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Effects of Thermophysical Properties On A Magnetohydrodynamic Flow Over A Flat Plate

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Abstract

The natural convective heat transfer of an incompressible, viscous, electrically conducting fluid past a vertical impermeable flat plate under the influence of a uniform magnetic field have been analyzed. Furthermore, using a similarity variable, the governing flow equations are transformed to non-linear coupled differential equations corresponding to a two point boundary value problem, which is solved using symbolic software Mathematica 8.0. A comparison of the solution technique is carried out with previous work and the results are found to be in good agreement. Numerical results for the coefficient of skin friction, the local Nusselt number, as well as the velocity and temperature profiles are presented for different physical parameters. The analysis of the obtained results show that the field of flow is significantly influenced by these parameters.

Keyword: Magnetohydrodynamics, heat transfer, impermeable surface, slip flow. natural convection, variable thermal conductivity, variable viscosity, similarity solution.

1. INTRODUTION

Many researchers have studied the effects of magnetic fields on induced or natural heat transfer problems, and have found Magnetohydrodynamics (MHD) to be useful. The knowledge of MHD is applied in Chemical Engineering in the production of drugs, chemicals, food and other products. Application is also found in petroleum industries, Geophysics since the earth as a planet is comprised of magnetic fields. Flow under the influence of a magnetic field can be applied in Engineering and metallurgy in order to continuously produce aluminum. The importance of fluid flow influenced by the presence of a magnetic field has prompted many researchers to investigate and study the subject. The Pioneer research work in this area can be traced to [1] who presented a theoretical result for the boundary layer flow over a flat plate in a uniform stream and on a circular cylinder. Later, numerical and analytical results to the classical Blasius problem were reported. The similarity solution for thermal boundary layer over a flat plate subject to convective surface boundary conditions was studied in [2]. He found out that the similarity solution for a constant convective coefficient of heat transfer does not exist. Instead, the solutions could be used as local similarity solutions. Ishak [3], expanded [2] with no modification to the flow equations but boundary conditions affected by suction and injection. He found that the surface shear stress increased due to suction, which increased the rate of heat transfer at the surface. The effects of chemical reaction on MHD flow with suction and injection was investigated in [4]. The flow equations were solved analytically using perturbation method. The team found that as the Grashof number increased so did velocity distribution. Particularly, temperature increases as the surface convection is increased for a wall that stretches and shrinks [5]. All of the aforementioned research work considered no-slip at the boundary. At the interface between the solid and fluid, there is zero velocity. This is known as the condition of no-slip. However, at very low pressure or for a flow system that is small, slip flow occurs. In micro electro mechanical system (MEMS), the region of imbalance at the interface between the solid and fluid is more accurately described using a slip flow model. There exists an exact solution to a modified micro-channel couette flow [6]. Thermal effects on the boundary layer flow over a flat plate was studied in [7]. The solutions were obtained numerically. The results obtained showed that as the slip increased so did the stream function. In summary for slip flows, as the shear stress at the wall decreases so did the slip velocity, this happens when the slip parameter is increased. In the review thus far, we have considered constant fluid properties. Experimental studies indicate that the thermophysical properties of fluids changes for large temperature variation: while the temperature increases, the transport phenomenon increases. As this happens, the physical properties across the thermal boundary layer is reduced. This consequently affects the heat transfer at the wall. Therefore in order to accurately predict the flow and heat transfer rates, it is imperative that the fluid properties are not taken to be constant all through the flow rather they should be seen as variables. Asogwa[8] studied the numerical solution of hydromagnetic flow past an infinite vertical porous plate in presence of constant suction and heat sink. He found out that the suction parameter, heat sink parameter, Hartmann number and permeability parameter significantly affected the flow. Temperature fields are only affected by the reduction in Reynolds number [9]. The effects of viscosity dissipation on a viscoelastic flow over a stretching surface has been investigated in [10]. It was found that the viscosity dissipation is largely dependent on the Brinkmann's constant of the material. In [11], the influence of viscosity dependent on temperature, on a hydrodynamic flow was studied. It was studied over a surface, moving continuously. The results were obtained numerically using Runge-Kutta method to the fourth order. The results obtained revealed that as the variable viscosity parameter decreases the temperature is increased. And the velocity increases as the variable viscosity parameter decreases. In [12], the effects of injection and suction on a boundary layer was considered. The results were obtained numerically using a method similar to the method used in [13]. It was reported that, in the boundary layer the rate at which heat was transferred is greatly govern by the suction and injection parameters. Thermal radiation on a MHD flow over a flat plate of uniform heat source and sink has been investigated in [14]. It was observed that the Nusselt number increases for pertinent flow parameters. In [15], the effects of blowing and suction on a thermal boundary layer was studied. An analytical solution was obtained using perturbation method, after which, graphical representations of pertinent fluid parameters were presented. Heat and mass transfer on a MHD flow over a semi-infinite flat plate in a porous medium was studied in [16]. The results obtained indicated that the concentration of the buoyancy effects was enhanced by the increase in Grashof number. The influence of the heat absorption on the system was that which reduced the temperature of the fluid; this resulted in decrease in the velocity of the fluid.

