilibrium, then $A = A_2 = 0$ so that

$0 = \pi - \mu N$

$\Rightarrow N = \frac{\pi}{\mu}$

It then follows that

if $\alpha\ (A + A_2) > 0$ then $\frac{dN}{dt} + \mu N \leq \pi$

i.e. $\frac{dN}{dt} + \mu N \leq \pi$ ..(2)

This is a first order non-homogeneous differential inequality, the solution of the above problem is obtained by finding the integrating factor (IF).

If $= e^{\int \mu dt} = e^{\mu t}$

$$e^{\mu t}\frac{dN}{dt} + \mu N e^{\mu t} = \pi e^{\mu t}$$

$$\frac{d}{dt}(Ne^{\mu t}) = \pi e^{\mu t}$$

Integrating both sides of the equation

$$\Rightarrow Ne^{\mu t} = \frac{\pi}{\mu} e^{\mu t} + C$$

When C is a constant of integration? Therefore

$$N = \frac{\pi}{\mu} + Ce^{-\mu t} \qquad ..(3)$$

Applying the initial conditions

When $t = 0 \;\; N(0) = N_0$, therefore

$$N_0 - \frac{\pi}{\mu} = C \qquad \Rightarrow N = \frac{\pi}{\mu} + (N_0 - \frac{\pi}{\mu})e^{-\mu t}$$

$$= \frac{\pi}{\mu}\left(1 - e^{-\mu t}\right) + N_0 e^{-\mu t} \qquad ..(4)$$

Applying Birkhof and Rota's theorem on differential inequality [30], we have that

as $t \to \infty$ then $N = \frac{\pi}{\mu}$.

The total population approaches $\frac{\pi}{\mu}$ as $t \to \infty . \frac{\pi}{\mu}$ is the carrying capacity of the system. Therefore the feasible solution set of the system (1) above will enter the region.

$$\left\{\Omega = (s + h + v + e + i + i_2 + a + a_1 + a_2) \in R_+^9 : s \geq 0, h \geq 0, v \geq 0, e \geq 0,\right.$$

$$\left. i \geq 0, i_2 \geq 0, a \geq 0, a_1 \geq 0, a_2 \geq 0, N = \frac{\pi}{\mu}\right\} \;\; ..(5)$$

In this region the model is biologically feasible. Thus the region is positive and attracting, every solution with the initial condition in Ω remains in the region for $t > 0$, so the model is well posed on Ω and the region is positively invariant therefore model is well posed and biologically meaningful.

3.2 POSITIVITY OF SOLUTIONS

Lemma 1: Let the initial data be $\left\{S(0) \geq 0,\ H(0) \geq 0,\ V(0) \geq 0,\ I(0) \geq 0,\ I_2(0) \geq 0,\ A(0) \geq 0,\ A_1(0) \geq 0,\ A_2(0) \geq 0\right\} \in R_+^9$. Then the solution, Set $\left\{s(t),\ h(t),\ v(t),\ i(t),\ i_2(t),\ a(t),\ a_1(t),\ a_2(t)\right\}$ of the normalized system are positive for all $t \geq 0.$

Proof:

We consider the first equation of the normalized system

We have

$$\frac{ds}{dt} = \pi + \rho v + \varepsilon h - r(n_1\beta_1 i + n_2\beta_2 i_2 + n_3\beta_3 a_2)s - (\delta_1 c + \delta_2 c + \mu)s$$

Or $$\frac{\dot{ds}}{dt} = \pi - \delta_1 cV + \varepsilon H - rks - (\delta_2 c + p + \mu)s$$

