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Experimental Study of Combination Synchronization of Three Van der Pol Oscillators Via Cyclic Coupling

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Abstract

In this study, combination synchronization of two drive and one response oscillator(s) as well as one drive to two response oscillator(s) via cyclic coupling are reported. The influence of the coupling strength on cyclic coupling of combined oscillators of Van der Pol oscillator as a case study was examined using linear feedback technique. Depending on the choice of coupling strength, a rich variety of synchronized regime is observed. We also study the analytical and numerical simulation to confirm the effectiveness of the scheme. An optimal value of critical coupling from experimental simulation of three oscillators which has better advantage over two oscillators. Finally, this type of synchronization can improve the security of communication by spliting or combined the transmitted signals from drive oscillator(s) or received signal(s) from response oscillator(s), respectively.

Keyword: Combination synchronization; Cyclic coupling; Linear feedback coupling; Numerical simulation

1. INTRODUTION

The concept of cyclic coupling involves two coupling strength in reverse directions when maximum control of synchrony is feasible [1]. The scheme established a mutual interaction between oscillator(s) when a signal is sent to another oscillator(s) via one pair of state variables and simultaneously receives a feedback via different state variables [1]. In synchrony of chaotic oscillators, the main task is to determine the critical coupling which depends on the type of coupling and the topology used [2- 4]. The numerical studies of one to one oscillator using linear stability analysis and the analytical solutions using master stability function (MSF) to determine the critical coupling for stable synchronization under cyclic coupling as well as diffusive coupling for different chaotic oscillators have been studied and compared [1]. In practice, oscillators are not actually coupled directly but synchrony can only occur due to interaction between the systems particles and the common medium. This is similar to the interaction between self-pulsating periodic and chaotic oscillations by controlling field cavity detuning and an ensemble of cold atoms interacting with coherent electromagnetic field [5]. However, a cyclic bidirectional interaction may occur in a natural situation such as neuronal interaction in the brain in which the signal sends to one neuron may fail until a feedback from another neuron is received via pair of dendrites [6]. Another example of mutual interaction that may occur during cyclic coupling is like pulling-by-hand by one individual while the other pushing-by-leg which eventually results in cyclic motion [1]. In other to provide rigid and secure information, combination or addition of two or more oscillators needs to be designed before coupling. The numerical simulation of two masters to one slave has been studied for active backstepping [7, 8] while the finite-time synchronization application to secure communication of three chaotic oscillators has equally been reported by Luo and Wang [9]. Recently, on cyclic coupling, the enhancement in synchrony for different identical oscillators has been investigated by Olusola *et. al.*[10]. The transition from homogenous to inhomogenous steady state of two van der Pol oscillators as limit cycle and two sprott oscillators as chaotic systems has been reported for both diffusive and cyclic coupling [11]. And recently, experimental evidence of chaos synchronization via cyclic coupling between Sprott and Rossler oscillators was reported for an improved synchrony [12]. To the best of the authors' knowledge, cyclic coupling of three oscillators via linear feedback scheme has not been studied. Motivated by the reports so far, coupling of three analog van der Pol – Duffing circuits with cubic function nonlinearity via diffusive and negative feedback scheme are investigated and reported in this paper as paradigmatic illustration. The rest of the paper is organized as follows. In section 2, combination synchronization scheme and cyclic coupling are discussed. Experimental implementation is studied in section 3, while section 4 concluded the paper.

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2.0 Models and Coupling Scheme

2.1 Combination synchronization Scheme

The combination arrangement for one or two drive Cubic-van der Pol oscillator(s) as a case study, synchronized to two or one response oscillator(s) respectively are designed in this work. One drive (x) to two responses (y and z) oscillators via negative feedback coupling is given as follows:

$$
\begin{aligned}
\dot{x} &= f_x(x), \\
\dot{y} &= f_y(y) + k(x - (y+z)), \\
\dot{z} &= f_z(z) + k(x - (y+z)),
\end{aligned}
\tag{1.1}
$$

Similarly, for two drives (y and z) to one response (x) systems, the equations can be described as follows:

$$
\begin{aligned}
\dot{x} &= f_x(x) + k((x+y) - z, \\
\dot{y} &= f_y(y), \\
\dot{z} &= f_z(z),
\end{aligned}
\tag{1.2}
$$

For diffusive coupling, one drive (x) to two responses (y and z) systems, the equations can be described as follows:

$$
\begin{aligned}
\dot{x} &= f_x(x) + k((y+z) - x), \\
\dot{y} &= f_y(y) + k(x - (y+z)), \\
\dot{z} &= f_z(z) + k(x - (y+z)),
\end{aligned}
\tag{1.3}
$$

Similarly, for two drives (y and z) to one response (x) systems, the equations can be described as follows:

$$
\begin{aligned}
\dot{x} &= f_x(x) + k((y+z) - x), \\
\dot{y} &= f_y(y) + k(x - (y+z)), \\
\dot{z} &= f_z(z) + k(x - (y+z)),
\end{aligned}
\tag{1.4}
$$

where $x = (x_1, x_2)$, $y = (y_1, y_2)$ and $z = (z_1, z_2)$ are the state vectors of systems while, f_x, f_y, f_z : $R^n \to R^n$ are three vector functions with controller or coupling parameter k which will be designed respectively.

Definition 1: If there exist three scaling matrices A, B, C $\in R^n$ and C $\neq 0$ such that $\lim_{t\to\infty} \| Ax + By - Cz \| = 0$, then the problem is combination synchronization. The drive-response systems (1.1) to (1.4) are realized via negative feedback and diffusive coupling respectively, where $\|.\|$ denotes the matrix norm.

***Remark* 1**: Definition 1 shows that the combination of the drive – response systems can be extended to three or more chaotic systems whether identical or different. The constant matrix A, B and C are called scaling matrices and further extended to the functional matrices of state variables x_A, y_A and z_A. Where $x_A(x_{1A}, x_{2A})$, $z_A(y_{1A}, y_{2A})$ and $z_A(z_{1A}, z_{2A})$ are the state vectors of systems (1.1) to (1.4), respectively.

