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Multiswitching Combination Synchronization in High Dimensional Hyperchaotic Systems

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Abstract

In this paper, we further examine and analyze multi-switching combination synchronization (MSCS) using a 5-dimensional hyperchaotic system. In the MSCS master-slave scheme in which the synchronization takes place in diverse/multiple combination directions, we show that for the 3D chaotic and 4D hyperchaotic systems maximum of 27 and 256 switches grouped into 5 and 15, respectively are allowable. We further obtained 3125 switches for the 5D system grouped into 52 parts using their dimensions. This shows the high variabilities of switches available in higher dimension for information encoding; and its security implication for signal transmission. We performed numerical simulations to demonstrate MSCS in the case of 5D hyperchaotic system.

Keyword: Multi-switching, combination synchronization, high-dimensional systems, hyperchaotic systems

1. INTRODUTION

Nonlinear deterministic dynamical systems exhibit sensitive dependence on initial conditions and the existence of this behavior has been confirmed in various fields such as sciences, medicines and engineering [1,2]. Such systems exhibit chaos as well as hyperchaos in high dimensions. Hyperchaotic systems though similar to the chaotic systems, are different because they possess two or more positive Lyapunov exponents [3] which determines the separation of nearby orbits in two or more directions and this makes them more complex and thereby more useful in chaoticdata encryption. One of the most fascinating attributes of such systems is synchronization because of its potential application in information, communication processing and security [1,4 6], among others. The original idea of synchronization as presented by Pecoraand Carroll in 1990 [7] is such that for coupled or interacting chaotic systems with state variables, $y_1(t)$ and $y_2(t)$,there is a complete or identical synchronization manifold$y_1(t) = y_2(t)$ if $\lim_{t\to\infty}\|y_1(t) - y_2(t)\| \to 0$for all $t \geq 0$. In a latter work, Mainleri and Rehacek [8]showed that it is possible for two chaotic systems to synchronize up to a scaling factor such that$\lim_{t\to\infty}\|y_1(t) - \alpha y_2(t)\| \to 0$

For all $t \geq 0$. This type of synchronization, known asProjective synchronization (PS) has gained prominentresearch attention because it yields a result in faster communication systems due to its scaling factors. Other works in this area includes functional projective synchronization

[9,10], generalized projective synchronization [11], complex projective synchronization [12,13], adaptive projective synchronization [14,15] and hybrid projective synchronization [16], among others. Notably, all these works are one driver- one response systems.

A more recent variation of PS was proposed by Luo et al [17]. In this case, two driver systems synchronise with a response system. This is known as combination synchronization. In this configuration communication signals can be split into two, each loaded and transmitted between

the drivers. Alternatively, each part could be ransmitted at different time intervals. Combination synchronization as attracted considerable research attention and developments in this direction include compound synchronization[18,19], double compound [20], equal combination [21],complex combination [22,23], combination-combination[24–28], and generalized reduced-order hybrid combination synchronization [29].

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Earlier, Ucar et al [30] had proposed the multi-switching synchronization of chaotic systems. In their work, slave state variables synchronize with different state variables of the master systems. Since then, works on multi-switching had been limited to switching of synchronizing systems with varieties of scaling functions [31–33], adaptive switching [26,34], switching of fractional order chaotic system[35], and multi-switching of driven hyperchaotic systems[36]. However, all these works addressed the single driver-single response systems until very recently when Vincent et al [1], proposed for the first time, a novel work on the double drivers-single response multi-switching synchronization for a 3-dimensional chaotic system. To the best of our knowledge, all works of this kind reported in literature has not involved a higher dimensional system of order five (5).

Furtherance to our earlier works [37-39], in this paper, we further examine and analyze multiswitching combinations synchronization (MSCS) using a 5D hyperchaotic systems, in which case, two drivers of the hyperchaotic 5D systems are synchronized with single response hyperchaotic system. Implementing MSCS in high-dimensional hyperchaotic systems would ensure that signals are transmitted in multi-directions, thereby eliminating predetermination and predictability. It is expected that this would enhance the security of transmitted information. Here, we report some new features that makes high-dimensional systems better models for MSCS. Unlike the 3D and 4D hyperchaotic systems which can generate maximum of 27 and 256 switches respectively, we obtained all the 3125 switches (grouped into 52 parts using their dimension) available for the 5D system, and this shows the high variabilities of switches available in higher dimension.