Where $k = (n_1\beta_1 i + n_2\beta_2 i_2 + n_3\beta_3 a_2) / N$

$$\frac{\dot{ds}}{dt} \geq -\left(kr + \left(\delta_1 c + \delta_2 c + \mu\right)\right)s \quad \Rightarrow \frac{1}{s}\frac{\dot{ds}}{dt} \geq -\left(rk + \left((\delta_1 c + \delta_2 c + \mu)\right)\right)$$

$$\frac{ds}{s} \geq -(rk + (\delta_1 c + \delta_2 c + \mu))dt$$ and integrating, we have

$$\log(S) \geq -(rk + (\delta_1 c + \delta_2 c + r + \mu))t + d$$

$$\Rightarrow s(t) \geq e^{-(rk + (\delta_1 c + \delta_2 c + r + \mu))t + d}$$

$$s(t) \geq e^d e^{-(rk + (\delta_1 c + \delta_2 c + r + \mu))t}$$

$$s(t) \geq S(0)e^{-(rk + (\delta_1 c + \delta_2 c + r + \mu))t}$$

where d is the constant of integration

$s(0) = s(t)$ (Using the initial condition from lemma) at $t = 0$

$$s(t) \geq s(0)e^{-(k_1 + (\delta_1 c + \delta_2 c + \mu))t} \geq 0$$

Since

$$e^{-(rk+(\delta_1 c+\delta_2 c+r+\mu))t} \geq 0 \text{ and } S(0) \geq 0 \quad ..(6)$$

Using the second equation of the normalized system, we have

$$\frac{dv}{dt} = -(p+\theta)v - (1-\theta_1)(n_1\beta_1 i + n_2\beta_2 i + n_3\beta_3 a_2)v - (\delta_1 c+\mu)v$$

if $1-\theta_1 = 0 \Rightarrow \theta_1 = 1$ then

$$\frac{dv}{dt} = -(p+\theta)v - (1-\theta_1)kv + \delta_1 cs - \mu v$$

$$= -(\rho+\theta)v - (1-\theta_1)kv - \mu v + \delta_1 cs$$

$$= -(\delta c+\mu)v + \delta_1 cs$$

$$\therefore \frac{dv}{dt} \geq -(\delta c+\mu)v$$

where $\delta c = p + \theta_{\mathbf{c}}$

So that

$$\frac{dv}{dt} \geq -(\delta c+\mu)v$$

After Integration and applying the initial conditions

$$v(t) \geq v(0)e^{-(\delta_1 c+\mu)t} \geq 0 \quad \text{for all } t$$

Since $e^{-(\delta_1 c+\mu)t} \geq 0 \quad \forall \ t \geq 0$ ..(7)

Again from the third equation of system (1), we have $\frac{dh}{dt} = (\delta_2 c)s - (1-\varphi)(n_1\beta_1 i + n_2\beta_2 i_2 + n_3\beta_3 a_2)h - (\varepsilon+\mu)h$

This gives that for all $\varphi = 1$

$$\frac{dh}{dt} \geq -(\varepsilon+\mu)h$$

By integrating and applying the initial conditions (from lemma1) we

$$h(t) \geq h(0)e^{-(\varepsilon+\mu)t} \geq 0. \quad (8)$$

Since $(\varepsilon+\mu) \geq 0 \, as \, \mu > 0$

From the fourth equation of the system from (1), we have.

$$\frac{de}{dt} = (1-\varphi)(n_1\beta_1 i_1 + n_2\beta_2 i_2 + n_3\beta_3 a_2)h + (1-\theta_1)(n_1\beta_1 i + n_2\beta_2 i_2 + n_3\beta_3 a_{2\beta_1})v$$

$$+r(n_1\beta_1 i + n_2\beta_2 i_2 + n_3\beta_3 a_2)S - (\theta+\mu)e$$

we discovered that

$\frac{de}{dt} \geq -(\varphi+\mu)$ On integration and applying the initial condition we have

$$e(t) \geq e(0)e^{-(\varphi+\mu)t} \geq 0 \quad ..(9)$$

since $(\varphi+\mu) \geq 0 \, and \, \mu > 0$

From the fifth equation of system (1), we have

$$\frac{di}{dt} = \varphi e + (1-\sigma_1)i_2 - (\tau c + \eta e_3 + \mu)i$$

we have that

$$\frac{di}{dt} \geq -(\tau c + \eta r_3 + \mu)i$$

$$\Rightarrow \frac{di}{i} \geq -(\tau c + \eta r_3) + \mu)dt$$

On integration, and using the initial conditions from lemma1 we have

$$i(t) \geq i(0)e^{-(\tau c+\eta r_3+\mu)t} \geq 0$$

As $\tau c + \eta r_3 + \mu \geq 0$ and $t \geq 0$ …(10)