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The Prandtl number is directly proportional to the fluid viscosity, it is again inversely proportional to the thermal conductivity of the fluid by definition. With thickness of the boundary layer depending on the Prandtl number, so, as the viscosity and the thermal conductivity of the fluid vary with temperature, in the same way does the Prandtl number. The Prandtl number has to be a variable [17, 18]. In addition, significant errors occur for regimens that use constant Prandtl number and other temperature dependent fluid parameters. In [19, 17], the locally similar equations were solved using a technique put forward in [20]. The results obtained showed that the thermal conductivity parameter increases, as the velocity and temperature increased. The effects of thermal radiation and thermophoresis on a hydromagnetic flow over a flat plate was investigated in [21]. It was reported that increasing Prandtl number decreased the thermal boundary layer. Similarly, the influence of viscous dissipation and internally decaying heat generation on a MHD flow over a flat plate was investigated in [22]. It was found that increasing the Prandtl number decreases both the rate of heat transfer and the temperature at the surface of the plate. Analyzing numerically the chemical reaction on a MHD flow past a flat plate, it was found that the Nusselt number decreased as a result of the increase in slip flow [23]. In [24], the effects of Navier slip together with Newtonian heating on a hydro magnetic flow was considered. The numerical solution of the resulting flow equations was obtained. He found that as the flow became more unsteady and Newtonian heating increased at the boundary layer, the slip parameter significantly affected the skin friction and transfer rate of heat. The present work is an extension of [18]. The free convective thermophysical effects of MHD flow past a flat plate is investigated. The flow equations are solved numerically, having been transformed using a suitable similarity variable. Graphical and tabular representations of important hydromagnetic features of the flow are presented.

1.0 Formulation of the problem

2.1 Analysis of Flow

A MHD flow of a steady, two-dimensional, electrically conducting, viscous and incompressible fluid is considered. The fluid of temperature T_∞ and uniform velocity U_∞ moves along a vertical impermeable flat plate that is heated. The flow is influenced by a uniform magnetic field.

2. 2 Governing Equations

Take x-axis along the plate to be the direction of flow and y-axis perpendicular to the x-axis. Figure 1 shows the co-ordinate system and the flow configuration.

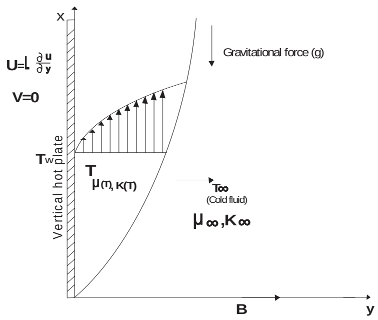

Figure 1 Sketch of flow geometry (*source:*[13]***)***

At the upper surface of the plate, cold fluid flows at temperature T_∞ and uniform velocity U_∞, while heat is transferred through convection to the lower surface of the plate causing the temperature to be $T_w(T_w > T_\infty)$. The fluid near the wall (hot fluid) transfers heat between the wall and the fluid farther out at coefficient h_f.We assume that thermal conductivity and viscosity of the fluid are dependent on temperature.

