

## Modelling Indoor Radon Through Dimensional Analysis

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### Abstract

*Dimensional analysis (DA) is frequently used by engineers and physicists to reduce the complexity of fundamental equations describing the behaviour of a system to the simplest and most economical form. This work proposes a refinement to an approach in the traditional paradigm for carrying out DA. The said refinement will enable the researcher to obtain some graphical displays that will enable him/her to predict, as accurately as possible, a functional relationship existing between the dependent and independent variables that are involved in a Buckingham  $\pi$ -theorem. With our technique, attempts to derive a model through which the rate of emission of indoor radioactive radon can be estimated have now materialized. In this article, we first provide an overview of DA and give basic recommendations for designing DA experiments. Next, we illustrated the use of technique and concluded by the fitness of our derived model. Radon is a radioactive gas whose effect on residents has not been adequately studied. This attempt is geared towards being able to quantify the average emitted radon for some common domestic household types (in the present case, a duplex building) in an averagely populated area.*

**Keyword:** Buckingham  $\pi$ -theorem; Traditional paradigm (DA); Dimensionless quantities; Interpolation of univariates; Irregularly spaced data; Splines; Radioactive (Indoor) Radon and R.

### 1. INTRODUCTION

Dimensional analysis (DA) is a method for reducing complex physical problems to their simplest (most economical) forms prior to quantitative analysis or experimental investigation. Its use in science and engineering is ubiquitous. Applications are many, including astrophysics, electromagnetic theory, radiation, aerodynamics, ship design, heat and mass transfer, mechanics of elastic and plastic structures, explosions, chemical reactions and processing, simulation of nuclear reactor accidents, biology, and even economics. DA reduces a problem's degrees of freedom to the minimum and thus suggests the most economical scaling laws. It can be particularly useful in exploratory investigations of novel phenomena for which the equations and boundary conditions have not yet been fully articulated [1].

The first benefit of DA is dimension reduction [2]. We will see that, by using DA, the number of independent variables, or factors, in a standard experiment can often be reduced by three or four. This leads to substantial savings at the experiment stage. The second major advantage associated with DA is that, because each factor in the reduced set is dimensionless, the model developed at the analysis stage will be completely scalable. This means, for example, that one could experiment with a small model turbine using DA and expect the results to translate to the full-scale counterpart. Scalability, however, is rare with standard experiments, where extrapolations are notoriously unreliable. The purpose of this article includes to review the basic DA methodology, and to develop strategies for analysing radon data and experiments.

A simple example, detailed in [3], involves characterizing the extent of deformation that occurs in elastic balls when they strike a wall. Here, the response variable is  $d$ , the diameter of the circular imprint left on the wall after a freshly dyed ball has impacted the wall. Physical considerations suggest that a set of five independent variables are required to characterize the response  $d$ :

$$d = f(V, \rho, D, E, \gamma) \quad (1)$$

Where  $V$  is the velocity of the ball;  $\rho$  is the material density of the ball;  $D$  is the diameter of the ball;  $E$  is the modulus of elasticity of the ball material; and  $\gamma$  is Poisson's ratio. The dimensions of the response variable and the five independent variables are all functions of (at most) three base dimensions, mass ( $M$ ), length ( $L$ ), and time ( $T$ ).

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For example, the dimension of both  $d$  and  $D$  is  $L$ , the dimension of  $V$  is  $LT^{-1}$ , and the dimension of  $E$  is  $ML^{-1}T^{-2}$ . Poisson's ratio,  $\gamma$ , is a dimensionless constant. Let  $k$  and  $p$  denote the number of base dimensions and the number of independent variables, respectively. The central result in DA, the Buckingham  $\pi$ -Theorem [4], tells us that an alternative to model (1) exists, which involves one dimensionless response and  $p - k = 5 - 3 = 2$  dimensionless independent variables. In this case, the result is

$$\pi_0 = g(\pi_1, \pi_2) \quad (2)$$

Where  $\pi_0 = \frac{d}{D}$ ,  $\pi_1 = \frac{E}{\rho V^2}$  and  $\pi_2 = \gamma$ . It is easy to verify mathematically that  $\pi_0$ ,  $\pi_1$  and  $\pi_2$  are dimensionless. As a result of

(2), the number of factors has been reduced from five to two, and, because all variables are dimensionless, the results are scalable. We refer to the function  $g$  as the DA model. From here, it is clear that we can, at least, achieve, without suffering any losses, dimensional reduction through the use of DA.

