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Solving Higher Dimensional Initial and Boundary Value Problem Using Laplace Homotopy Perturbation Decomposition Method (LHPDM)

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Abstract

In this paper, higher dimensional initial and boundary value problems of variable coefficients were solved by means of relatively new technique which is called the Laplace Homotopy Perturbation Decomposition Method (LHPDM) by combining the Laplace Homotopy Perturbation and Decomposition Method. Several examples were given to verify the accuracy and efficiency of the proposed technique. The analysis was accompanied by numerical examples. The obtained results demonstrate the reliability and the efficiency of the method.

Keyword: Boundary value problem, Homotopy transformation, Initial value problem, Non-linear problems, Pertubation decomposition.

1. INTRODUTION

The numerical and analytical solutions of higher dimensional initial boundary value problems of variable coefficients, linear and nonlinear, are of considerable significance for applied sciences. A considerable size of research work has been invested in these scientific applications. Several techniques including the spectral, characteristics, decomposition and variational homotopy perturbation have been used for solving these problems. It is worth mentioning that the origin of variational iteration method can be traced back to[1] but the true potential of the VIM was explored in [2]. Since the beginning of 1980s, the Adomian's decomposition method has been applied to a wide class of functional equations [3]. In these methods the solution is given in an infinite series usually converging to an accurate solution.
Inspired and motivated by the on-going research in this area, the Laplace Homotopy Perturbation Decomposition Method (LHPDM) is applied for solving the higher dimensional initial boundary value problems. This method is an elegant combination of Laplace homotopy and the Adomian's decomposition methods for solving higher dimensional initial boundary value problems. This idea has been used implicitly in [4], for solving quadratic Riccati differential and Klein-Gordon equations.
In a subsequent work, the elegant coupling of Adomian'spolynomials with the correction functional of variational iteration method for solving various classes of singular and non-singular initial and boundary value problems were developed in [5,6]. Several examples are given to illustrate the reliability and performance of the proposed method.

2.0 Methodology

To illustrate our method, we consider the general form of equation

$$Du(x,y,t)+Lu(x,y,t)+R(x,y,t)+N(x,y,t)=g(x,y) \quad (1)$$

With the initial condition

$$u(x,y,0)=h(x,y)\,,\quad u_t(x,y,0)=f(x,y) \quad (2)$$

Where D is an high-order derivative of time (t), such that $D=\frac{\partial^2}{\partial t^2}$, L is the high-order linear derivative of distance (x), R is a linear differential operator of lower order to L, N represents the non-linear operator and g is the source term.
Firstly, we apply Laplace Transformation, then equation (1) becomes;

$$\mathcal{L}\{Du(x,y,t)+Lu(x,y,t)+R(x,y,t)+N(x,y,t)\}=\mathcal{L}\{g(x,y)\} \quad (3)$$

Making $\ell\{Du(x,y,t)\}$ the subject of the formulae, we obtain

$$\ell\{Du(x,y,t)\}=\ell\{g(x,y)+Lu(x,y,t)+R(x,y,t)+N(x,y,t)\} \quad (4)$$

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Then, decomposing the L.H.S with respect to time (t), leaving the R.H.S so that we can have;

$$\ell\{u(x,y,t)\} = \frac{1}{s^2}\ell\{g(x,y) + Lu(x,y,t) + R(x,y,t) + N(x,y,t)\} + \frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t) \quad (5)$$

(i.e, a twice Laplace decomposition of velocity u with respect to time (t))
Then, applying homotopy with perturbation parameter P, we have

$$u(x,y,t) = u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots \quad (6)$$

So that we would have the following after substituting the value for velocity in the system of equation above; (7)

$$\ell\begin{Bmatrix} u_0(x,y,t) + Pu_1(x,y,t) + \\ P^2u_2(x,y,t) + \ldots \end{Bmatrix} = \begin{bmatrix} \frac{1}{s^2}\ell\begin{Bmatrix} g(x,y) + \\ L(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) \\ + R(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) \\ + N(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) \end{Bmatrix} \\ + \frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t) \end{bmatrix}$$