***Remark* 2**: If $A = B$, then the combination synchronization will be turned into a chaos control problem.

The non-identical cubic function oscillators via negative feedback coupling which involves one drive to two responses systems in Eqns. (1.1) to (1.2), respectively, are given as follows:

$$
\begin{aligned}
&\dot{x}_{1A} = x_{2A} \\
&\dot{x}_{2A} = (\varepsilon_1 - x_{1A}{}^2) x_{2A} - \alpha x_{1A} - \beta x_{1A}{}^3 + f_{1A} \cos\omega_{1A} t \\
&\dot{y}_{1B} = y_{2B} \\
&\dot{y}_{2B} = (\varepsilon_2 - y_{1B}{}^2) y_{2B} - \alpha y_{1B} - \beta y_{1B}{}^3 + f_{2B} \cos\omega_{2B} t \\
&+ k[x_{1A} - (y_{1B} + z_{1C}] \\
&\dot{z}_{1C} = z_{2C} \\
&\dot{z}_{2C} = (\varepsilon_3 - z_{1C}{}^2) z_{2C} - \alpha z_{1C} - \beta z_{1C}{}^3 + f_{3C} \cos\omega_{3C} t \\
&+ k[(x_{1A} - (y_{1B} + z_{1C})]
\end{aligned}
\tag{1.5}
$$

While two drives (y and z) to one response (x) systems for negative feedback as stated in Eqns. (1.3) to (1.4), respectively, are given as follows:

$$\begin{aligned}
\dot{x}_{1A} &= x_{2A} \\
\dot{x}_{2A} &= (\varepsilon_1 - x_{1A}{}^2)x_{2A} - \alpha x_{1A} - \beta x_{1A}{}^3 + f_{1A}\cos\omega_{1A}t \\
&+ k[(y_{1B} + z_{1C}) - x_{1A}] \\
\dot{y}_{1B} &= y_{2B} \\
\dot{y}_{2B} &= (\varepsilon_2 - y_{1B}{}^2)y_{2B} - \alpha y_{1B} - \beta y_{1B}{}^3 + f_{2B}\cos\omega_{2B}t \\
\dot{z}_{1C} &= z_{2C} \\
\dot{z}_{2C} &= (\varepsilon_3 - z_{1C}{}^2)z_{2C} - \alpha z_{1C} - \beta z_{1C}{}^3 + f_{3C}\cos\omega_{3C}t
\end{aligned} \tag{1.6}$$

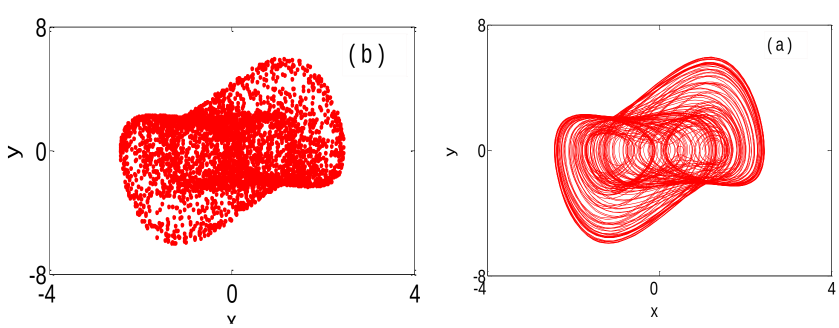
Similarly for diffusive coupling, one drive (x) to two slaves (y and z) systems is described as follows:

$$\begin{aligned}
\dot{x}_{1A} &= x_{2A} \\
\dot{x}_{2A} &= (\varepsilon_1 - x_{1A}{}^2)x_{2A} - \alpha x_{1A} - \beta x_{1A}{}^3 + f_{2A}\cos\omega_{2A}t \\
&+ k[(y_{1B} + z_{1C}) - x_{1A}] \\
\dot{y}_{1B} &= y_{2B} \\
\dot{y}_{2B} &= (\varepsilon_2 - y_{1B}{}^2)y_{2B} - \alpha y_{1B} - \beta y_{1B}{}^3 + f_{2B}\cos\omega_{2B}t \\
&+ k[x_{1A} - (y_{1B} + z_{1C})] \\
\dot{z}_{1C} &= z_{2C} \\
\dot{z}_{2C} &= (\varepsilon_3 - z_{1C}{}^2)z_{2C} - \alpha z_{1C} - \beta z_{1C}{}^3 + f_{3C}\cos\omega_{3C}t \\
&+ k[(x_{1A} - (y_{1B} + z_{1C})]
\end{aligned} \tag{1.7}$$

While two drives (with state variables y and z) to one response (x) systems are given as follows:

$$\begin{aligned}
\dot{x}_{1A} &= x_{2A} \\
\dot{x}_{2A} &= (\varepsilon_1 - x_{1A}{}^2)x_{2A} - \alpha x_{1A} - \beta x_{1A}{}^3 + f_{2A}\cos\omega_{2A}t \\
&+ k[(y_{1B} + z_{1C}) - x_{1A}] \\
\dot{y}_{1B} &= y_{2B} \\
\dot{y}_{2B} &= (\varepsilon_2 - y_{1B}{}^2)y_{2B} - \alpha y_{1B} - \beta y_{1B}{}^3 + f_{2B}\cos\omega_{2B}t \\
&+ k[x_{1A} - (y_{1B} + z_{1C})] \\
\dot{z}_{1C} &= z_{2C} \\
\dot{z}_{2C} &= (\varepsilon_3 - z_{1C}{}^2)z_{2C} - \alpha z_{1C} - \beta z_{1C}{}^3 + f_{3C}\cos\omega_{3C}t \\
&+ k[(x_{1A} - (y_{1B} + z_{1C})]
\end{aligned} \tag{1.8}$$

where $\varepsilon_{1,2,3} > 0$, α and β are constant parameters, $\omega_{1,2,3}$ and $f_{1,2,3}$ are angular frequencies and amplitudes respectively. State vectors $x_{1,2}$, $y_{1,2}$ and $z_{1,2}$ represent the circuitry voltages of the oscillators.