A uniform magnetic field is applied perpendicular to the plate parallel to the y-axis and varies in strength as a function of x, such that **B** = (0, B(x)). The external electric field is assumed to be zero. The induced magnetic field is negligible when compared with the applied magnetic field. This is so because of the assumed smallness of magnetic Reynolds number. In same manner, viscous dissipation, pressure gradient and Joule heating effects in comparison with the effects of heat source/sink are neglected. The flow is due to buoyancy forces necessitated by the density gradients created by temperature differences in the medium.

As a result of the aforementioned conditions and assumptions, the governing flow equations subject to the Boussineq's approximations are given by:

Equation of continuity: (2.1)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Equation of motion: (2.2)

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta(T - T_\infty) - \sigma B^2(u - U_\infty)$$

Energy equation: (2.3)

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$

Where u, v are velocity components along x, y-axis respectively, ρ = density of the fluid, μ =coefficient of viscosity, g = acceleration due to gravity, β =coefficient of thermal expansion, T & T_∞ = fluid temperature within the boundary and in the free stream respectively. σ = electric conductivity, B = uniform magnetic field or magnetic strength, C_p = specific heat at constant pressure and k = thermal conductivity.

As the temperature increases, the viscosity decreases. This affects the rate of heat transfer at the surface of the plate. Hence, it is necessary that the thermal conductivity and viscosity is properly taken into account. To accurately predict the flow and rate of heat transfer, the temperature dependent viscosity in [21] is adopted;

$$\mu(T) = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)} = \frac{1}{A(T - T_r)} \qquad (2.4)$$

Where,

$$A = \frac{\gamma}{\mu_\infty} \; and \; T_r = T_\infty - \frac{1}{\gamma}$$

γ = thermal property of the fluid,

A and T_r are constants whose values depend on the γ and reference state.
Similarly, the thermal conductivity in [25] is adopted

$$k(T)=k_\infty\left(1+\varepsilon\left(\frac{T-T_\infty}{T_w-T_\infty}\right)\right) \quad (2.5)$$

Where
ε = variable thermal conductivity parameter and
k_∞ = thermal conductivity of the fluid far away from the plate.

2.3 Boundary Conditions

I. on the upper surface of the plate (y=0)

$$u(x,0)=L\frac{\partial u}{\partial y}(x,0) \quad v(x,0)=0 \quad -k(T)\frac{\partial T}{\partial y}(x,0)=h_f\left(T_w-T(x,0)\right) \quad (2.6)$$

$$(\textit{partial slip} \quad \textit{impermeable surface} \quad \textit{convective surface heat flux})$$

II. matching with the free stream $(y\to\infty)$

$$u(x,\infty)=U_\infty \qquad T(x,\infty)=T_\infty \quad (2.7)$$

where L = slip length and the subscripts w and ∞ are respectively wall and boundary layer edge.

2.4 Introduction to Dimensionless Form

The dimensionless variables for ψ and T with respect to a similarity variable are introduced following [23]as:

$$\psi=\sqrt{\upsilon_\infty U_\infty x}\,f(\eta) \quad (2.8)$$

$$T=T_\infty+(T_w-T_\infty)\theta(\eta) \quad (2.9)$$

$$\eta=y\sqrt{\frac{U_\infty}{\upsilon_\infty x}} \quad (2.10)$$

Where,
ψ is the stream function,
η is the similarity variable,
$\upsilon_\infty=\mu_\infty/\rho$, υ_∞ = kinematic viscosity of the ambient fluid,

μ_∞ = dynamic viscosity at ambient temperature, f and θ are dimensionless stream function and temperature respectively. Adopting the dimensionless form of temperature θ in [18]:

$$\theta=\frac{T-T_r}{T_w-T_\infty}+\theta_r \quad (2.11)$$

Where

$$\theta_r=-\frac{1}{\gamma(T_w-T_\infty)}=\frac{(T_r-T_\infty)}{(T_w-T_\infty)}$$