Perturbing the system on the R.H.S in term of velocity u, so we may have;

$$\ell\begin{Bmatrix} u_0(x,y,t) + Pu_1(x,y,t) + \\ P^2u_2(x,y,t) + \ldots \end{Bmatrix} = \begin{bmatrix} \frac{1}{s^2}\ell\begin{Bmatrix} g(x,y) \\ + PL(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) \\ + PR(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) \\ + PN(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) \end{Bmatrix} \\ + \frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t) \end{bmatrix} \quad (8)$$

Expandingthe system,we have

$$\ell\begin{Bmatrix} u_0(x,y,t) + Pu_1(x,y,t) + \\ P^2u_2(x,y,t) + \ldots \end{Bmatrix} = \begin{bmatrix} \frac{1}{s^2}\ell\begin{Bmatrix} g(x,y) \\ + (LPu_0(x,y,t) + LP^2u_1(x,y,t) + LP^3u_2(x,y,t) + \ldots) \\ + (RPu_0(x,y,t) + RP^2u_1(x,y,t) + RP^3u_2(x,y,t) + \ldots) \\ + (NPu_0(x,y,t) + NP^2u_1(x,y,t) + NP^3u_2(x,y,t) + \ldots) \end{Bmatrix} \\ + \frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t) \end{bmatrix} \quad (9)$$

Then, the equation becomes

$$\begin{pmatrix} u_0(x,y,t) + Pu_1(x,y,t) + \\ P^2u_2(x,y,t) + \ldots \end{pmatrix} = \ell^{-1}\begin{bmatrix} \frac{1}{s^2}\ell\begin{Bmatrix} g(x,y) \\ + (LPu_0(x,y,t) + LP^2u_1(x,y,t) + LP^3u_2(x,y,t) + \ldots) \\ + (RPu_0(x,y,t) + RP^2u_1(x,y,t) + RP^3u_2(x,y,t) + \ldots) \\ + (NPu_0(x,y,t) + NP^2u_1(x,y,t) + NP^3u_2(x,y,t) + \ldots) \end{Bmatrix} \\ + \frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t) \end{bmatrix} \quad (10)$$

Employing the Adomian Polynomial with respect to the non-linear operator still in (10), we make;

$$N(u_0(x,y,t) + Pu_1(x,y,t) + P^2u_2(x,y,t) + \ldots) = \sum_{n=0}^{\infty} A_n = F(u) \quad (11)$$

Where

$$A_n = \frac{1}{n!}\left(\frac{d^n}{d\lambda^n}\right)\sum_{n=0}^{\infty}(\lambda^i u_i)_{\lambda=0} \quad (12)$$

Then we can have;

$$A_0 = F(u_0) \quad (13)$$

$$A_1 = u_1F(u_0) \quad (14)$$

$$= u_2F'(u_0) + \frac{u_1^{\ 2}}{2}F''(u_0)$$

$$A_3 = u_3F'(u_0) + u_1u_0F''(u_0) + \frac{u_1^{\ 3}}{3!}F'''(u_0) \quad (15)$$

Hence(12) becomes

$$\begin{pmatrix} u_0(x,y,t) + Pu_1(x,y,t) + \\ P^2u_2(x,y,t) + \ldots \end{pmatrix} = \ell^{-1}\begin{bmatrix} \frac{1}{s^2}\ell\begin{Bmatrix} g(x,y) \\ + (LPu_0(x,y,t) + LP^2u_1(x,y,t) + LP^3u_2(x,y,t) + \ldots) \\ + (RPu_0(x,y,t) + RP^2u_1(x,y,t) + RP^3u_2(x,y,t) + \ldots) \\ + (PA_0 + P^2A_1 + P^3A_2 + \ldots) \end{Bmatrix} \\ + \frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t) \end{bmatrix} \quad (16)$$