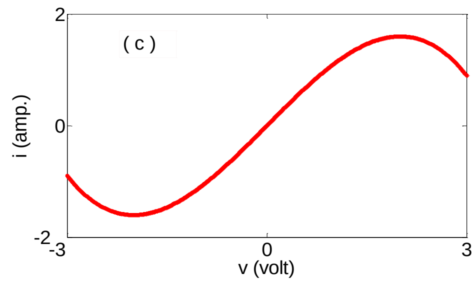


Figure 1: Phase portrait (a) and Poincare map (b) for cubic-van der Pol oscillator and ($i - v$) characteristics (c) with constant parameters: $\varepsilon = 1.25$, $\alpha = 0.8$, $\beta = 1.05$, $f = 4.0$ and $\omega = 0.55$

Numerical solutions of Eqns. (1.5) to (1.8) were obtained using the Fourth- order Runge-Kutta routine with time step-size 0.02, and fixing the parameter values as in Figure 1. In Figure 2(a-b), we display the non-synchronous results for one drive to two response negatively feeedback coupled van der Pol oscillators (x_1) vs ($y_1 + z_1$) and two drive to one negatively feedback coupled van der Pol oscillators ($y_1 + z_1$) vs (x_1) when coupling constant k is 0 (which implies no coupling effect on the coupled systems) and 10, respectively. Similarly, Figure 3 (a-b) also display the results when the oscillators are diffusively coupled. In Figures 4 and 5, complete synchronization actually occurs between $k = 58$ and 59 respectively when the oscillators are negatively coupled, and error dynamics of the oscillators converges to zero which implies that the oscillators (1.5) and (1.6) have achieved combination synchronization. Similar trend also follows when Eqns. (1.7) and (1.8) were diffusively coupled.

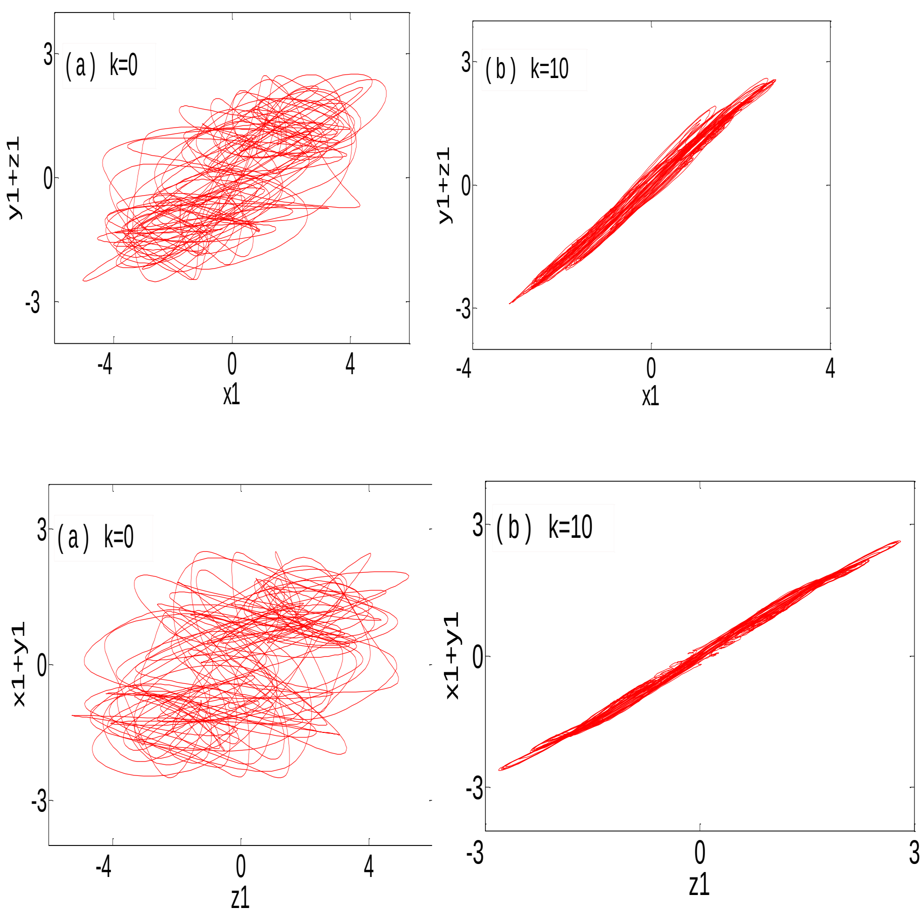


Figure 2: Phase portraits for nonsynchronous state for one (x_1) to two ($y_1 + z_1$) and two ($x_1 + y_1$) to one (z_1) negative feedback coupled oscillators at coupling strength $k = 0$ and 10 respectively.

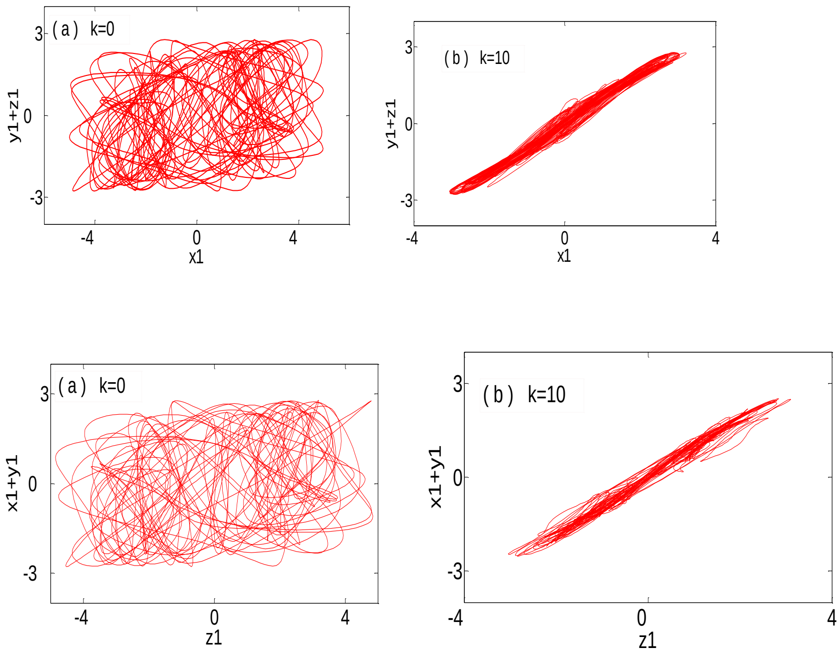


Figure 3: Phase portraits for nonsynchronous state for one (x_1) to two ($y_1 + z_1$) and two ($x_1 + y_1$) to one (z_1) diffusively coupled oscillators at coupling strength $k = 0$ and 10 respectively.