θ_r = variable viscosity parameter.
Thus,

$$\mu=-\mu_\infty\frac{\theta_r}{(\theta-\theta_r)}=\mu_\infty\frac{\theta_r}{(\theta_r-\theta)} \quad (2.12)$$

The viscosity and temperature characteristics of the fluid together with temperature difference (T_w-T_∞) determines the value of θ_r .
Equation (2.4) has been adopted by numerous researchers to model boundary layers with viscosity that is dependent on temperature. The choice of equation (2.4) is suitable for this work since it holds for wider range of temperatures.
Using equations (2.8) and (2.9), we have

$$u=\frac{\partial\psi}{\partial y}=U_\infty f', \qquad v=-\frac{\partial\psi}{\partial x}=-\frac{1}{2}\left(\frac{\upsilon_\infty U_\infty}{x}\right)^{\frac{1}{2}}(f-\eta f') \quad (2.13)$$

Here, primes denote differentiation with respect to η. Now for equations (2.13), the continuity equation (2.1) is satisfied automatically. Substituting equations (2.5), (2.9), (2.12) and (2.13) where necessary in equations (2.2) and (2.3), we get

$$\frac{\theta_r}{\theta_r-\theta}f'''+\frac{\theta_r}{(\theta_r-\theta)^2}f''\theta'+\frac{1}{2}f''f+\frac{Gr_x}{\mathrm{Re}^2{}_x}\theta-Ha^2(f'-1)=0 \quad (2.14)$$

$$(1+\varepsilon\theta)\theta''+\varepsilon\theta'^2+\frac{1}{2}\Pr_\infty f\theta'=0 \quad (2.15)$$

where

$\Pr_\infty=\frac{\mu_\infty C_p}{k_\infty}$ *is the ambient* Pr*andtl number*

The corresponding boundary conditions are:

$$f'(0)=\delta f''(0), \qquad f(0)=0 \qquad \theta'(0)=-a\left(\frac{1-\theta(0)}{1+\varepsilon\theta(0)}\right) \quad (2.16)$$

$$f'(\infty)=1 \quad \textit{and} \quad \theta(\infty)=0 \quad (2.17)$$

where

$Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\upsilon_\infty}$ *is the local Grashof number*, $\mathrm{Re}_x = \frac{U_\infty x}{\upsilon_\infty}$ *is the local Reynolds number,*

$Ha = B\sqrt{\frac{\sigma x}{\rho U_\infty}}$ *is the magnetic parameter*, $\delta = L\left(\frac{U_\infty}{\upsilon_\infty x}\right)^{\frac{1}{2}}$ *is the slip parameter and*

$a = \frac{h_f}{k_\infty}\left(\frac{\upsilon_\infty x}{U_\infty}\right)^{\frac{1}{2}}$ *is the surface convection parameter*

Notice that Gr_x , Ha, δ and a are dependent on x. All the aforementioned parameters must be constant if equation (2.14) and equation (2.15) must have a similarity solution. If we assume that B(x), h_f , β, are proportional to $x^{-\frac{1}{2}}$ [7, 25, 26], then, Ha, Gr_x , δ and a would not depend on x. Therefore, we assume that $B(x) = B_0 x^{-\frac{1}{2}},\ h_f = a x^{-\frac{1}{2}},\ \beta = b x^{-\frac{1}{2}}$ where B_0, a and b are constants.

Observe that δ is a function of x, so for flow with slip over flat plate does not possess self similar solution. However, the mass and momentum conservation is still preserved using this approach, and as such, it is still valid to study the flow dynamics within the boundary layer [6]. Thus the solution of equation (2.14) subject to equations (2.16) for fixed value of δ would be locally similar. Locally similarity approach implies that the dimensionless quantity δ is determined for any values of x and the upstream history of the flow will be ignored, except as far as it influences the similarity variable.