Comparing the system in term of the perturbation parameter P, we have

$$P^0 : u_0 = \ell^{-1}\left[\frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t)\right] \quad (17)$$

$$P^1 : u_1 = \ell^{-1}\left\{\frac{1}{s^2}\ell\{g(x,y) + (LPu_0(x,y,t)) + (RPu_0(x,y,t)) + (PA_0)\}\right\} \quad (18)$$

$$P^2 : u_2 = \ell^{-1}\left\{\frac{1}{s^2}\ell\{g(x,y) + (LPu_1(x,y,t)) + (RPu_1(x,y,t)) + (PA_1)\}\right\} \quad (19)$$

$$P^3 : u_3 = \ell^{-1}\left\{\frac{1}{s^2}\ell\{g(x,y)+(LPu_2(x,y,t))+(RPu_2(x,y,t))+(PA_2)\}\right\} \quad (20)$$

$$P^n : y_n = \ell^{-1}\left\{\frac{1}{s^2}\ell\{g(x,y)+(LPu_{n-1}(x,y,t))+(RPu_{n-1}(x,y,t))+(PA_{n-1})\}\right\} \quad (21)$$

Then, $u(x,y,t)=\sum_{n=0}^{\infty}u_n$ i.e

$$u(x,y,t)=u_0+u_1+u_2+... \quad (22)$$

3.0 Numerical Examples

In this section, we will provide some examples and we use LHPDM to illustrate the effectiveness of our method.

Example 1

Considering the two dimensional initial boundary value problem.

$$u_{tt}=\frac{1}{2}y^2u_{xx}+\frac{1}{2}x^2u_{yy} \quad (23)$$

With the boundary condition

$u(0,y,t)=y^2e^{-1}$ $\qquad u(1,y,t)=(1+y^2)e^{-1}$

$u(x,0,t)=x^2e^{-1}$ $\qquad u(0,1,t)=(1+x^2)e^{-1}$

Satisfying the initial condition

$u(1,y,t)=x^2+y^2$ $\qquad u_t(x,y,0)=-(x^2+y^2)$ (24)

Solution

Firstly, we find the Laplace of the equation

$$\ell\{u_{tt}\}=\ell\left\{\frac{1}{2}y^2u_{xx}+\frac{1}{2}x^2u_{yy}\right\} \quad (25)$$

Decomposing as illustrated, we have

$$\ell\{u\}=\frac{h(x,y)}{s}+\frac{f(x,y)}{s^2}+\frac{1}{s^2}\ell\left\{\frac{1}{2}y^2u_{xx}+\frac{1}{2}x^2u_{yy}\right\}, \quad (26)$$

then finding the Laplace transform of u in (3.2.3), we obtained

$$u=\ell^{-1}\left\{\frac{h(x,y)}{s}+\frac{f(x,y)}{s^2}+\frac{1}{s^2}\ell\left\{\frac{1}{2}y^2u_{xx}+\frac{1}{2}x^2u_{yy}\right\}\right\} \quad (27)$$

According to procedures of homotopy and expansion from the basics, we have the iterative values of u to be as follows;

$$P^0 : u_0 = \ell^{-1}\left\{\frac{1}{s^2}f(x,t)+\frac{1}{s}g(x,t)\right\}$$

$$P^0 : u_0 = (x^2+y^2)-(x^2+y^2)t \quad (28)$$

$$P^1 : u_1 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{\frac{1}{2}y^2u_{0xx}+\frac{1}{2}x^2u_{0yy}\right\}\right\}$$

$$P^1 : u_1 = (x^2+y^2)\left(\frac{t^2}{2}-\frac{t^3}{3!}\right) \quad (29)$$

$$P^2 : u_2 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{\frac{1}{2}y^2u_{1xx}+\frac{1}{2}x^2u_{1yy}\right\}\right\}$$

$$P^2 : u_2 = (x^2+y^2)\left(\frac{t^4}{4!}-\frac{t^5}{5!}\right) \quad (30)$$

$$P^3 : u_3 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{\frac{1}{2}y^2u_{2xx}+\frac{1}{2}x^2u_{2yy}\right\}\right\}$$