There are also disadvantages and risks associated with the use of DA. First, specification of the independent variables requires knowledge of the basic mechanisms (physical, chemical, biological, etc.) of the system. Since statistical modellers are unlikely to possess this knowledge, close collaboration with the pertinent experts is required. Second, the form of the new model  $g$  is often unknown and is usually nonlinear. For this reason, the use of low-order polynomial models to approximate  $g$  at the analysis stage may be inadequate, and it is unclear how to proceed from a design standpoint. Engineering texts and standard references [3, 5] actually suggest using full factorial designs involving 10 equally spaced points for each factor. With the two factors involved in the ball-deformation example, this would imply the need for 100 runs for a single replicate. Clearly, more efficient experimental approaches can be suggested. Another difficulty that sometimes arises is that the reduced set of experimental factors is often comprised of derived quantities that may have various base quantities in common, and, as a result, the region of DA experimentation may be irregular. Fourth, and perhaps most importantly, the scientist must know, a priori, the complete set of independent variables describing the behaviour of a system. If the independent variables are wrongly specified (e.g., if one variable is missing), the results of the DA experiment may be completely unusable. Finally, DA, and thus all results described herein, applies only to continuous experimental factors [6].

## 2.0 Dimensional Analysis (DA) Overview

Here, we shall give an overview of the DA process. We begin by motivating the use of DA with some simple examples. We then review the concepts of dimension, physical quantities, base quantities, and derived quantities. Finally, we describe the DA process, which involves dimension reduction via the Buckingham  $\pi$ -Theorem, and normalization of the reduced variable set.

### 2.1 Basic Ideas

The bulk of the literature on applications of DA has, historically, been based on the fields of engineering and physics. Thus, the typical perception is that DA is a tool used for describing and simplifying the natural laws of the universe. However, we all use the concepts of DA, virtually on a daily basis. For example, we intuitively compare financial investments (real estate, stocks, bonds) using the concept of rate of return, which is a dimensionless quantity of profit/initial investment. Although only a minor simplification, rate of return identifies a "natural law" governing the behaviour of investors across a range of financial instruments. Other everyday examples include education (graduation rates), biology (population growth rates), and nutrition (percentage of recommended daily nutrient amounts).

Although these examples are overly simplistic when compared with the typical DA construct, they do reveal some of the essential features of DA, namely, the use of dimension reduction (two variables, profit and initial investment, are combined to form rate of return); the use of dimensionless variables (rate of return has no dimension); and the scalability of results (small stock investments can be directly compared with large housing investments). Before we introduce the DA process as applied in the physical sciences, we review key characteristics of physical quantities and physical equations.

### 2.2 Dimension, Base Quantities, Derived Quantities, and Dimensionless Quantities

Physical quantities can be characterized as either base quantities or derived quantities. In the physical sciences, the SI system (Système International) defines standard base quantities as length, time, mass, temperature, current, number of elementary particles, and luminous intensity. The base quantity length could be measured in different units, such as feet or meters, but in either case the base quantity (or base dimension) is length. A derived quantity of the first kind is a quantity that is constructed from products of powers of base quantities. In the ball-deformation example of the introduction, the base quantities are mass ( $M$ ), length ( $L$ ), and time ( $T$ ). Since the dimension of velocity,  $V$ , is  $LT^{-1}$ , velocity is a derived quantity of the first kind.

It is a standard practice in engineering to use square brackets to denote the dimension of a physical quantity. For example, since the underlying dimension of the diameter  $d$  in the ball deformation example is length ( $L$ ), we write  $[d] = L$ . Similarly, we have  $[V] = LT^{-1}$ .

It has been shown that not all formulas can be used to represent physical quantities. Because base quantities all have a physical origin, the ratio of the measurements any two base quantities does not change if the base unit changes. This principle was first articulated by [7] as Bridgman’s principle of absolute significance of relative magnitude. Bridgman further showed that this principle will hold for a physical quantity  $Q$  having a monomial formula only if it assumes the power-law form:

