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The Dynamics of Periodically Driven Two-level Atom Cavity Coupling

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Abstract

The dynamics of a periodically driven two-level atom cavity coupling driven by both the sawtooth and the triangular wave functions is investigated. The crossings of the Floquet quasi-energies of the two-level atom-cavity system was obtained. The Floquet quasi-energies of the sawtooth driven system avoided crossing themselves unlike the those of the triangular function driven system. A result that would be of immense importance in quantum dot lasers, solar cells and single electron transistors fabrication and quantum computing.

Keyword: Cavity-electron, Floquet modes, sawtooth function, time-dependent Hamiltonian

1. INTRODUTION

A particle tunneling between two potential wells is a typical example of a quantum two-level system (qubit) which can be realised by confining an electron to a pair of coupled quantum-dots [1-2].

Floquet quasi-energies of a two-level atom cavity coupling had been studied widely [3-7] due to the relevance of this subject to quantum computing [8-10]. Several methods of obtaining the quasi-energies of periodically driven two-level systems abound [11-13] and these solutions had shown that for some driving functions, the quasi-energies cross one another leading to what is often regarded as coherent destruction of tunneling (CDT) [14-16] while for some others, there is avoided crossing. A whole gamut of periodic functions had been explored with a lot of emphases always placed on the sinusoidal driving functions [3]. However, the crossings of the quasi-energies of two very similar periodic functions, the sawtooth and the triangular wave functions, had not been examined closely.

The sawtooth and the triangular wave functions are similar functions in the family of periodic functions. One would naturally expect both functions to behave similarly and in particular have the same or nearly the same quasi-energies spectrum and with these energies crossing at identical locations when used to drive an atom coupled to a two-level cavity (i.e. a double quantum-dot system). However, this study finds something intriguing about their behaviours.

The crossings of the Floquet quasi-energies of the Hamiltonians containing both the sawtooth and triangular functions constitute the kernel of this study. Although Creffield [3] did enormous work on periodic functions, the study was however silent about the dynamics of the sawtooth function driven atom-cavity coupling. Interestingly, this function presents differently from the triangular function that was reported. The intrinsic sawtooth function *signal* available in SciPy would be used to drive the coupled system and the calculations would be performed using the QuTiP application [17]. The crossings of the Floquet eigenenergies of the sawtooth and triangular functions would be obtained in addition to those of some other periodic functions. Moreover, the occupational probabilities of the wavefunctions of the two functions as they evolve with time will also be presented both in the Floquet and Lindblad formalisms.

2.0 Methodology

The Hamiltonian H of a two-level periodically driven system coupled to the z-direction is given by Creffield [3]:

$$H(t) = -\frac{1}{2}\Delta\sigma_x - \frac{1}{2}\epsilon\sigma_z + \frac{1}{2}Ef(t)\sigma_z, \tag{1}$$

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where the σ_α ($\alpha = x, y, z$) are the Pauli spin matrices, Δ is the tunneling rate , ε is the energy bias in the absence of the driving field and E is the amplitude of the external driving field $f(t)$. The Hamiltonian Eq.(1) can be taken to be describing a system with an unperturbed diagonal Hamiltonian:

$$H_0 = -\frac{1}{2}\Delta\sigma_x - \frac{1}{2}\varepsilon\sigma_z, \tag{2}$$

subject to a periodic, $[f(t)=f(t+T)]$, time-dependent perturbation $H_1(t)$:

$$H_1(t)=\frac{1}{2}Ef(t)\sigma_z. \tag{3}$$

The solution $\Psi(t)$ of the time-dependent Hamiltonian Eq.(2) is obtained from the Schrödinger equation

$$H(t)\Psi(t)=i\hbar\frac{\partial}{\partial t}\Psi(t): \tag{4}$$

$$\Psi_\alpha(t) = \exp\left(-i\frac{\varepsilon_\alpha}{\hbar}t\right)\Phi_\alpha(t). \tag{5}$$

Here ε_α is the quasi-energy levels, $\Psi_\alpha(t)$ are the Floquet states and the periodic $\Phi_\alpha(t)$ are the Floquet modes. The ε_α have unique values within the interval 0 and $2\pi/T$ inclusive with T being the period. With Eq. (5) inserted into Eq. (4), the eigenvalue equation for the Floquet modes is easily obtained as:

$$H_F(t)\Phi_\alpha(t)=\varepsilon_\alpha\Phi_\alpha(t), \tag{6}$$

where

$$H_F(t)=H(t)-i\hbar\delta_t \tag{7}$$

can be solved numerically for the Floquet states and quasi-energies. Eq. (2) and Eq. (7) are solved using the techniques described in [17] for different driving fields f(t). The function describing the triangular wave is [3]:

$$f(t)=\begin{cases} 1-4t/T & \text{for } 0\le t\le T/2 \\ -3+4t/T & \text{for } T/2<t\le T \end{cases}; \tag{8}$$

and is implemented using the SciPy intrinsic function *signal-sawtooth* with a width of 0.5. Setting the width to 1.0 results in the sawtooth function. This slight variations results in the difference observed in fig. 1(a) and (b). The square wave driving function is defined as

$$f(t)=\Theta(t)-2\Theta(t-T/2). \tag{9}$$

The result of using Eq (9) in Eq (6) is shown in fig. 1(d) and this agrees with that of Ref. [3].