Considering the Sixth equation of the normalized equation

$$\frac{di_2}{dt} = \tau c i - ((1-\sigma_1)+\mu)i_2$$

After integrating, and use the initial conditions, we have

$$i_2(t) \geq i_2(0)e^{-((1-\delta_1)+\mu)t} \geq 0$$

since $((1-\delta_1)+\mu) \geq 0$ ..(11)

From the seventh equation of the normalized equation, we have

$$\frac{da_1}{dt} = r_1 ca - \left((1-\sigma_2)+\mu\right)a_1$$

we noted that $$\frac{da_1}{at} \geq -\left((1-\sigma_2)+\mu\right)a_1$$

$$\Rightarrow \frac{da_1}{a_1} \geq -\left((1-\sigma_2)+\mu\right)dt$$

upon integration and subsequent use of initial conditions

$$\log(a_1) = -\left((1-\sigma_2)+\mu\right)t$$

$$a_1(t) \geq a_1(0)e^{-((1-\sigma_2)+\mu)t} \geq 0$$

since $$\left((1-\sigma_2)+\mu\right) \geq 0 \quad ..(12)$$

From eight equation of the normalized model system (1) we have,

$$\frac{da}{dt} = \eta ei + (1-\sigma_2)a_1 - (r_1 c + \alpha + r_2 + \mu)a$$

This gives that,

$$\frac{da}{dt} \geq -(r_1 c + \alpha + r_2 + \mu)a$$

After integration and applying the initial conditions

$$a(t) \geq a(0)e^{-(r_1 c+\alpha+r_2+\mu)t} \geq 0$$

for at $$t = 0, a(t) = a(0), and (r_1 + a + r_2 + \mu) \geq 0 \quad (13)$$

Lastly from the ninth equation of the normalized equation, we have

$$\frac{da_2}{dt} = r_2 a - (\alpha + \mu)a_2$$

This gives $$\frac{da_2}{dt} \geq -(\alpha+\mu)a_2$$

After integration, and the application of initial conditions

$$a_2(t) \geq a_2(0)e^{-(\alpha+\mu)t} \geq 0 \quad ..(14)$$

Since $$(\alpha+\mu)t > 0 \text{ and } \mu \geq 0$$

Therefore all the solution of equation are positive for all $t > 0$

Therefore it is true that,

$$s(t) \geq 0,\ v(t) \geq 0,\ h(t) \geq 0$$

$$i(t) \geq 0,\ i_2(t) \geq 0,\ a(t) \geq 0,\ a_1(t) \geq 0,$$

$$a_2(t) \geq 0\ V(t) \geq 0$$

We shall present in our next volume on how to determine the existence of disease free equilibrium since our model is invariant and positive. The disease free equilibrium point will help us to determine the basic reproduction number using the next generation approach.

4.0 MODEL SIMULATIONS.

In this section we simulate our various models from (1) to see the nature and numerical solution to them. We use Matlab 7.1 programming language for our simulation using the following parameter values as indicated in the following table.

Table 3 Parameter values

S/No.	Parameter	Description	Estimated Value	Sources
1	π	Recruitment rate	10,000	[29]
2	ρ	Rate of preventive vaccination	0.01	[31]
3	ω	Individuals that use condom	$0 \leq \omega \leq 1$	[32]
4	μ	Natural death rate	$0.015 \leq \mu \leq 0.025$	[32]
5	α_1	Disease induced death rate	$0.4 \leq \alpha_1 \leq 0.5$	[32]
6	δ_1	Preventive vaccination rate in the population	0.45	[29]
7	δ_2	Rate of condom usage in the population	$0 \leq \delta_2 \leq 1$	[32]
8	θ	Warning rate of the vaccine	0.025	[29]
9	ε	Rate at which condom are used wrongly	0.04	[29]
10	φ	Condom efficiency rate	0.50	Estimated
11	θ_1	Vaccine efficiency	$0 \leq \theta_1 \leq 1$	[31]
12	ϕ	Progression rate of latent individual to infectious class	$0.06 \leq \phi \leq 0.45$	[33]
13	c	Public health campaign rate	$0 \leq c \leq 10$	[33]
14	σ_1	Rate at which people usually stop taking treatment (therapy)	$0.05 \leq \sigma_1 \leq 0.45$	Estimated