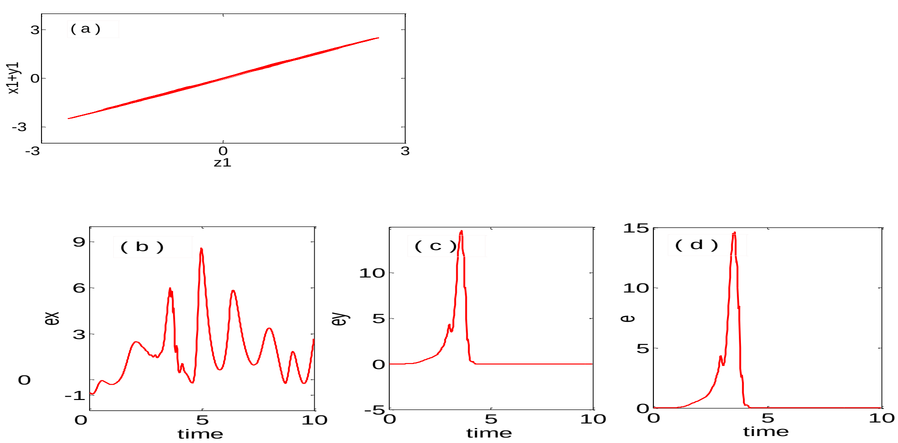
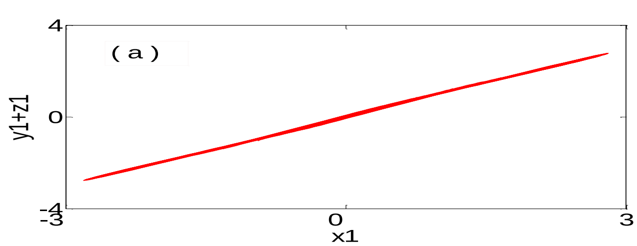


Figure 4: Complete synchronization (a) and error dynamics $e_x = [z_1 - (x_1 + y_1)]$, $e_y = [(x_2 + y_2) - z_2)]$ and $e = \sqrt{e_x{}^2 + e_y{}^2}$ for two to one coupled oscillators when the coupling strength (k) is 59.19



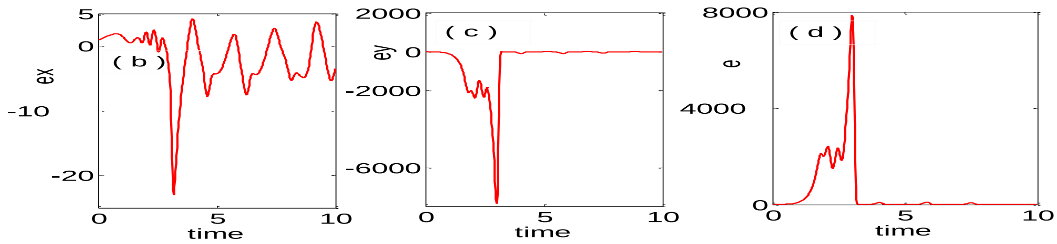


Figure 5: Complete synchronization (a) and error dynamics $e_x = [(y_1 + z_1) - x_1]$, $e_y = [x_2 - (y_2 + z_2)]$ and $e = \sqrt{e_x^{\ 2} + e_y^{\ 2}}$ for one to two coupled oscillators when the coupling strength is 58.

2.2 Design of coupled Cubic-van der Pol circuit

The experimental design of cubic-van der Pol oscillator is shown in Figure 6, where the circuit parameters are resistors R_{1-7}, capacitors C_{1-2}, AD633JN U_{1-3} as analog multiplier, TL084CN U_{4A-6A} as operational amplifier, and operational voltage of positive and negative power supplies were set to $\pm 12V$. Each system was designed with a cubic-nonlinear function of current-voltage characteristics $I(V) = -aV + bV^3$ as shown in Figure 1(c), where $(a, b > 0)$. Different phase portrait of attractors shown in Figure 7, are generated from Figure 6 due to regulation in internal noise and circuitry parameters. The coupled systems (1.5) - (1.8) are transformed into circuits A and B interconnected via linear feedback technique to circuit C. Let V_1 and V_2 be the voltages across the capacitor C involved in the cubic-van der Pol circuits A, B and C, respectively. Application of Kirchhoff's rule on schematic diagram of van der Pol circuit [12] when combined and coupled together yields:

$$\begin{aligned}
&LC\frac{d^2V_{1A}}{d\tau^2} + [R_{1A}C + L(-a + 3bV_{1A}^{\ 2})]\frac{dV_{1A}}{d\tau} + (-aR_{1A} + 1)V_{1A} + R_1 bV_{1A}^{\ 3} = 0 \\
&LC\frac{d^2V_{1B}}{d\tau^2} + [R_{1B}C + L(-a + 3bV_{1B}^{\ 2})]\frac{dV_{1B}}{d\tau} + (-aR_{1B} + 1)V_{1B} + R_1 bV_{1B}^{\ 3} + \\
&R_C R_f C(V_{1A} - (V_{1B} + V_{1C})) = 0 \\
&LC\frac{d^2V_{1C}}{d\tau^2} + [R_{1C}C + L(-a + 3bV_{1C}^{\ 2})]\frac{dV_{1C}}{d\tau} + (-aR_{1C} + 1)V_{1C} + R_{1C} bV_{1C}^{\ 3} + \\
&R_C R_f C(V_{1A} - (V_{1B} + V_{1C})) = 0
\end{aligned} \tag{1.9}$$