2.5 Variable Prandtl Number

The standard definition of the Prandtl number Pr, denotes that it is a function of thermal conductivity, specific heat and viscosity [18]. Since the fluid parameters continue to differ across the boundary layer, Pr is given by:

$$\mathrm{Pr} = \frac{\mu C_p}{k} = \frac{\frac{\theta_r}{\theta - \theta_r}\mu_\infty C_p}{k_\infty(1+\varepsilon\theta)} = \frac{1}{\left(1 - \frac{\theta}{\theta_r}\right)(1+\varepsilon\theta)}\mathrm{Pr}_\infty \qquad (2.18)$$

with equation (2.18), equation (2.14) becomes

$$(1+\varepsilon\theta)\theta'' + \varepsilon\theta'^2 + \frac{1}{2}\mathrm{Pr}\left(1 - \frac{\theta}{\theta_r}\right)(1+\varepsilon\theta) f\,\theta' = 0 \qquad (2.19)$$

Equation (2.19) is the dimensionless form of the energy equation for varying Prandtl number.

2.6 Skin Friction Coefficient and Nusselt Number

The coefficient of skin friction (rate of shear stress) and the Nusselt number (rate of transfer) are quantities of practical interests.

I. The local skin friction coefficient C_f , is given by the formula:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty} \qquad (2.20)$$

where

$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ *is the local wall shear stress*

Using equation (2.13) and equation (2.12), equation (2.20) becomes

$$C_f = 2\left(\frac{U_\infty^{\frac{3}{2}}}{(\upsilon_\infty x)^{\frac{1}{2}}} f''(0)\right)\left(\mu_\infty \frac{\theta_r}{\theta_r - \theta(0)}\right)\frac{1}{\rho U_\infty} = 2\,\mathrm{Re}_x^{-1/2}\left[\frac{\theta_r}{\theta_r - \theta(0)}\right] f''(0) \qquad (2.21)$$

$$or \quad C_f^{\ *} = \frac{\theta_r}{\theta_r - \theta(0)} \quad where \quad C_f^{\ *} = \frac{1}{2}\mathrm{Re}_x^{-1/2} \qquad (2.22)$$

II. The local Nusselt number Nu_x Rahman (2011):

$$Nu_x = \frac{1}{2}\mathrm{Re}_x^{1/2}\frac{1}{\theta(0)} \qquad (2.23)$$

$$or \quad Nu_x^{\ *} = \theta(0) \quad where \quad Nu_x^{\ *} = \frac{1}{2}\mathrm{Re}_x^{1/2} Nu_x^{\ -1} \qquad (2.24)$$

2.0 Numerical Solutions

The set of equations (2.14) and (2.15) fall under the category of non-linear equations and with equations (2.14) and (2.15) are coupled, the equations cannot be solved analytically thus a numerical solution is to be found. Notice that the set of equations (2.14) -(2.17) form a two-point boundary value problem (BVP). This can be solved numerically using the shooting method. Notice also that the boundary condition at the end (i.e. the final boundary) is at infinity. We need a finite value for $\eta \to \infty$. Let that finite value be η_∞. Thus the numerical solutions of equation (2.14) and (2.15) subject to the boundary conditions equation (2.16) and (2.17)are obtained using symbolic software MATHEMATICA 8.0.

We proceed to confirm the validity of our model. To do this, we set the fluid properties to be constants in order to make them particular forms of the equations known in literature, thereafter a comparison is made. Considering the case of no-slip at the boundary, absence of buoyancy force and magnetic field, and constant fluid properties i.e. ($\theta_r \to \infty,\ \varepsilon = 0,$), this present work matches that in [2] and [3]. To actually test our reduced model for accuracy, a comparison of the numerical solutions for various values of the Prandtl number and surface convection parameter $\theta_r \to \infty,\ \varepsilon = 0,\ Ha = 0,\ Gr_x = 0\ and\ \delta = 0$ are calculated for $-\theta'(0)$ and presented on Table 1.