15	τ	Treatment rate of infection individual	$0.06 \le \tau \le 0.2$	[33]
16	η	Rate of progression to full blown aids	0.1	[29]
17	r_1	Rate at which those in the AIDS class receive treatment due to effective campaign	$0.1 \le r_1 \le 0.4$	[33]
18	δ_2	Rate of progress to AID of those who stop treatment	0.4	[29]
19	K_i	Force of infection	$0 \le K_i \le 1$	[33]
20	e	Reduction in developing symptom	0.19	[31]
21	r_1	Rate of exposure to infection of those who stop treatment	0.05	Estimated
22	r_2	Rate at which population of those in the AIDS class refused to receive the therapy	0.025	[27]
23	α_2	Diseased induced death of those who refused therapy	$0.3 \le \alpha_2 \le 0.45$	[27]

Using the table above we simulate and generate the following graphs with the following initial conditions $S(0) = 10,000,\ H(0) = 3,000\ V(0) = 2,500\ E(0) = 1,500$.

$I(0) = 1000\ I_2(0) = 600\ A(0) = 500\ A_1(0) = 400\ A_2(0) = 200$

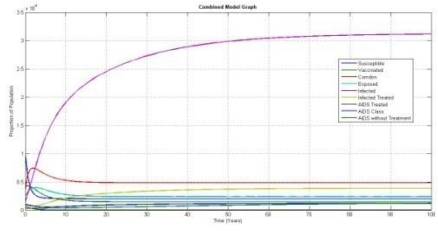

FIGURE 2: VARIATION OF PROPORTION OF TOTAL POPULATION IN DIFFERENT CLASES WITH TIME.

Figure 1 shows ρ the variation of total population with time with the parameter values. $\pi = 2000,\ \delta_2 = 0.5,\ c = 0.5,\ \varepsilon$=0.10, $r = 0.25,\ k = 0.5\ \rho = 0.01,\ \delta_1 = 0.45,\ \mu = 0.02,\ \varphi = 0.4,\ \theta_1 = 0.5,$

$\phi = 0.743,\ \tau = 0.13,\ \eta = 0.1,\ \sigma_1 = 0.08,\ \sigma_2 = 0.4,\ r_1 = 0.05,\ r_2 = 0.025,\ \alpha_1 = 0.45,\ \alpha_2 = 0.375,$

$e = 0.19$

It is observed that the proportions of susceptible population decrease with time and then reaches its equilibrium position after about 5 years. This is due to public health campaign for the population to use condom effectively anytime they may have sex with opposite sex and vaccination for effective protection with the parameters δ_2 and δ_1 respectively. Therefore infection becomes less endemic in the population. Initially the proportion of those in the condom class increase but due to effective use of public health campaign it diminishes after some time. The vaccination class also increased and reaches an equilibrium position than the condom use class this is because vaccination protect the individual more than condom as failure rate of condom due to improper usage may occurs and people tend to prefer to publicly go for vaccination than to buy condom because of the stigma attached to the name and use of condom. Exposed class increased a little and reaches its equilibrium position after some time. The infected population rise sporadically and reaches its equilibrium position after some time, this is due to inability of the population to use condom effectively and failure to present themselves for vaccination, but when the publicity or awareness become strong it then reduces the rate that population got infected. The infected with treatment class reduces but because some people may have some apathy toward taking their daily HAART, the graph shoot up a little because some may be faithful and finally attained its equilibrium position after some time. The Aids class, the Aids with treatment class rises and maintain an equilibrium classed due to effective use of therapy of those in the treatment class and death due to any other cause but those that refused to take therapy, as we can see, the graph drop to zero because of death due to HIV/AIDS, what it means is that, if infected and the person refused to take the therapy, death due to HIV/AIDS is uncontrolled and compulsory.