for one master and two slaves oscillators with diffusive coupling, and

$$\begin{aligned}
&LC\frac{d^2V_{1A}}{d\tau^2} + [R_{1A}C + L(-a + 3bV_{1A}^{\ 2})]\frac{dV_{1A}}{d\tau} + (-aR_{1A} + 1)V_{1A} + R_1 bV_{1A}^{\ 3} \\
&+ R_C R_f C((V_{1B} + V_{1C}) - V_{1A}) = 0 \\
&LC\frac{d^2V_{1B}}{d\tau^2} + [R_{1B}C + L(-a + 3bV_{1B}^{\ 2})]\frac{dV_{1B}}{d\tau} + (-aR_{1B} + 1)V_{1B} + R_1 bV_{1B}^{\ 3} + \\
&R_C R_f C(V_{1A} - (V_{1B} + V_{1C})) = 0 \\
&LC\frac{d^2V_{1C}}{d\tau^2} + [R_{1C}C + L(-a + 3bV_{1C}^{\ 2})]\frac{dV_{1C}}{d\tau} + (-aR_{1C} + 1)V_{1C} + R_{1C} bV_{1C}^{\ 3} + \\
&R_C R_f C(V_{1A} - (V_{1B} + V_{1C})) = 0
\end{aligned} \tag{2.0}$$

for one master and two slaves oscillators with negative feedback coupling.

Similar transformation can be performed on two masters coupled to one slave with linear feedback coupling (diffusive and negative feedback). With the following changes to the variables and parameters:

$\varepsilon_1 = a\rho - \frac{R_{1A}}{\rho}$, $\varepsilon_2 = a\rho - \frac{R_{1B}}{\rho}$, $\varepsilon_3 = a\rho - \frac{R_{1C}}{\rho}$, $\rho = \sqrt{\frac{L}{C}}$, $\tau = t\sqrt{LC}$, $V_{ref} = \frac{1}{\sqrt{3b\rho}}$, $\mu A_d \frac{R_f}{\rho}$, $\alpha_1 = 1 - aR_{1A}$,

$\alpha_2 = 1 - aR_{1B}$, $\alpha_3 = 1 - aR_{1C}$, $\beta_1 = R_{1A}bV_{ref}^2$, $\beta_2 = R_{1B}bV_{ref}^2$, $\beta_3 = R_{1C}bV_{ref}^2$, $x = \frac{V_{1A}}{V_{ref}}$, $y = \frac{V_{1B}}{V_{ref}}$, $z = \frac{V_{1C}}{V_{ref}}$

and $k = \frac{R_f}{\rho}$

the electronic equations for cubic van der Pol circuit can be written in differential form as shown in Eqn. (1.5) - (1.8).

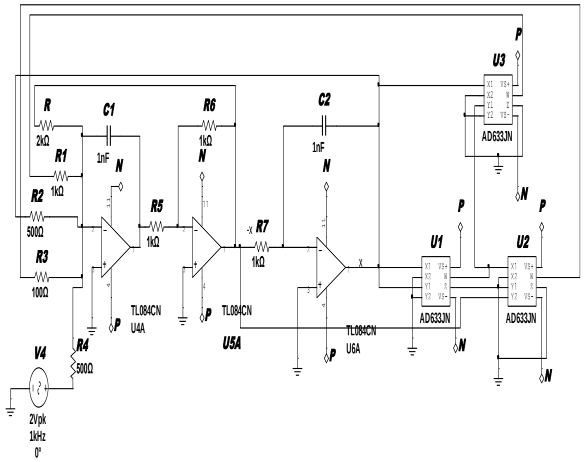


Figure 6: Analog design of cubic-nonlinear Van der Pol Oscillator

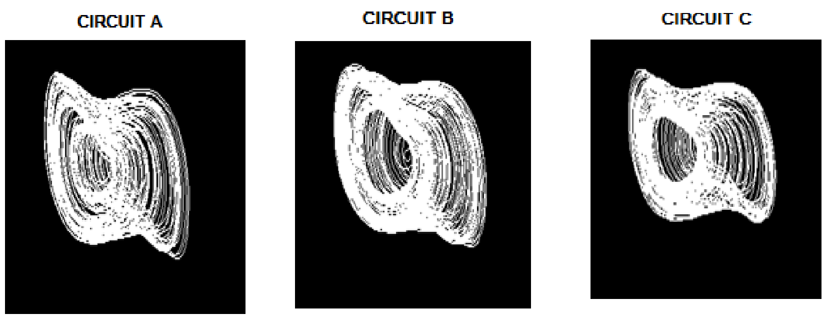


Figure 7: Phase portrait attractors V_2 vs V_1 for Circuit A: $R_A = 2k\Omega$, $R_2 = 500\Omega$ and $R_4 = 500\Omega$, Circuit B: $R_B = 3k\Omega$, $R_2 = 400\Omega$ and $R_4 = 500\Omega$ and Circuit C: $R_C = 2k\Omega$, $R_2 = 600\Omega$ and $R_4 = 500\Omega$

2.3 Cyclic coupling Scheme

The idea of cyclic coupling in chaotic oscillators for both negative feedback and diffusive scheme involves pairs of variables engaged in the coupling such that there is a different possible topologies of cyclic coupling out of which some are independent while the other topologies are symmetric for identical oscillators. As example, for three, two dimensional oscillators, the state vectors can be denoted as $x = [x_1 \quad x_2]^T$, $y = [y_1 \quad y_2]^T$ and $z = [z_1 \quad z_2]^T$. Table 1 indicates some dimensions with possible topologies.