Table 1 Different values of a and Pr when $\theta_r \to \infty,\ \varepsilon = 0,\ Ha = 0,\ Gr_x = 0\ and\ \delta = 0$ ***for*** $-\theta'(0)$

A	Present results		[2]		[3]	
	Pr =0.1	Pr=0.72	Pr =0.1	Pr=0.72	Pr =0.1	Pr=0.72
20	0.146106	0.291329	0.1461	0.2913	0.139056	0.291329
10	0.145047	0.287146	0.1450	0.2871	0.138096	0.287146
5	0.142973	0.279131	0.1430	0.2791	0.136215	0.279131
1	0.128299	0.228178	0.1283	0.2282	0.122830	0.228178
0.80	0.124311	0.215864	0.1243	0.2159	0.119170	0.215864
0.60	0.118189	0.198051	0.1182	0.1981	0.113533	0.198051
0.40	0.107593	0.169994	0.1076	0.1700	0.103720	0.169994
0.20	0.0847866	0.119295	0.0848	0.1193	0.082363	0.119295
0.10	0.0595439	0.0747242	0.0594	0.0747	0.058338	0.074724
0.05	0.0373213	0.0427669	0.0373	0.0428	0.036844	0.042767

From Table 1 we can deduce that the numerical technique used in this work is justified. This is so because the data given in [2] and [3] and those of the present study show excellent agreement.

3.1 Results and Discussion

Numerical computations have been carried out to analyze the results using the method described in the preceding section for different values of the slip parameter δ, surface convection parameter a, viscosity parameter θ_r, Grashof number Gr_x, thermal conductivity parameter ε, Reynolds number Re_x, Hartmann number Ha and Prandtl number Pr within the boundary layer. Only positive numbers of Grashof number are chosen, since we are considering a cooling problem. Take $\theta_r = 3, \varepsilon = 0.5, Ha = 1, Gr_x = 10, \delta = 0.5, a = 0.5, Re_x = 1\ and\ Pr = 1$ to be the default values of the parameters unless otherwise specified.

3.2 Computational Results for Fluid Flow

The non-dimensional velocity profiles and temperature profiles for both slip and no-slip flows within the boundary layer corresponding to values of the surface convection parameter a, thermal conductivity parameter ε, Hartmann number Ha, Grashof number Gr_x, viscosity parameter θ_r, Reynolds number Re_x and Prandtl number Pr are presented in the following figures.

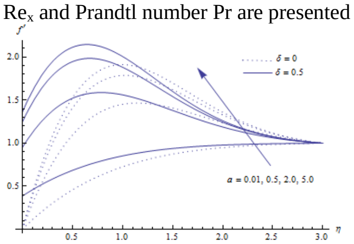

Figure 2 Velocity profiles for different values of surface convection and slip parameter

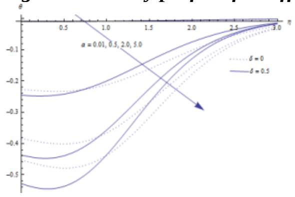

Figure 3 Temperature profiles for different values of surface convection and slip parameter

Figure 2 and Figure 3 show the effect of surface convection parameter, a on the velocity and temperature profiles respectively. For increasing values of a, we see that the velocity increases for both slip and no-slip flows. Whereas the temperature decreases in the boundary layer region and is maximum at the surface of the plate for both slip and no-slip flows.

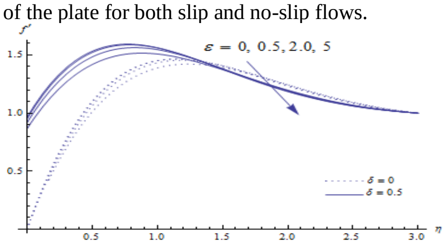

Figure 4 Velocity profiles for different values of thermal conductivity parameter and slip parameter

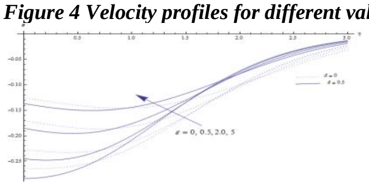

Figure 5 Temperature profiles for different values of Thermal conductivity and slip parameter

The effect of thermal conductivity parameter ε, on the velocity and temperature profiles are shown on Figure 4 and Figure 5 respectively. The velocity decreases with increase in ε shown on Figure 4 for both slip and no-slip flows. From Figure 5 we see that the temperature increases with increase in ε but farther away from the plate the temperature profiles overlap and decreases from then on for both slip and no-slip flows.