$$Q = \alpha \prod_{j=1}^n Z_j^{d_j} \tag{3}$$

Where  $Z_j$  is the numerical value of the  $j^{\text{th}}$  base quantity and the coefficients  $\alpha, d_1, d_2, \dots, d_n$  are real numbers. Thus, all physical quantities have power-law form (3) and no other form represents a physical quantity. A generalized form of (3) recognizes that any given base quantity may appear more than once in the expression. For example, length may be used to represent both a radius and a height of a cylinder. Letting  $k_j$  denote the number of times that the  $j^{\text{th}}$  base quantity appears in the formula, letting  $Z_{ji}$  denote the  $i^{\text{th}}$  instance of the  $j^{\text{th}}$  base quantity  $Z_j$ , and letting  $d_{ji}$  denote the power to which the  $i^{\text{th}}$  instance of that base quantity is raised, the generalized form is

$$Q = \alpha \prod_{j=1}^n \prod_{i=1}^{k_j} Z_{ji}^{d_{ji}} \tag{4}$$

Denote the dimension of  $Z_{ji}$  by  $D_{ji}$ . That is,  $[Z_{ji}] = D_{ji} = D_j$ . It follows that the dimension of  $Q$  is

$$[Q] = \prod_{j=1}^n D_j^{\sum_{i=1}^{k_j} d_{ji}} = \prod_{j=1}^n D_j^{d_j} \tag{5}$$

Where  $d_j = \sum_{i=1}^{k_j} d_{ji}$ . If the units chosen for the  $j^{\text{th}}$  dimension are changed by a factor  $C_j$  for  $j = 1, 2, \dots, n$ , it follows from (5) that the value of  $Q$  becomes  $Q^* = c^{-1}Q$ , where  $c = \prod_{j=1}^n c_j^{d_j}$ . Finally, we say that a derived quantity is dimensionless, if its value does not change along with the units of the base quantities [6].

**3. Method of Repeating Variables and the Buckingham  $\pi$ -Method**

There are several examples of the usefulness and power of DA. Now we are ready to learn how to generate the nondimensional parameters, i.e., the  $\pi$ ’s. There are several methods that have been developed for this purpose, but the most popular (and simplest) method is the method of repeating variables, an interesting example that is due to [3] was highlighted in section 1. However, we now want to give the algorithmic steps involved with the method and further illustrate with more examples, most especially those that often result in modified  $\pi$  (i.e.  $\pi_{\text{modified}}$ ). There are six steps, listed concisely in sub-section 3.1, and in more detail in Table 1. These steps are explained in further detail as we work through the following problem:

Consider a ball falling in a vacuum experiencing the gravitational pull such that its equation of motion is given as

$$\frac{d^2Z}{dt^2} = -g \tag{6},$$

Where  $t$  is the time variable in seconds,  $Z$  is the elevation of the ball from the ground whilst  $Z_0$  is its initial velocity;  $w$  is the component of velocity whilst  $w_0$  is its initial velocity in the  $Z$ -direction. Let us pretend that we neither know that Equation (6) appropriately describes this problem, nor do we know much physics concerning falling objects. In fact, suppose that all we know is that the instantaneous elevation  $Z$  of the ball must be a function of time  $t$ , initial vertical speed  $w_0$ , initial elevation  $Z_0$ , and gravitational constant  $g$ . The beauty of dimensional analysis is that the only other thing we need to know is the primary dimensions of each of these quantities. As we go through each step of the method of repeating variables, we shall now explain some of the concepts involved with the technique in more detail.

**3.1 The Steps involved with the algorithm**

**Step 1:** List the parameters (dimensional variables, nondimensional variables, and dimensional constants) and count them. Let  $n$  be the total number of parameters in the problem, including the dependent variable (i.e. the subject of the formula). Make sure that any listed independent parameter is indeed independent of the others, i.e., it cannot be expressed in terms of them. (e.g., do not include radius  $r$  and area  $A = \pi r^2$ , since  $A$  is dependent on  $r$ .)

**Step 2:** List the base (primary) dimensions for each of the  $n$  parameters.

**Step 3:** Guess the reduction  $j$ . As a first guess, set  $j$  equal to the number of primary dimensions represented in the problem. The expected number of  $\pi$ ’s (i.e.  $k$ ) is equal to  $n$  minus  $j$ , in-line with the ball deformation example in section 1 (i.e. with  $n = p$  and  $j = k$ ). If at this step or during any subsequent step, the analysis does not work out, verify that you have included enough parameters in step 1. Otherwise, go back and reduce  $j$  by one and try again.

Step 4: Choose  $j$  repeating parameters that will be used to construct each  $\pi$ . Since the repeating parameters have the potential to appear in each  $\pi$ , be sure to choose them wisely.