$$P^3 : u_3 = (x^2+y^2)\left(\frac{t^6}{6!}-\frac{t^7}{7!}\right) \quad (31)$$

$$P^4 : u_4 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{\frac{1}{2}y^2u_{3xx}+\frac{1}{2}x^2u_{3yy}\right\}\right\}$$

$$P^3 : u_3 = (x^2+y^2)\left(\frac{t^8}{8!}-\frac{t^9}{9!}\right) \quad (32)$$

We have the approximate solution of u to be

$u(x,y,t)=\sum_{n=0}^{\infty}u_n$ i.e

$$u(x,y,t)=u_0+u_1+u_2+... \quad (33)$$

$$u(x,y,t)=(x^2+y^2)\left(1-t+\frac{t^2}{2}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}+...\right)$$

For the exact solution

$$u(x,y,t) = (x^2 + y^2)e^{-t} \quad (34)$$

Example 2

Considering the two dimensional initial boundary value problem.

$$u_{tt} = \frac{1}{45}x^2u_{xx} + \frac{1}{45}y^2u_{yy} + \frac{1}{45}z^2u_{zz} - u \quad (35)$$

With the boundary condition

$u_x(0,y,z,t) = 0$ $\quad u_x(1,y,z,t) = 6y^6z^6\sinh(t)$

$u_y(x,1,z,t) = 6x^6z^6\sinh(t)$ $\quad u_z(x,y,0,t) = 0$

$u_y(x,0,z,t) = 0$ $\quad u_z(x,y,1,t) = 6x^6y^6\sinh(t)$

Satisfying the initial condition

$u(x,y,z,0) = 0$ $\quad u(x,y,z,0) = x^6y^6z^6$ (36)

Solution

According to procedures from the basics, we have the iterative values of u to be as follows;

$$P^0 : u_0 = \mathcal{L}^{-1}\left\{\frac{1}{s^2} f(x,t) + \frac{1}{s} g(x,t)\right\}$$

$$P^0 : u_0 = x^6y^6z^6t \quad (37)$$

$$P^1 : u_1 = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\,\ell\left\{\frac{1}{45}x^2u_{0xx} + \frac{1}{45}y^2u_{0yy} + \frac{1}{45}z^2u_{0zz} - u_0\right\}\right\}$$

$$P^1 : u_1 = x^6y^6z^6\frac{t^3}{3!} \quad (38)$$

$$P^2 : u_2 = \ell^{-1}\left\{\frac{1}{s^2}\,\ell\left\{\frac{1}{45}x^2u_{1xx} + \frac{1}{45}y^2u_{1yy} + \frac{1}{45}z^2u_{1zz} - u_1\right\}\right\}$$

$$P^2 : u_2 = 3x^6y^6z^6\frac{t^5}{5!} \quad (39)$$

$$P^3 : u_3 = \ell^{-1}\left\{\frac{1}{s^2}\,\ell\left\{\frac{1}{45}x^2u_{2xx} + \frac{1}{45}y^2u_{2yy} + \frac{1}{45}z^2u_{2zz} - u_2\right\}\right\}$$

$$P^3 : u_3 = 9x^6y^6z^6\frac{t^7}{7!} \quad (40)$$

$$P^4 : u_4 = \ell^{-1}\left\{\frac{1}{s^2}\,\ell\left\{\frac{1}{45}x^2u_{3xx} + \frac{1}{45}y^2u_{3yy} + \frac{1}{45}z^2u_{3zz} - u_3\right\}\right\}$$

$$P^4 : u_4 = 27x^6y^6z^6\frac{t^9}{9!} \quad (41)$$

We have the approximate solution of u to be

$u(x,y,z,t) = \sum_{n=0}^{\infty} u_n$ i.e.

$$u(x,y,z,t) = u_0 + u_1 + u_2 + \ldots$$

$$u(x,y,z,t) = x^6y^6z^6\left(t + \frac{t^3}{3!} + \frac{3t^5}{5!} + \frac{9t^7}{7!} + \frac{27t^9}{9!}\right) \quad (42)$$