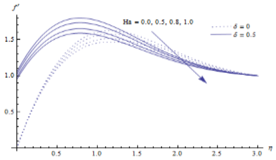

Figure 6 Velocity profiles for different values of Hartmann number and slip parameter

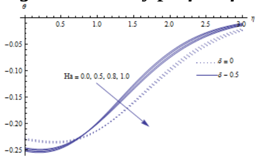

Figure 7 Temperature profiles for different values of Hartmann number and slip parameter

Figure 6 and Figure 7 displays the effect of the Hartmann number on the velocity and temperature profiles respectively. We see from both plots that the velocity and temperature profiles decreases monotonically. Interestingly, on Figure 7, far away from the plate these temperature profiles overlap leading to increase in temperature.

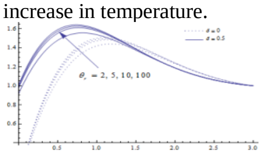

Figure 8 Velocity profiles for different values of Viscosity and slip parameter

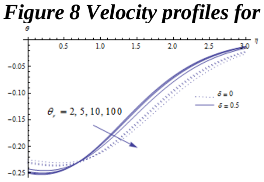

Figure 9 Temperature profiles for different values of Viscosity and slip parameter

We see from Figure 8 that the velocity increase with the increase in the viscosity parameter for both slip and no-slip flow. On the other hand, from Figure 9 the temperature decreases. Away from the plate, the temperature profiles overlap resulting in decreasing temperatures for increasing values of the viscosity parameter for both slip and no-slip flow.

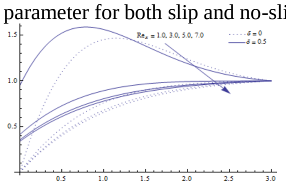

Figure 10 Velocity profiles for different values of Reynolds number and slip parameter

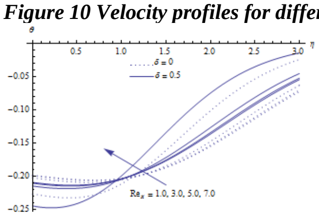

Figure 11 Temperature profiles for different values of Reynolds number and slip parameter

Figure 10 and Figure 11 show the effect of Reynolds number on the velocity and temperature profiles respectively. For increasing values of Reynolds number, we see that the velocity decreases for both slip and no-slip flows. Whereas the temperature increases in the boundary layer region and decreases far away from the plate for both slip and no-slip flows shown in Figure 11.

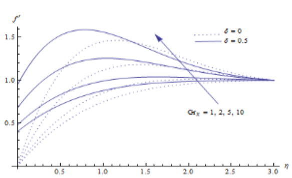

Figure 12 Velocity profiles for different values of Grashof number and slip parameter

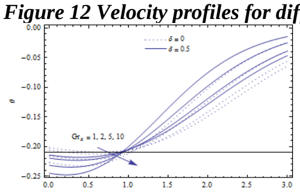

Figure 13 Temperature profiles for different values of Grashof number and slip parameter

The effect of Grashof number Gr_x, on the velocity and temperature profiles are shown on Figure 12 and Figure 13 respectively. For increasing values of Gr_x, we see that the velocity increases for both slip and no-slip flows. Whereas the temperature decreases monotonically in the boundary layer region and increases far away from the plate for both slip and no-slip flows

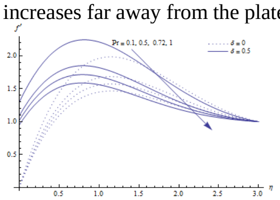

Figure 14 Velocity profiles for different values of Prandtl number and slip parameter

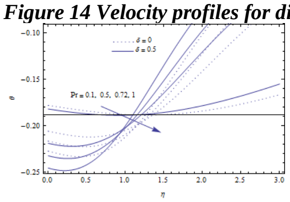

Figure 15 Temperature profiles for different values of Prandtl number and slip parameter

Figure 14 and Figure 15 show the effect of Prandtl number on the velocity and temperature profiles respectively. For increasing values of the Prandtl number the velocity decreases for both slip and no-slip flows. Whereas the temperature decreases in the boundary layer region and is maximum at the surface of the plate for both slip and no-slip flows.