Step 5: Generate the  $\pi$ 's one at a time by grouping the  $j$  repeating parameters with one of the remaining parameters, forcing the product to be dimensionless. In this way, construct all  $k$   $\pi$ 's. By convention the first  $\pi$ , designated as  $\pi_1$ , is the dependent  $\pi$  (the one on the left side of the list). Manipulate the  $\pi$ 's as necessary to achieve established dimensionless groups.

Step 6: Check that all the  $\pi$ 's are indeed dimensionless. Write the final functional relationship in the form in which  $\pi_1 = f(\pi_2, \pi_3, \dots, \pi_k)$  (This is a step-by-step method for finding the dimensionless  $\pi$  groups when performing a dimensional analysis).

With respect to equation 1, the above-specified six steps will now be taken;

Step 1: There are five parameters (dimensional variables, nondimensional variables, and dimensional constants) in this problem;  $n = 5$ . They are listed in functional form, with the dependent variable being identified as the "subject of the formula" involving all other variables (that are independent) and constants:

$$Z = f(t, w_0, Z_0, g), \quad n = 5 \quad (7)$$

Where the primary dimensions and their associated symbols are; Mass (m), Length (L), Time (t), Temperature (T), Electric current (I), Amount of light (C) Amount of matter (N).

Step 2: The primary dimensions of each parameter are listed underneath them. The dimensions are also written with their appropriate exponents in order to simplify the pertinent algebra that will be done later. That is;

$$\begin{array}{ccccc} Z & t & w_0 & Z_0 & g \\ (L) & (t^1) & (Lt^{-1}) & (L) & (Lt^{-2}) \end{array}$$

Step 3: As a "start-off" guess,  $j$  is set equal to 2, the number of primary dimensions represented in the problem (this guess is borne out of the fact that the entire five variables have to do with L and t). That is;  $j = 2$  (the guess) and if this guess is correct, the number of  $\pi$ 's predictable through the Buckingham  $\pi$ -theorem is  $k = n - j = 5 - 2 = 3$ .

Step 4: We need to choose two repeating parameters (since  $j = 2$ ). This part is somewhat "inexplicable". Consequently, several guidelines about choosing repeating parameters are listed in Table 1 below. By following the guidelines of Table 1, the "best" choices of the two repeating parameters are  $w_0$  and  $Z_0$ .

Step 5: Now we combine these repeating parameters into products with each of the remaining parameters, one at a time, to create the  $\pi$ 's. The first  $\pi$  is always the dependent  $\pi$  and is formed with the dependent variable Z. That is;

$$\pi_1 = Z w_0^{\alpha_1} Z_0^{\beta_1} \quad (8)$$

where  $\alpha_1$  and  $\beta_1$  are constant exponents that need to be determined. We apply the primary dimensions of step 2 into Equation (8) and "force" the  $\pi$  to be dimensionless by setting the exponent of each primary dimension to zero. That is;

$$\text{Dimensions of } \pi_1 = [\pi_1] = \{L^0 t^0\} = \{Z w_0^{\alpha_1} Z_0^{\beta_1}\} = \{L^1 (L t^{-1})^{\alpha_1} L^{\beta_1}\} \quad (9)$$

Since primary dimensions are by definition independent of each other, we equate the exponents of each primary dimension independently, in equation (9), to solve for exponents  $\alpha_1$  and  $\beta_1$ . That is; with respect to

- i. Time:  $\{t^0\} = \{t^{-\alpha_1}\} \rightarrow -\alpha_1 = 0, \quad \alpha_1 = 0$
- ii. Length: Equation  $\{L^0\} = \{L^1 L^{\alpha_1} L^{\beta_1}\} \rightarrow 1 + \alpha_1 + \beta_1 \rightarrow \beta_1 = -1$

Hence equation (8) becomes

$$\pi_1 = \frac{Z}{Z_0} \quad (10)$$

In similar fashion we create the first independent  $\pi$  ( $\pi_2$ ) by combining the repeating parameters with independent variable t. With respect to the first independent  $\pi$ :

$$\pi_2 = t w_0^{\alpha_2} Z_0^{\beta_2} \quad (11)$$

With the exponents, as constants to be determined;

$$\text{Dimension of } \pi_2 = [\pi_2] = \{L^0 t^0\} = \{t w_0^{\alpha_2} Z_0^{\beta_2}\} = \{t (L t^{-1})^{\alpha_2} L^{\beta_2}\} \quad (12)$$