For the exact solution $x^6y^6z^6\left(t + \frac{t^3}{3!} + \frac{3t^5}{5!} + \frac{9t^7}{7!} + \frac{27t^9}{9!}\right)$ (43)

Example 3

Considering the two dimensional initial boundary value problem with a non-linear operator.

$$u_{tt} = 2(x^2 + y^2) + \frac{15}{2}\left(xu^2{}_{xx} + yu^2{}_{yy}\right) \quad (44)$$

With the boundary condition

$u(0,y,t) = y^2t^2 + yt^6$ $\quad u(1,y,t) = (1 + y^2)t^2 + (1 + y)t^6$

$u(x,0,t) = x^2t^2 + xt^6$ $\quad u(0,1,t) = (1+x^2)t^2 + (1+x)t^6$

Satisfying the initial condition

$u(x,y,0) = 0$ $\quad u_t(x,y,0) = 0$ (45)

Solution

Firstly, we find the Laplace of the equation;

$$\ell\{u_{tt}\} = \ell\left\{2(x^2+y^2) + \frac{15}{2}\left(xu^2{}_{xx} + yu^2{}_{yy}\right)\right\} \quad (46)$$

Decomposing as illustrated, we have

$$\ell\{u\} = \frac{h(x,y)}{s} + \frac{f(x,y)}{s^2} + \frac{1}{s^2}\ell\left\{2(x^2+y^2) + \frac{15}{2}\left(xu^2{}_{xx} + yu^2{}_{yy}\right)\right\}, \quad (47)$$

then we have

$$u = \ell^{-1}\left\{\frac{h(x,y)}{s} + \frac{f(x,y)}{s^2} + \frac{1}{s^2}\ell\left\{2(x^2+y^2) + \frac{15}{2}\left(xu^2{}_{xx} + yu^2{}_{yy}\right)\right\}\right\} \quad (48)$$

According to procedures of homotopy and expansion from the basics, and obviously, the nonlinear operator requires us to introduce the Adomian Polynomial series which will provide us with the result;

$$P^0 : u_0 = \ell^{-1}\left\{\frac{1}{s^2}f(x,t) + \frac{1}{s}g(x,t)\right\}$$

$$P^0 : u_0 = 0 \quad (49)$$

$$P^1 : u_1 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{2(x^2+y^2) + \frac{15}{2}(xA_0 + yB_0)\right\}\right\} \quad (50)$$

$$P^2 : u_2 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{2(x^2+y^2) + \frac{15}{2}(xA_1 + yB_1)\right\}\right\} \quad (51)$$

$$P^3 : u_3 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{2(x^2+y^2) + \frac{15}{2}(xA_2 + yB_2)\right\}\right\} \quad (52)$$

$$P^4 : u_4 = \ell^{-1}\left\{\frac{1}{s^2}\ell\left\{2(x^2+y^2) + \frac{15}{2}(xA_3 + yB_3)\right\}\right\} \quad (53)$$

The polynomial formulae gives the following for the respective iterates of A and B