The coefficient of skin friction and Local Nusselt number are parameters of practical interest. The former indicates the rate of physical wall shear stress while the later indicates the rate heat is transferred. Proportional to the skin friction and Nusselt number are the values f''(0) and $\theta(0)$ respectively, which are calculated and presented in Table 2 below.

Table 2 Values of f ''(0), $\theta(0)$ and $-\theta'(0)$ for different values of δ and θ_r when ε = 0.5, a = 0.5, Pr =1 and Ha = 1

Parameters			$\delta = 0$			$\delta = 0.5$		
θ_r	ε	a	f''(0)	$\theta(0)$	$-\theta'(0)$	$-\theta'(0)$	$\theta(0)$	f''(0)
3	0	0.01	1.17235	0.0213943	0.00978606	0.0098728	0.0192724	0.771424
3	0	0.5	3.23949	0.470512	0.264744	0.28385	0.432299	1.9245
3	0	1.0	3.78292	0.6312237	0.368763	0.407627	0.592373	2.24013
3	0	10.0	4.60861	0.942261	0.577394	0.680739	0.931926	2.7443
3	0.2	0.01	1.17215	0.0213501	0.00974489	0.0097705	0.0192366	0.771329
3	0.2	0.5	3.21992	0.459457	0.247526	0.266301	0.422402	1.91504
3	0.2	1.0	3.77235	0.617041	0.340891	0.377619	0.578677	2.23548
3	0.2	10.0	4.65989	0.936929	0.531172	0.62633	0.92577	2.77875
3	0.8	0.01	1.17159	0.0212197	0.0096442	0.00966084	0.0191307	0.771045
3	0.8	0.5	3.16168	0.431718	0.211199	0.228569	0.397493	1.8861
3	0.8	1.0	3.72782	0.581189	0.285887	0.317478	0.544283	2.21402
3	0.8	10.0	4.75788	0.9216560	0.450949	0.531286	0.908267	2.84554
5	0.2	0.01	1.17397	0.0213393	0.00974502	0.00977019	0.0192247	0.77228
5	0.2	0.5	3.33834	0.455652	0.249442	0.268527	0.418043	1.96832
5	0.2	1.0	3.97526	0.611767	0.34591	0.383875	0.572194	2.32919
5	0.2	10.0	5.12303	0.934542	0.551499	0.656961	0.922187	3.01768

From Table 2, we see that the skin friction decreases with increase in θ_r and δ. But the reverse effect is observed as a increases. However, as a and θ_r increases, the Nusselt number increase. But the Nusselt number decreases as δ increases.

3.0 CONCLUSION

Effects of thermophysical properties for a hydromagnetic flow of a viscous incompressible electrically conducting fluid over a vertically flat plate with partial slip at the surface of the boundary in the presence of the convective boundary condition is studied. Viscosity and thermal conductivity are considered to be temperature dependent. Non-dimensional velocity and temperature profiles within the boundary layer are displayed. Presented in tabular form is the skin friction and the rate of heat transfer from the plate to the fluid for different values of the pertinent parameters governing the flow. The following conclusions can be made from the present study:

1. The velocity profiles are increasing for increasing values of the surface convection parameter a, Grashof number Gr_x, and viscosity parameter θ_r. It decreases for decreasing values of the thermal conductivity parameter ε, Hartmann number Ha, Reynolds number Re_x and Prandtl number Pr.
2. The temperature profiles are decreasing for increasing values of the surface convection parameter a, Hartmann number Ha, viscosity parameter θ_r, and Prandtl number Pr. It increases for increasing values of the thermal conductivity parameter ε, Grashof number Gr_x and Reynolds number Re_x.
3. The coefficient of skin friction decreases with increase in variable viscosity parameter and slip parameter but for increase in surface convection parameter the effect is reversed.
4. Nusselt number increases for increasing surface convection parameter and variable viscosity parameter. But decreases for increasing slip parameter.

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