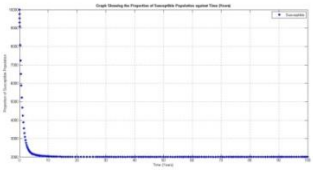

FIGURE 3: GRAPH OF SUSCEPTIBLE POPULATION WITH TIME.

Fig 2 shows that, the susceptible population drops drastically and attained an equilibrium position, due to effective public health campaign, encouraging people to go for vaccination and effective use of condom against the dreaded HIV/AIDS. Therefore the number of susceptible will decline after 10 years if the population are faithful to the use of approved condom using the value of the parameters as specified above and the initial condition.

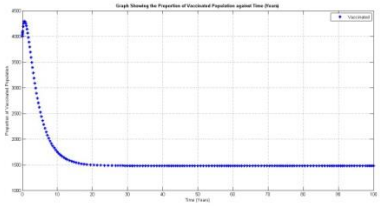

FIGURE 4: GRAPH OF VACCINATION POPULATION WITH TIME

The vaccination class increases and drop drastically and attained an equilibrium position and remain steady after a long period. This is due to effective public health campaign at the beginning, creating awareness for protective vaccination this give rise to a drop in the diseases prevalence within the population upon attaining the equilibrium position the population of the vaccinated class remain steady.

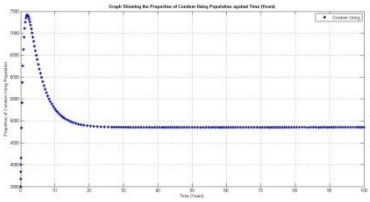

FIGURE 5: PROPORTION OF CONDOM USERS POPULATION WITH TIME

The graph shows a significant increase and drop slowly indicating the stigma attached to the use of condom especially in Nigeria setting, if you are found in public place with condom, it connote infidelity on the part of the users. It also attained the equilibrium position and then remain stable for a long time reducing to number of those to be exposed with the parameter θ_1 .

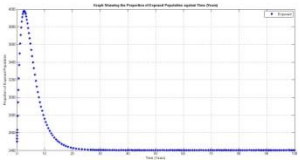

FIGURE 6: PROPORTION OF EXPOSED POPULATION WITH TIME.

The exposed classed behaved like the population of those in the vaccination and condom class. It rises significantly because of lack of awareness and when the public health campaign (c) was significantly used the exposed population drops as a result of effective use of condom and vaccination converge

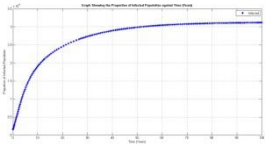

FIGURE 7: PROPORTION OF THE POPULATION OF INFECTED CLASS WITH TIME.

Figure 6 shows a rise in the infected class due to the fact that initially the educational campaign for the population to go for vaccination and usage of government approved condom just commenced and awareness may be very low initially but later when the awareness was created and the policy implemented the infected class reaches an equilibrium position and become stable over time.

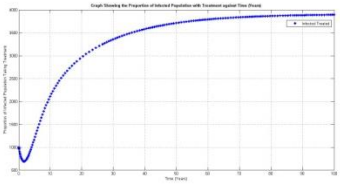

FIGURE 8: PROPORTION OF INFECTED CLASS WITH TREATMENT WITH TIME.

The graph show a drop in the class and shoot up , due to the failure of the patient taking the therapy because initially patients may not be willing to go for therapy but due to effective campaign the infected people presented themselves for therapy which lead to a rise in the graph as shown above, the little drop in the graph above is due to ignorance on the part of the patient on need for the drug use,but if the patient maintain a high level of discipline they will not die of HIV/AIDS. They may die of a natural phenomenon not HIV/AIDS disease. The HAART drugs if effectively applied will boost the immune system and reduces the viral load of the infected.

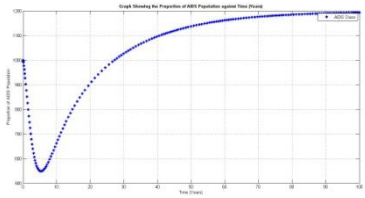

FIGURE 9: PROPORTION OF AIDS POPULTION WITH TIME.