3.0 Results

The comparison of the triangular and sawtooth functions are shown in fig. 1. The results of the calculation for different types of driving functions is is shown in fig. 2. In fig. 2(a) the driving function is a triangular wave, (b) is a sawtooth function, (c) and (d) are sinusoidal functions (e) is the square wave function. The quasi-energies crossed for certain amplitudes of the driving functions fig. 2(a), (c)-(e). However, this is not the case for the sawtooth function fig. 2(b). There is obviously avoided crossings of the quasi-energies for the entire range of the driving amplitudes in the sawtooth functions fig. 2(b).

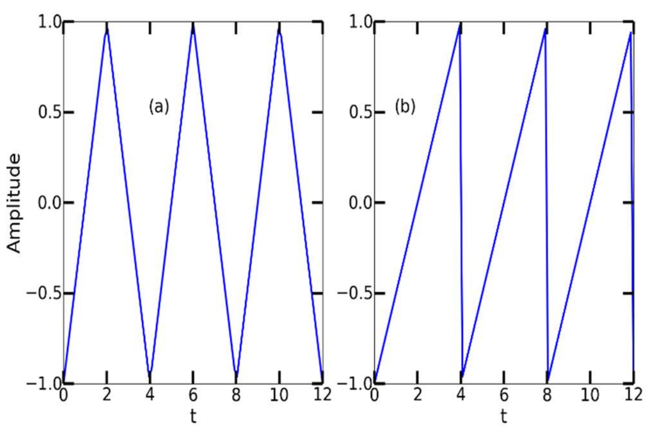

Figure 1: Comparison of the (a) triangular and (b) saw tooth

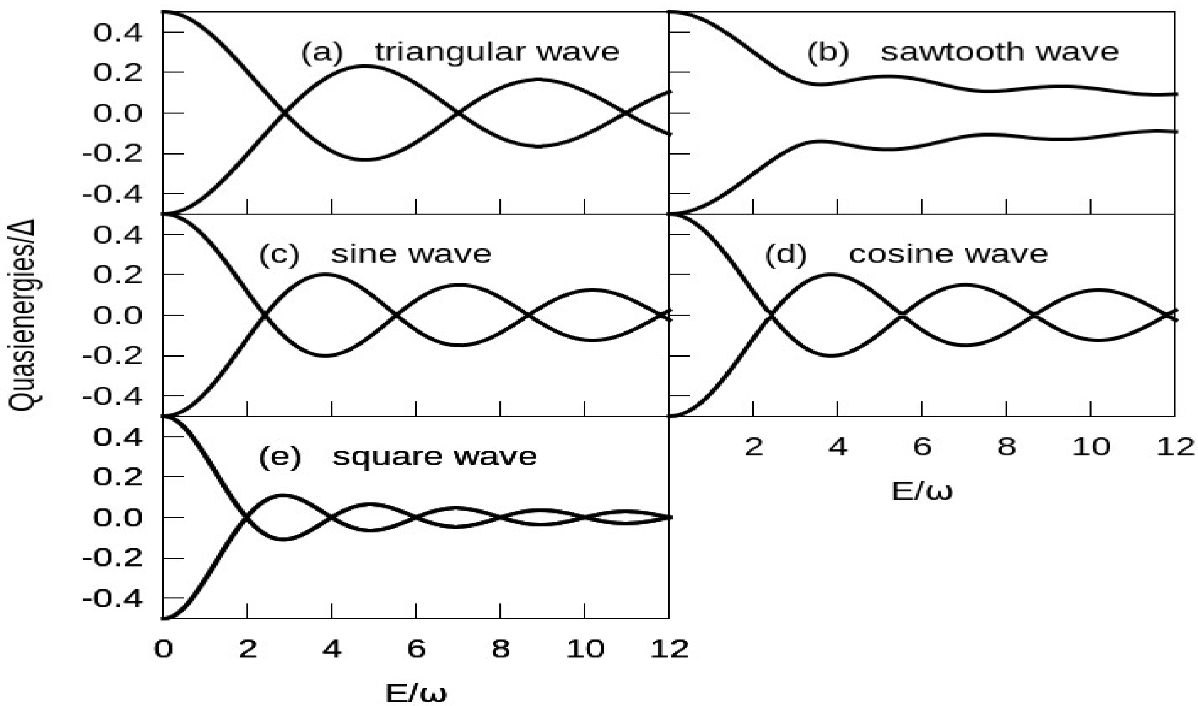

Figure 2: The variation of the quasi-energies of with the amplitude of the driving functions. The frequency of (a)-(e) is ω=8. Although (a) and (b) are similar functions, the quasi-energies of the triangular driving cross three times within the range of E/ω shown unlike (b) where crossing was avoided.