$$A_0 = u^2{}_{0xx} \quad (54)$$

$$A_1 = 2u^2{}_{0xx}u_{1xx} \quad (55)$$

$$A_2 = 2u_{1xx}u^2{}_{0xx} + u^2{}_{1xx} \quad (56)$$

$$A_3 = 2\left(u^2{}_{0xx}u_{3xx} + u_{1xx}u_{2xx}\right) \quad (57)$$

and

$$B_0 = u^2{}_{0yy} \quad (58)$$

$$B_1 = 2u^2{}_{0yy}u_{1yy} \quad (59)$$

$$B_2 = 2u_{1yy}u^2{}_{0yy} + u^2{}_{1yy} \quad (60)$$

$$B_3 = 2\left(u^2{}_{0yy}u_{3yy} + u_{1yy}u_{2yy}\right) \quad (61)$$

Hence,

$$P^1 : u_1 = (x^2+y^2)t^2 \quad (62)$$

$$P^2 : u_2 = (x^2+y^2)t^2 \quad (63)$$

$$P^3 : u_3 = (x^2+y^2)t^2 + (3x^3 + xy^2 + yx^2 + 3y^3)t^6 \quad (64)$$

$$P^4 : u_4 = (x^2+y^2)t^2 + 2(x+y)t^6 \quad (65)$$

We have the approximate solution of u to be

$u(x,y,t) = \sum_{n=0}^{\infty} u_n$ i.e

$$u(x,y,t) = u_0 + u_1 + u_2 + \ldots \quad (66)$$

$$u(x,y,t) = (x^2+y^2)t^2 + (3x^3 + xy^2 + yx^2 + 3y^3 + 2x + 2y)t^6$$

For the exact solution (67)

$$u(x,y,t) = (x^2+y^2)t^2 + (x+y)t^6$$

4.0 Result

The computational results obtained for solving higherdimensional initial boundary value problems using our proposedLaplace Homotopy Perturbation Decomposition Method (LHPDM), in the three examples or, (i.e at when t= 0, 0.05 and 0.1) for some values of x=0, 0.1, 0.3,0.5, 0.7, 0.9, y=0.2, 0.4, 0.6, 0.8,1.0 and z=0.15,0.35,0.55,0.75,0.95 is shown in the tables below;

Example 1

Table 4.1: The comparison of the exact solution, VIDM and LHPDM at when t=0

X	y	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	5.0 E10^{-2}	5.0 E10^{-2}	5.0 E10^{-2}	0
0.3	0.4	2.5 E10^{-1}	2.5 E10^{-1}	2.5 E10^{-1}	0
0.5	0.6	6.1 E10^{-1}	6.1 E10^{-1}	6.1 E10^{-1}	0
0.7	0.8	1.13	1.13	1.13	0
0.9	1.0	1.81	1.81	1.81	0

Table 4.2: The comparison of the exact solution, VIDM and LHPDM at when t=0.05

X	Y	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	4.75615 E 10^{-2}	5.0 E10^{-2}	5.0 E10^{-2}	5.12710 E 10^{-2}
0.3	0.4	2.37807 E 10^{-1}	2.5 E10^{-1}	2.5 E10^{-1}	5.12710 E 10^{-2}
0.5	0.6	5.80249 E 10^{-1}	6.1 E10^{-1}	6.1 E10^{-1}	5.12710 E 10^{-2}
0.7	0.8	1.07488	1.13	1.13	5.12710 E 10^{-2}
0.9	1.0	1.72172	1.81	1.81	5.12710 E 10^{-2}

Table 4.3: The comparison of the exact solution, VIDM and LHPDM at when t=0.1

X	Y	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	4.52419 E 10^{-2}	5.0 E10^{-2}	5.0 E10^{-2}	1.05171 E 10^{-1}
0.3	0.4	2.26209 E 10^{-1}	2.5 E10^{-1}	2.5 E10^{-1}	1.05171 E 10^{-1}
0.5	0.6	5.51951 E 10^{-1}	6.1 E10^{-1}	6.1 E10^{-1}	1.05171 E 10^{-1}
0.7	0.8	1.02247	1.13	1.13	1.05171 E 10^{-1}
0.9	1.0	1.63776	1.81	1.81	1.05171 E 10^{-1}

Example 2

Table 4.4: The comparison of the exact solution, VIDM and LHPDM at when t=0

X	Y	z	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	0.15	0	0	0	0
0.3	0.4	0.35	0	0	0	0
0.5	0.6	0.55	0	0	0	0
0.7	0.8	0.75	0	0	0	0
0.9	1.0	0.95	0	0	0	0