By equating the exponents, we obtain;  $\alpha_2 = 1, \beta_2 = -1$ . Thus rendering;

$$\pi_2 = \frac{t w_0}{Z_0} \quad (13)$$

Finally we create the second independent  $\pi (\pi_3)$  by combining the repeating parameters with  $g$  and forcing the  $\pi$  to be dimensionless.

$$\text{Second independent } \pi : \pi_3 = g w_0^{\alpha_3} Z_0^{\beta_3} \quad (14)$$

$$\text{Dimensions of } \pi_3 : [\pi_3] = \{L^0 t^0\} = \{g w_0^{\alpha_3} Z_0^{\beta_3}\} = \{L t^{-2} (L t^{-1})^{\alpha_3} L^{\beta_3}\} \quad (15)$$

Equating exponents, we have;  $0 = 1 + \alpha_3 + \beta_3, 0 = -2 - \alpha_3$  which implies that;

$$\alpha_3 = -2, \beta_3 = 1 \text{ thus rendering; } \pi_3 = \frac{g Z_0}{w_0^2} \quad (16)$$

All three  $\pi$ 's have been found, but at this point it is customary to examine them to see if any manipulation is required. With respect to this case, we recognize that the third  $\pi$  must be raised to the power of  $\frac{-1}{2}$ , in order to be of the same form, as an

established dimensionless parameter, called the Froude number (Fr);

$$\text{Modified } \pi_3 : \pi_{3, \text{modified}} = \left( \frac{g Z_0}{w_0^2} \right)^{-\frac{1}{2}} = \frac{w_0}{\sqrt{g Z_0}} = Fr \quad (17).$$

Manipulations such as the one in equation (17) are often necessary to put the  $\pi$ 's into proper established form. The  $\pi$  of equation (16) is not wrong, and there is certainly no mathematical advantage of equation (17) over equation (16). Instead, we would like to just say that equation (17) is more "scientifically acceptable" than equation (16), since it is a named, established non-dimensional parameter that is commonly used in the literature. In table 2 are listed some guidelines for manipulation of your non-dimensional  $\pi$  groups into established non-dimensional parameters. Table 2 lists some established non-dimensional parameters, most of which are named after a notable scientist or engineer. Whenever possible, one should manipulate one's  $\pi$ 's as necessary in order to convert them into established non-dimensional parameters.

Table 1: Guidelines for choosing repeating parameters in step 4 of the method of repeating variables

Guideline	Comments and Application to Present Problem
1. Never pick the dependent variable. Otherwise, it may appear in all the $\pi$ 's which is undesirable.	In the present problem we cannot choose $z$ , but we must choose from among the remaining four parameters. Therefore, we must choose two of the following parameters: $t, w_0, Z_0$ and $g$ .
2. The chosen repeating parameters must not by themselves be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the $\pi$ 's.	In the present problem, any two of the independent parameters would be valid according to this guideline. For illustrative purposes, however, suppose we have to pick three instead of two repeating parameters. We could not, for example, choose $t, w_0$ and $Z_0$ , because these can form a $\pi$ all by themselves ( $\frac{t w_0}{Z_0}$ ).
3. The chosen repeating parameters must represent all the primary dimensions in the problem.	Suppose for example that there were three primary dimensions (m, L, and t) and two repeating parameters were to be chosen. You could not choose, say, a length and a time, since primary dimension mass would not be represented in the dimensions of the repeating parameters. An appropriate choice would be a density and a time, which together represent all three primary dimensions in the problem.
4. Never pick parameters that are already dimensionless. These are $\pi$ 's already, all by themselves.	Suppose an angle $\theta$ were one of the independent parameters. We could not choose $\theta$ as a repeating parameter since angles have no dimensions (radian and degree are dimensionless units). In such a case, one of the $\pi$ 's is already known, namely $\theta$ .