Table1: Systems dimensions and there cyclic topologies

System Dimensions	Possible Topologies	Independent Toplogies	Symmetric Toplogies
2-D	2	1	1
3-D	6	3	3
4-D	12	6	6
5-D	20	10	10
6-D	30	15	15

Considering the topologies in Table 1, two pairs of variables for combined cubic-van der Pol oscillators as shown in Eqn. 1.1 and Eqn.1.2, then one possible pairing could be: for one drive to two response, $x_1 \to y_1 + z_1$, $x_2 \leftarrow y_2 + z_2$ and $x_1 + y_1 \to z_1$, $x_2 + y_2 \leftarrow z_2$ for two drive to one response.

The directions of coupling in the expression above denote how the concept of cyclic coupling is achieved. Hence, introducing stability/coupling parameter k_1 and k_2, the stability matrix (M) can be written in terms of output function of each oscillator that is engaged in the coupling (H), linear coupling Hurwitz matrix of the oscillating oscillator (A) and the identity matrix (I) as given in the relation:

$$M = A + \psi I - k_i H_i - k_j H_j \qquad (1.9)$$

The stability criteria for complete synchronization are derived from the negative real parts of the eigenvalues λ of the matrix M according to Routh-Hurwitz stability criterion. Here, k_i and ψ are real valued and non-negative.

3.0 Experimental Implementation

In this paper, our focus is basically on experimental synchronization of combination arrangement of cubic-van der Pol circuits via negative feedback and diffusive coupling. We also investigate the influence of coupling strength on cyclic coupling of the oscillators. The set-up involves one drive coupling with two response circuits as well as two drives and one response circuits for possibility of synchrony. In Appendix I and II, the output voltage signals of circuit B and C are added together with an adder to form drive and coupled with circuit A, while the relayed voltage is sent back to the input voltages of the individual circuit via voltage follower (operational amplifiers). In Appendix I, negative feedback coupling is applied unlike diffusive coupling used in Appendix II which leads to changes in chaotic dynamics of each oscillator by varing the coupling resistor. However,
the circuit exhibit chaotic dynamics in the absence of coupling and changes in dynamical behaviour as coupling strength increasing which may lead to complete synchronization.
Similarly, the arrangement of chaotic circuits which involves one drive against two response oscillators using negative feedback (Appendix III) or diffusive coupling (Appendix IV) lead to CS.

3.1 Two masters coupled with one slave

De-synchronization between systems occurs when the coupling is very weak (i.e at higher resistance) while synchronization occurs when coupling strength is high. For example, low coupling strength ($R_C = 2k\Omega$) in Figure 8 (a-c) which consist of two masters and one slave show uncorrelated dynamics for the layed output (a) and relayed inputs (b and c) voltages or signals. At high coupling strength or low resistance value as shown in Figure 8 (d-f), the dynamics completely synchronized for both layed and relayed output voltages specifically for $R_C \leq 100\Omega$.

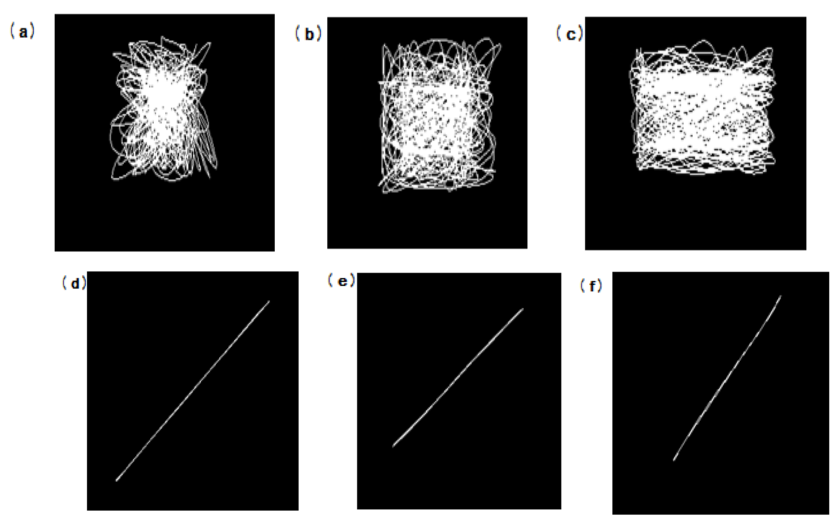


Figure 8: De-synchronization (a-c) and complete synchronization (e-f) for negative feedback coupling of two masters to one slave cubic function van der Pol circuits.

The diffusive coupling of two masters to one slave Oscillators presented in Figure 9 also shows uncorrelated synchronization at low coupling strength (a-c) just like the one observed in negative feedback coupling. With increasing the coupling strength, the layed output voltage completely synchronized (d) but desynchronized (e and f) relayed input voltages. In other words, the two master circuits sending out one of their state variables and receiving them as input voltages at the same time and at the same coupling strength through the other variable.

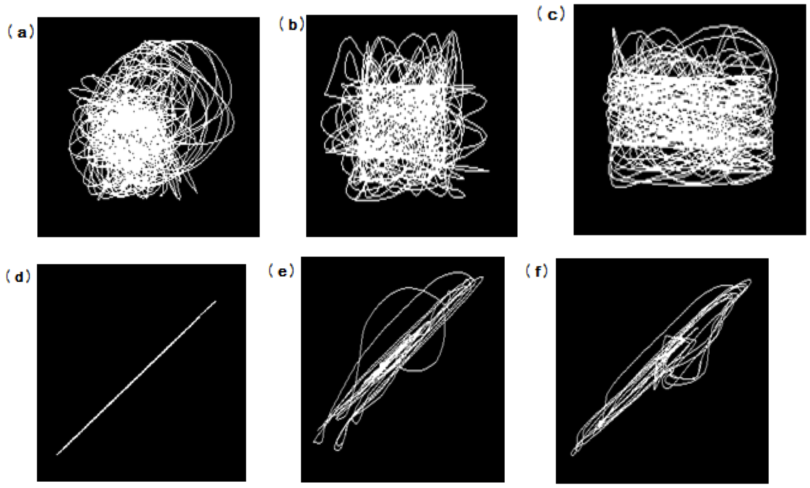


Figure 9: De-synchronization (a-c) and (e-f) for both layed and relayed voltages, complete synchronization occur at (d) for only layed output voltages for diffusive coupling of two masters to one slave cubic function van der Pol circuits

3.2 One master coupled with two slaves

In case of one master to two slaves, the design involves negative feedback coupling, which also follows the same trajectory as observed in two masters to one slave coupling scheme. The dynamics still reveals uncorrelated output and input voltages at low coupling strength (say, $R_C = 2k\Omega$) as presented in Figure 10 (a-c), but complete synchrony at high coupling strength (say, $R_C \le 100$) as shown in Figure 10 (d-f). In Figure 11, similar trend of unsynchronized attractors occur for both layed and relayed output and input voltages (a-c) at low coupling strength when diffusively coupled, but complete synchronization only occurs at higher coupling strength (say, $R_C \le 1\Omega$) for layed output voltage (d) with incomplete synchronization for the relayed input voltage (e and f).