Table 4.5: The comparison of the exact solution, VIDM and LHPDM at when t=0.05

x	Y	z	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	0.15	3.64651 E 10^{-17}	3.64651 E 10^{-17}	3.64651 E 10^{-17}	8.333408E10^{-3}
0.3	0.4	0.35	2.74566 E10^{-10}	2.74566E10^{-10}	2.74566E10^{-10}	8.33403 E10^{-4}
0.5	0.6	0.55	1.0094 E10^{-6}	1.0094 E10^{-6}	1.0102 E10^{-6}	8.3340 E10^{-4}
0.7	0.8	0.75	2.7456 E10^{-4}	2.7456 E10^{-4}	2.7480 E10^{-4}	8.3350 E10^{-4}
0.9	1.0	0.95	1.9541 E10^{-2}	1.9541 E10^{-4}	1.9557 E10^{-2}	8.3340 E10^{-4}

Table 4.6: The comparison of the exact solution, VIDM and LHPDM at when t=0.1

X	Y	z	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	0.15	7.3022 E10^{-17}	7.3022 E10^{-17}	7.3265 E10^{-17}	3.3344 E10^{-3}
0.3	0.4	0.35	5.4982 E10^{-10}	5.4982 E10^{-10}	5.5165 E10^{-10}	3.3344 E10^{-3}
0.5	0.6	0.55	2.0213 E10^{-6}	2.0213 E10^{-6}	2.0280 E10^{-6}	3.3344 E10^{-3}
0.7	0.8	0.75	5.4982 E10^{-4}	5.4982 E10^{-4}	5.5165 E10^{-4}	3.3344 E10^{-3}
0.9	1.0	0.95	3.9131 E10^{-2}	3.9131 E10^{-2}	3.9261 E10^{-2}	3.3344 E10^{-3}

Example 3

Table 4.7: The comparison of the exact solution, VIDM and LHPDM at when t=0

X	y	EXACT	VIDM	LHPDM	ABS ERR
0.1	0.2	0	0	0	0
0.3	0.4	0	0	0	0
0.5	0.6	0	0	0	0
0.7	0.8	0	0	0	0
0.9	1.0	0	0	0	0

Table 4.8: The comparison of the exact solution, VIDM and LHPDM at when t=0.05

X	y	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	1.2500 E10^{-4}	1.2500 E10^{-4}	3.7500 E10^{-4}	2.5000 E10^{-4}
0.3	0.4	6.2501 E10^{-4}	6.2501 E10^{-4}	1.8750 E10^{-4}	1.2500 E10^{-3}
0.5	0.6	1.5250 E10^{-3}	1.5250 E10^{-3}	4.5750 E10^{-3}	3.0500 E10^{-3}
0.7	0.8	2.8250 E10^{-3}	2.8250 E10^{-3}	8.4751 E10^{-3}	5.6501 E10^{-3}
0.9	1.0	4.5250 E10^{-3}	4.5250 E10^{-3}	9.0501 E10^{-3}	9.0501 E10^{-3}

Table 4.9: The comparison of the exact solution, VIDM and LHPDM at when t=0.1

X	y	EXACT	VIDM	LHPDM	ABS-ERR
0.1	0.2	5.003 E10^{-4}	5.003 E10^{-4}	4.5006 E10^{-3}	1.0000 E10^{-3}
0.3	0.4	2.5007 E10^{-3}	2.5007 E10^{-3}	2.5018 E10^{-3}	5.0010 E10^{-3}
0.5	0.6	6.1011 E10^{-3}	6.1011 E10^{-3}	5.8304 E10^{-3}	1.2203 E10^{-2}
0.7	0.8	1.1301 E10^{-2}	1.1301 E10^{-2}	1.3130 E10^{-2}	2.2605 E10^{-2}
0.9	1.0	1.8101 E10^{-2}	1.8101 E10^{-2}	2.1001 E10^{-2}	3.6209 E10^{-2}

5.0 CONCLUSION

In this work, Laplace Homotopy Perturbation Decomposition Method (LHPDM) is used for solving higher dimensional initial boundary value problems with variable co-efficients. The proposed technique is successfully implemented by using the initial conditions only. It has a significant advantage as an alternative solving method amongst the existing techniques of solving the higher dimensional initial boundary value problem.

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