5. Never pick two parameters with the same dimensions or with dimensions that differ by only an exponent.	In the present problem, two of the parameters, $Z$ and $Z_0$ , have the same dimension (length). We cannot choose both of these parameters. (Note that dependent variable $Z$ has already been eliminated by guideline 1). Suppose one parameter has dimensions of length and another parameter has dimensions of volume. In dimensional analysis, volume contains only one primary dimension (length) and is not dimensionally distinct from length—we cannot choose both of these parameters.
6. Whenever possible, choose dimensional constants over dimensional variables so that only one $\pi$ contains the dimensional variable.	If we choose time $t$ as a repeating parameter in the present problem, it would appear in all three $\pi$ 's. While this would not be wrong, it would not be wise since we know that ultimately we want some non-dimensional height as a function of some non-dimensional time and other nondimensional parameter(s). From the original four independent parameters, this restricts us to $w_0, Z_0$ and $g$ .
7. Pick common parameters since they may appear in each of the $\pi$ 's.	In fluid flow problems we generally pick a length, a velocity, and a mass or density. It is unwise to pick less common parameters like viscosity $\mu$ or surface tension $\sigma_s$ , since we would in general not want $\mu$ or $\sigma_s$ to appear in each of the $\pi$ 's. In the present problem, $w_0$ and $Z_0$ are wiser choices than $g$ .
8. Pick simple parameters over complex parameters whenever possible.	It is better to pick parameters with only one or two basic dimensions (e.g., a length, a time, a mass, or a velocity) instead of parameters that are composed of several basic dimensions (e.g., an energy or a pressure).

Table 2: Guidelines for manipulation of the  $\pi$ 's resulting from the method of repeating variables.

Guideline	Comments and Application to Present Problem
1. We may impose a constant (dimensionless) exponent on a $\pi$ or perform a functional on a $\pi$ .	We can raise a $\pi$ to any exponent $n$ (changing it to $\pi^n$ ) without changing the dimensionless stature of the $\pi$ . For example, in the present problem, we imposed an operation exponent of $\frac{-1}{2}$ on $\pi_3$ . Similarly we can perform the functional operation $\sin(\pi)$ , $\exp(\pi)$ , etc., without influencing the dimensions of the $\pi$ .
2. We may multiply a $\pi$ by a pure (dimensionless) constant.	Sometimes dimensionless factors of $\frac{1}{2}$ , 2, 4, etc., are included in a $\pi$ for convenience. This is perfectly okay since such factors do not influence the dimensions of the $\pi$ .
3. We may form a product (or quotient) of any $\pi$ with any problem to replace one of the $\pi$ 's.	We could replace $\pi_3$ by $\pi_3\pi_1$ , $\frac{\pi_3}{\pi_2}$ , etc. Sometimes such manipulation is necessary to convert our $\pi$ into an established $\pi$ . In many cases, the established $\pi$ would have been produced if we would have chosen different repeating parameters.
4. We may use any of guidelines 1 to 3 in combination.	In general, we can replace any $\pi$ with some new $\pi$ such as $A\pi_3^B \sin(\pi_1^C)$ , where $A$ , $B$ , and $C$ are pure constants.

<u>Guideline</u>	<u>Comments and Application to Present Problem</u>
5. We may substitute a dimensional parameter in the $\pi$ with other parameter(s) of the same dimensions.	For example, the $\pi$ may contain the square of a length or the cube of a length, for which we may substitute a known area or volume, respectively, in order to make the $\pi$ agree with established conventions.

Step 6: We are finally ready to write the functional relationship between the non-dimensional parameters;

Relationship between  $\pi$ 's: 
$$\pi_1 = f(\pi_2, \pi_3) \rightarrow \frac{Z}{Z_0} = f\left(\frac{w_0 t}{Z_0}, \frac{w_0}{\sqrt{g Z_0}}\right) \quad (18)$$

#### 4.0 The Buckingham $\pi$ -Theorem and Indoor Radon

A collection of past works on the indoor radon concentration are in existence already [8, 9, 10, 11]. In a few of them [8, 11] the use of an equipment (i.e. Radon Scout™) which estimates the indoor radon concentration amidst pertinent co-varying variables, such as; Temperature (T), Pressure (P), Ventilation (V), Humidity (H) and the associated Error (E) was inevitable. By setting aside the E, in the meantime, a list of both the dependent and independent variables, with their respective dimensions, is as follows;

$$\begin{matrix} R_{conc} & T & P & H & V \\ \{t^{-1}L^{-3}\} & \{T^1\} & \{M_{air}^1 L^{-1} t^{-2}\} & \{M_{water}^1 L^{-3}\} & \{L^3 t^{-1}\} \end{matrix} \quad (19)$$

Where the utilized dimensions and their associated symbols are; Mass (M), Length (L), Time (t) and Temperature (T). If for the sake of convenience, we disregard  $M_{air}^1$  and  $M_{water}^1$ , then (19) becomes;

$$\begin{matrix} R_{conc} & T & P & H & V \\ \{t^{-1}L^{-3}\} & \{T^1\} & \{L^{-1}t^{-2}\} & \{L^{-3}\} & \{L^3 t^{-1}\} \end{matrix} \quad (20)$$