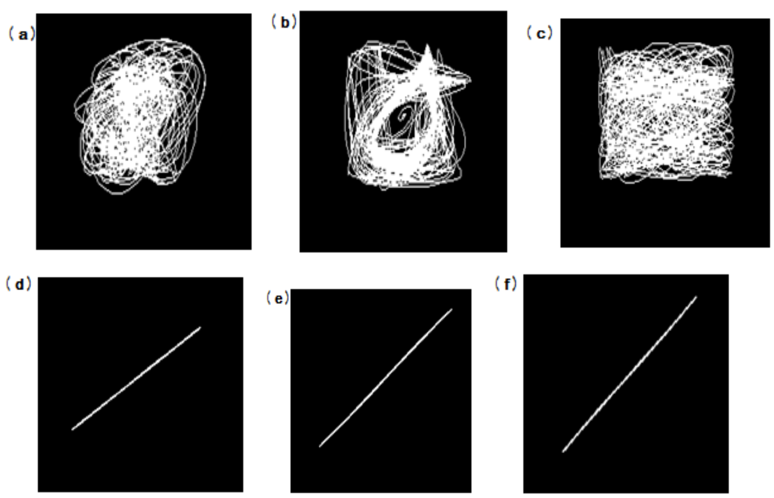
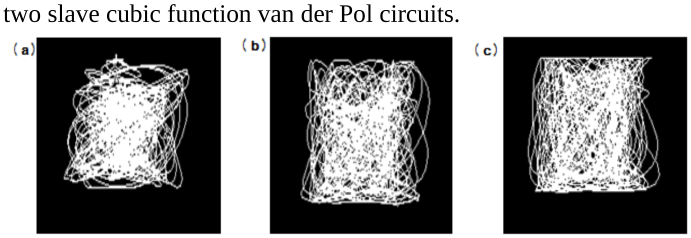


Figure 10: Desynchronization (a-c) and complete synchronization (e-f) for negative feedback coupling of one master to two slave cubic function van der Pol circuits.



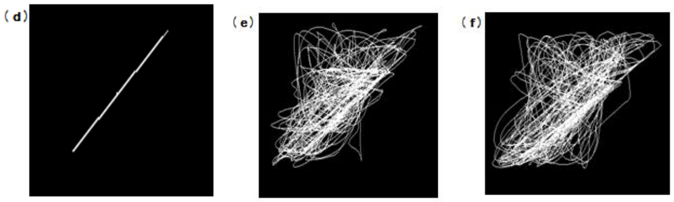
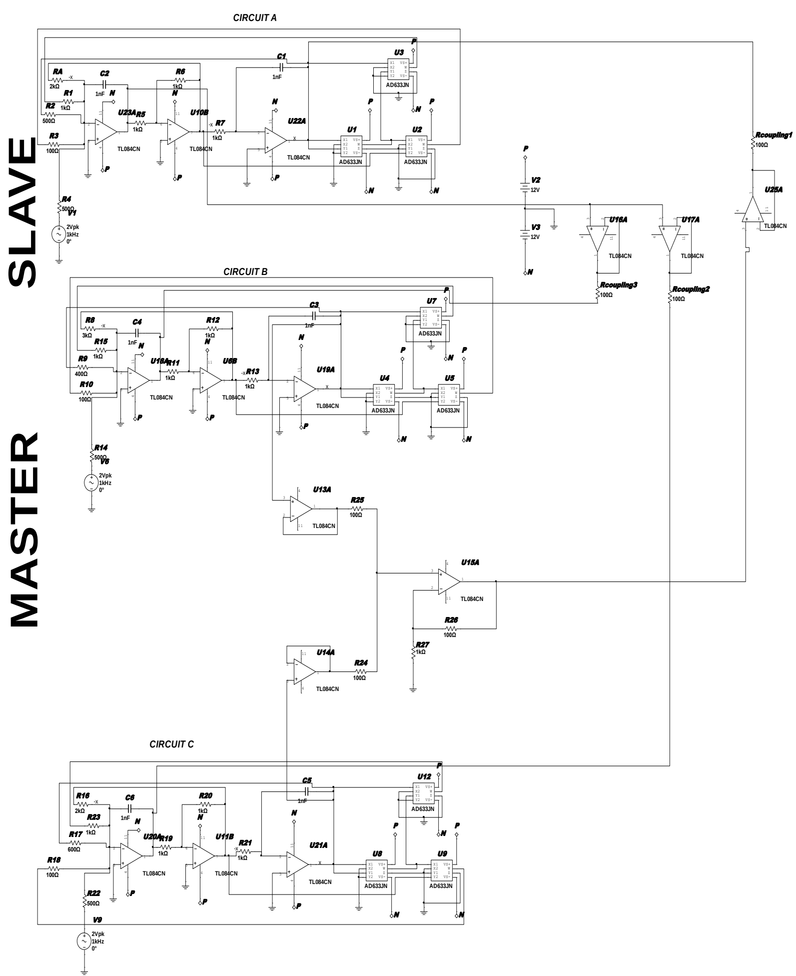


Figure 11: Desynchronization (a-c) and (e-f) for both layed and relayed voltages, complete synchronization occur at (d) for only layed output voltages for diffusive coupling of one master to two slaves cubic function van der Pol circuits