Figure 4.13 show a significant drop and rise steadily until an equilibrium position is attained and remain stable. The decline is due to effective campaign which drop the population initially but after 5 years and due to stigmatization on the AID patient the population shoot up once again. This show that people in this class will die of the disease due to their status

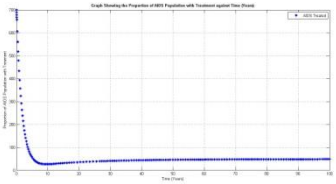

FIGURE 10: PROPORTION OF INFECTED CLASS WITH THERAPY (HAART).

The use of HAART with significant decrease the number of those in this classes and reaches its equilibrium position and remains asymptotically stable for a long time. The HAART if effectively apply prevent HIV/AIDS death.

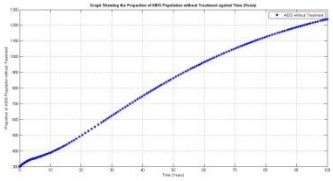

FIGURE 11: HIV/AIDS POPULATION WITHOUT TREATMENT.

Figure 10 indicate that as the patient did not go for therapy the viral load will increase linearly over a long period of time and eventually death of those suffering from HIV/AID, this increase the number of people entering into this class. The only source of defence mechanism is the natural immune systems which are been weaken daily by the activities of the lentivirus cells. The population of those in this class will certainly die of the disease, this will happen because their viral load is on the steady increase.

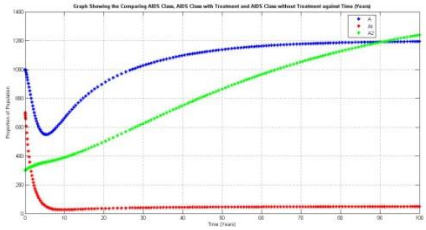

FIGURE 12: COMPARISON OF AIDS CLASS, AIDS CLASS WITH TREATMENT AND WITHOUT TREATMENT.

This show the interaction with various classes for more clear view. After a long period the AIDS with treatment rises above AIDS class without treatment indicating that the treatment has the power to prevent HIV/AIDS death.

5.0 CONCLUSION

Based on the results of this study we conclude that the most effective way to control the spread of the disease (HIV/AIDS) within the population is to use effective public health campaign on the radio, television, newspapers, in churches and mosque, even in schools on the need for individual who cannot abstained from sex to use government approved and certified condom and educate the population to present them self for preventive vaccine, when available. The need for faithfulness on part of married couples also to be encouraged in churches and family members should caution their children on the need to abstain from sex before marriage. And if an individual is HIV positive he/she should be willing to prevent him/her self for ARV treatment and therapy to reduce the viral load and hence prolong their life.

More so, individual within the population at large should be educated on consequences of contracting HIV/AIDS. Government should also introduce VTC to enable people know their HIV status. If the population can show positive attitude through the use of all these control strategies, then in no distance time the presence and effects of HIV/AIDS will be history. We therefore recommend that since HIV/AIDS is a global pandemic and is also delaying human existence in all countries of the world strengthen the existence control strategies and inculcating the new one is very necessary and compulsory in order to curb the spread of this dreaded disease. Thu we recommend that:

1. The HIV/AIDS public health campaign program for all developing countries especially the sub-Sahara African should be developed to a training centre.
2. Stigmatization bill of people living with HIV/AIDS should be passed to punish anyone who is involved, to the HIV/AIDS patients.
3. The use of condom should be legalized and government should donate free condoms to our youth in schools, hotel brutels and motels.
4. Prostitution should be considered a criminal offence and those involved should be cleared from the street. operators of hotel brutel and motels should sign an memorandum of understanding with the government not to allow prostitute within their organization.
5. HIV / AIDS centres for screening and treatment should be established across the countries and the treatment should be made free and to ensure anybody who present themselves for ARV treatment receives it. This will reduce the viral load of the infectious and therefore reduces the AIDS epidemic within the population.
6. Government should enforce all religious leaders to at least preach in organization on the dangers of HIV/AIDS if contracted once a month and also on the need for the susceptible population to use the control strategies to curb the spread of HIV/AIDS.

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