Consequently;  $n = 5, j = 3, k = n - j = 5 - 3 = 2$ . With  $j = 3$ , we need to choose three repeating parameters. Through the use of table 1, we chose T, P and V. Hence;

Dependent  $\pi$ :  $\pi_1 = R_{conc} T^{\alpha_1} P^{\beta_1} V^{\gamma_1}$

$$\rightarrow \{t^0 L^0 T^0\} = \{t^{-1} L^{-3} T^{\alpha_1} (L^{-1} t^{-2})^{\beta_1} (L^3 t^{-1})^{\gamma_1}\} \quad (21)$$

By equating exponents, we have;  $\alpha_1 = 0, \beta_1 = \frac{-6}{7}, \gamma_1 = \frac{5}{7}$ , thus resulting into;

$$\pi_1 = R_{conc} P^{\frac{-6}{7}} V^{\frac{5}{7}} = \frac{R_{conc} V^{\frac{5}{7}}}{P^{\frac{6}{7}}} = R_{conc} \left(\frac{V^5}{P^6}\right)^{\frac{1}{7}} \quad (22)$$

With respect to the only independent  $\pi$ :  $\pi_2 = HT^{\alpha_2} P^{\beta_2} V^{\gamma_2}$ , which implies that;

$$\{t^0 L^0 T^0\} = \{L^{-3} T^{\alpha_2} (L^{-1} t^{-2})^{\beta_2} (L^3 t^{-1})^{\gamma_2}\} = \{t^{-2\beta_2 - \gamma_2} L^{-3 - \beta_2 + 3\gamma_2} T^{\alpha_2}\} \quad (23)$$

By equating exponents, we have;  $\alpha_2 = 0, \beta_2 = \frac{-3}{7}, \gamma_2 = \frac{6}{7}$ . Hence giving;

$$\pi_2 = HP^{\frac{-3}{7}} V^{\frac{6}{7}} = H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}} \quad (24)$$

Presenting, our result in the form;  $\pi_1 = f(\pi_2)$  gives;

$$R_{conc} \left(\frac{V^5}{P^6}\right)^{\frac{1}{7}} = f\left(H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}}\right) \rightarrow R_{conc} = \left(\frac{P^6}{V^5}\right)^{\frac{1}{7}} f\left(H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}}\right) \quad (25)$$

By dividing throughout by  $\left(\frac{P^6}{V^5}\right)^{\frac{1}{7}}$ , we obtain;  $R_{conc} \left(\frac{V^5}{P^6}\right)^{\frac{1}{7}} = f\left(H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}}\right)$ . Now by setting

, with;  $V = 0.5$  (for an averagely ventilated Bedroom),  $P = \frac{p}{1000}$ ,  $H = \frac{h}{100}$  and regressing (linear) y on x, we obtained the

following results  $Y = R_{conc} \left(\frac{V^5}{P^6}\right)^{\frac{1}{7}}, X = H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}}$  through Microsoft Excel;

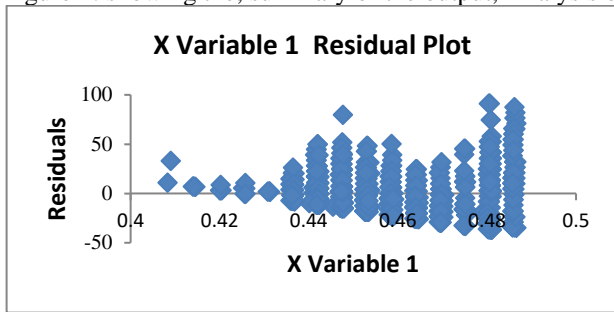
**SUMMARY OUTPUT**

Regression Statistics	
Multiple R	0.441719
R Square	0.195116
Adjusted R Square	0.194432
Standard Error	19.94117
Observations	1179