4. Conclusion

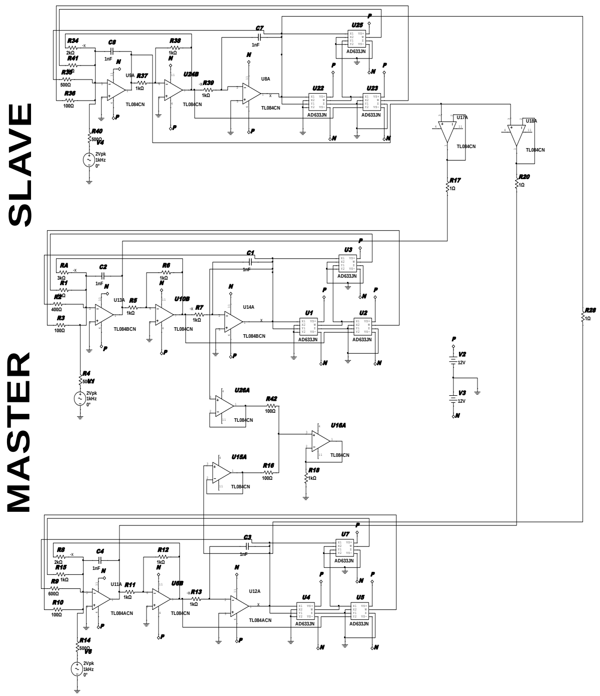
In this paper, the main motivation was to examine the dynamics evolves when two combined chaotic oscillators, interact with the third oscillator via cyclic coupling. The influence of the same coupling strength on synchronization behaviour of cubic function Van der Pol circuits as a case study has been discussed. The coupled oscillators were analyzed analytically and numerically. The experimental circuit coupling of two masters with one slave as well as one master to two slaves using negative feedback and diffusive schemes has been reported. Our experimental results show correlation in voltages ranging from de-synchronization to complete synchronization when changing the coupling strength value. The relayed outputs were not, but actually acted as dynamical relay between the response and the drive voltages of the circuits. The combined scheme has more advantage than one to one synchrony and possibility of implementation in electronics which may have wider application in secure communication and cryptography.

Appendix I



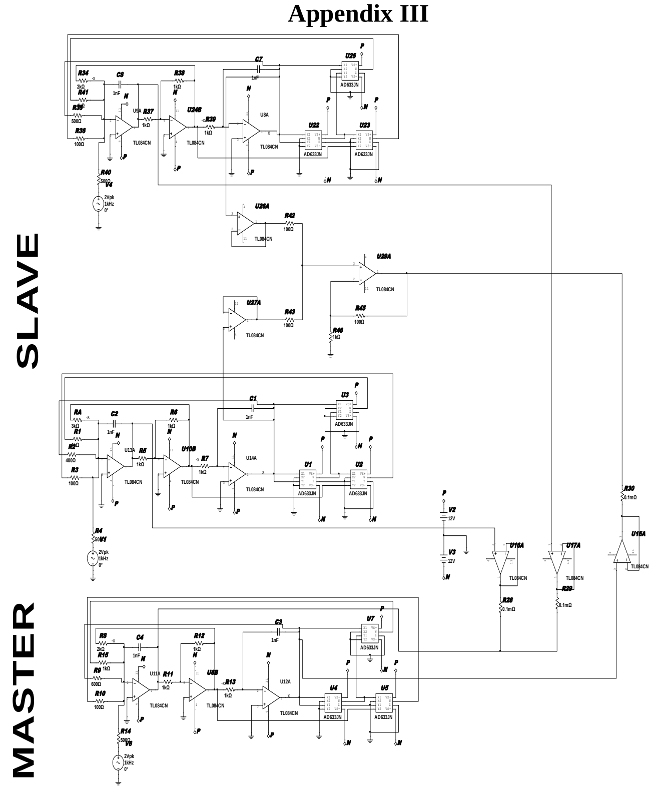
Negative feedback coupling of combination scheme of two masters against one slave circuits via cyclic coupling.

Appendix II



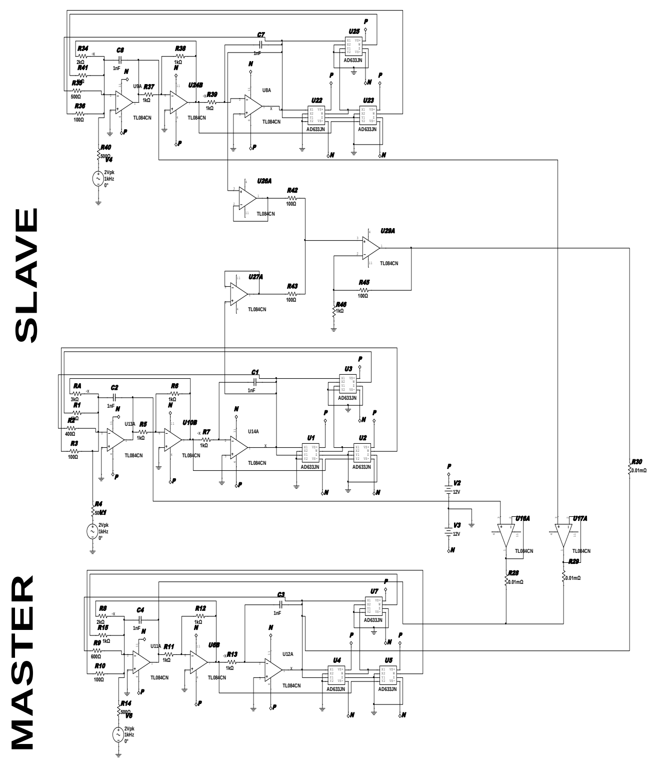
Diffusive coupling of combination scheme of two masters against one slave circuits via cyclic coupling.

Appendix III



Negative feedback coupling of combination scheme of one master against two slave circuits via cyclic coupling.

Appendix IV



Diffusive coupling of combination scheme of one master against two slave circuits via cyclic coupling.

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