The frequency of the driving function is kept constant throughout the calculation, see the constant width of the vertical lines in fig. 3, however the driving amplitude was allowed to vary.

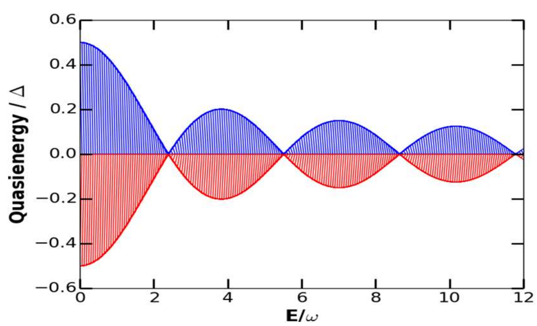

Figure 3: Quasi-energy of a cavity-atom coupling driven by a sine wave function showing the constant frequency (width of the vertical lines) and the varying amplitude which is reminiscence of amplitude modulated singals.

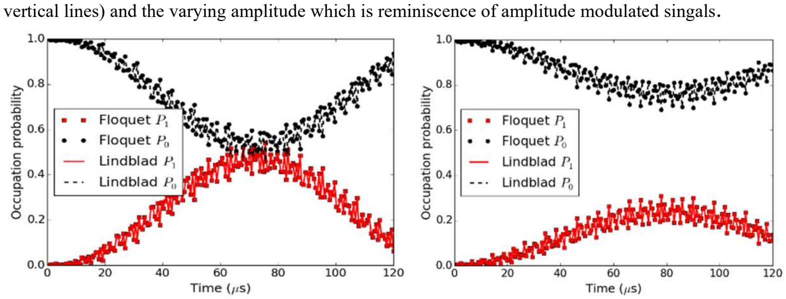

Figure 4: Occupational probabilities of the sawtooth wave function driven system (left panel) and the triangular wave driven system (right panel). The square are the calculated Floquet modes for the left quantum dot while the black dots are those for the atoms in the right quantum dot. The solid and dashed lines are their counterpart calculated using the Lindblad master equation solver. With the system driven by a sawtooth function, there is obviously equal probability ≈ 0.5 of the particle being in the left quantum dot as well as being in the right quantum dot simultaneously. But for the triangular driving function, (right), the particle in the right quantum dot remains there thereby ruling out the possibility of it been found in the left quantum-dot.

4.0 Discussion

The eigenenergies of the triangular-function-driven system crossed three times (at 2.8915, 6.9879 and 10.9397 which compares nicely with 2.92519, 7.02525, 10.9864 of Ref.[3]) within the amplitudes of the driving function explored fig. 2(a). However, the eigenenergies of the sawtooth-function-driven system avoided crossing one another fig. 2(b). The crossings of the quasi-energies of the sine-, cosine-, and square-wave functions driven systems are located as indicated in Table 1. These crossings signify the blockade of tunneling by the system. In essence, it can be said that there is unhindered flow atoms from the left quantum dot to the right quantum dot in a system driven by the sawtooth function.

At the beginning of the simulation, the particles were localized at the left and right quantum dots. After some time there are 50% chances of electrons being simultaneously domiciled in both the left and right quantum dots in the cavity driven by the sawtooth function at some point (approx. 70 μs) of the driving fig. 4 (left panel). The right panel shows a higher probability (approx. 75%) of the particles being confined to their original locations for the triangular function driven cavity.

Table 1: Observed crossings of the quasi-energies for the sine, cosine and square driving functions.

Function	Crossing
Sine	**2.46, 5.52, 8.64, 11.82**
Cosine	**2.4242, 5.5758, 8.6061, 11.7576**
Square	**1.9394, 6.0606, 8.0000, 12.0000**

5.0 Conclusion

The dynamics of the coupling of the two-level atom to the cavity wherein it is confined has been investigated. It has among other things explored the behaviour of the sawtooth-function-driven system and the triangular-function driven system and finds that despite the semblance of the two functions, their behaviours are radically different. Additionally, the ground and excited states of the triangular-function-driven system shows equal occupational probabilities at some stage of their evolution whereas the situation was quite different for the sawtooth-function-driven system.

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