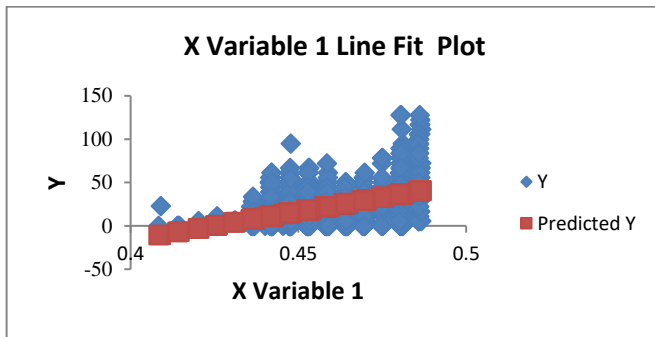
ANOVA					
	df	SS	MS	F	Significance F
Regression	1	113458.2	113458.2	285.3217	1.75386E-57
Residual	1177	468034.3	397.6502		
Total	1178	581492.5			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	277.820891	17.7763848	-15.6286	3.56E-50	-312.6978296	-242.94395
X Variable 1	653.749185	38.7029225	16.89147	1.75E-57	577.8147651	729.683605

Figure 1: showing the; summary of the output, Analysis of Variance (ANOVA) and the pertinent table of coefficients.



(a)



(b)

Figure 2: showing plots; (a) which is the residual plot, (b) the predicted line (line of best fit).

Consequently,  $Y = 653.75X - 277.82$ , is a line of best fit for;

And hence;

$$f(X) = f\left(H\left(\frac{V^6}{P^3}\right)^{\frac{1}{7}}\right) = 653.75\left(H\left(\frac{V^6}{P^3}\right)^{\frac{1}{7}}\right) - 277.82$$



$$R_{conc} = \left(\frac{P^6}{V^5}\right)^{\frac{1}{7}} f \left( H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}} \right) = \left(\frac{P^6}{V^5}\right)^{\frac{1}{7}} \left( 653.75 \left( H \left(\frac{V^6}{P^3}\right)^{\frac{1}{7}} \right) - 277.82 \right) \tag{26}$$

4. Discussion and Conclusion

In this work, we have done a ‘thorough’ review on dimensional analysis and used the Buckingham  $\pi$ -theorem to derive the functional relationship (equation (25)) between the radon concentration ( $R_{conc}$ ) of our study area and the pertinent covariates (i.e. Pressure (P), Ventilation (V), Relative Humidity (H) of the area). Figures 1 ((a) and (b)) and 2 show that a linear representation is acceptable. This is a universal relationship that is not location (study area) dependent. We also found out, in the course of our analysis and with respect to our study area, that the quantity  $\left(\frac{p^6}{V^5}\right)^{\frac{1}{7}}$  in equation (26) is approximately equal to

1.64, in numerical value when  $V = 0.5$  (i.e. when the study area is averagely ventilated). The equation can be used to predict the radon concentration of our study area days, after the study period as long as the ventilation is kept constant (i.e. at 0.5). This prediction capability is of immense value because it enables us to be able to move the study equipment to other areas without losing information on the studied areas. The equation (26) is purposely kept linear (i.e. simple) for easy determination of the radon concentration. However, if a polynomial of a higher degree is desired, a package like Matlab can be utilized (i.e. by using an appropriate **polytool** command) in the derivation of such a polynomial as illustrated in figure 3, below, using a simple sample of size 10 from our study data, for instance, the commands (four lines of codes);

```
x=[0.447 0.453 0.441 0.436 0.442 0.436 0.447 0.453 0.453 0.452]
y=[0 10.99 5.49 22.02 39.08 0 21.98 61.05 33.61 10.98]
polyfit(x,y,2)
polytool(x,y,2,0.025)
```

On the command window of Matlab will give the results;

```
1.0e+004 *
 8.2333 -7.2265 1.5870
```

And

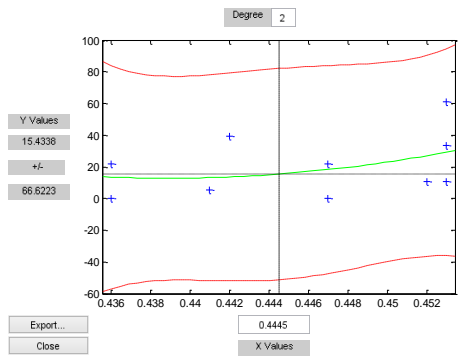


Figure 3: showing the pertinent commands as well as the results in the course of fitting a quadratic polynomial for the sample from our study data.

With the respective coefficients multiplied by  $10^4$ , the quadratic polynomial for the sample from our study data is  $Y = f(X) = 8.2333X^2 - 7.2265X + 1.5870$ , whilst X and Y remain as specified in equation (26).

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