The rest of the paper is organised as follows: Section 2; System descriptions, section 3, definition and formulation of 5D multis-witching combination synchronization 5DMSCS, in section 4, we designed the controllers for the 5D hyperchaotic systems, the numerical simulations were presented in section 5 while concluding remarks were made in section 6.

2. System description

For the purpose of this study, we consider the following prototype 5D hyperchaotic system proposed by Yang andChen [40].

$$\begin{aligned}
\dot{x} &= a\,(y - x) + p,\\
\dot{y} &= cx - xz + w,\\
\dot{z} &= -bz + xy, \qquad (1)\\
\dot{p} &= -hp - xz,\\
\dot{w} &= -k_1 x - k_2 y,
\end{aligned}$$

where $a, b, h \neq 0$ and c are the system parameters and h, k_1, k_2 are three control parameters of the system. System(3) has five Lyapunov exponents, three of which are positive Lyapunov exponents for a given set of system parameters; implying the existence of hyperchaos [37]. The phaseportraits of the system in different planes $xy, xz, xp, xw, yz, yp, yw$ and zp for $a = 10, b = \frac{8}{3}, c = 28, h = 2.25, k_1 = -0.12, k_2 = 11.3$ are as shown in figure 1.

3. Definition and formulation of MSCS

Let us consider the followingmasters-slave n dimensionalchaotic systems, where the master systems are given by

$$\begin{aligned}
\dot{x}_{1d1} &= f_{1x}(x_{1d1}, x_{2d1}, x_{3d1}, \ldots, x_{nd1}),\\
\dot{x}_{2d1} &= f_{2x}(x_{1d1}, x_{2d1}, x_{3d1}, \ldots, x_{nd1}), \qquad (2)\\
\vdots \quad & \quad \vdots\\
\dot{x}_{nd1} &= f_{n}(x_{1d1}, x_{2d1}, x_{3d1}, \ldots, x_{nd1}).
\end{aligned}$$

and

$$\begin{aligned}
\dot{y}_{1d2} &= g_{1y}(y_{1d2}, y_{2d2}, y_{3d2}, \ldots, y_{nd2}),\\
\dot{y}_{2d2} &= g_{2y}(y_{1d2}, y_{2d2}, y_{3d2}, \ldots, y_{nd2}), \qquad (3)\\
\vdots \quad & \quad \vdots\\
\dot{y}_{nd2} &= g_{ny}(y_{1d2}, y_{2d2}, y_{3d2}, \ldots, y_{nd2}).
\end{aligned}$$

and the controlled slave system is given by

$$\begin{aligned}
\dot{z}_{1r} &= h_{1z}(z_{1r}, z_{2r}, z_{3r}, \ldots, z_{nr}) + u_1,\\
\dot{z}_{2r} &= h_{2z}(z_{1r}, z_{2r}, z_{3r}, \ldots, z_{nr}) + u_2, \qquad (4)\\
\vdots \quad & \quad \vdots\\
\dot{z}_{nr} &= h_{nz}(z_{1r}, z_{2r}, z_{3r}, \ldots, z_{nr}) + u_n.
\end{aligned}$$

where $\dot{x}_{1d1}, \ldots \dot{x}_{ndn}, \dot{y}_{1d2}, \ldots \dot{y}_{ndn}$, and $\dot{z}_{1r}, \ldots \dot{z}_{nr}$ are the driver systems and response system respectively, $x_{1d1}, \ldots x_{nd1}, y_{1d2}, \ldots y_{nd2}$ and $z_{1r}, \ldots z_{nr}\ \varepsilon R^n$ are the state space vectors of the driver systems and the response system, respectively, f_{nx}, g_{ny} and $h_{nz} : R^n \to R^n$are continuous vector functions composed of linear and nonlinear components; and $u_i\ (i * = 1, 2, \ldots n) : R^n \to R^n$ is a nonlinear control function.

Definition 1 [17]

If there exists three constant matrices A, B and $C\ \varepsilon\ R^n$ and $C \neq 0$, such that $\lim_{t\to\infty} \|Cz_{nr} - Ax_{nd1} - By_{nd2}\| = 0$

where $\|\cdot\|$is the matrix norm, A, B, C are scaling matricesand $Cz_{nr} - Ax_{nd1} - By_{nd}$ is the error state with respectto n then systems (2), (3) and (4) are said to be in combination synchronization.

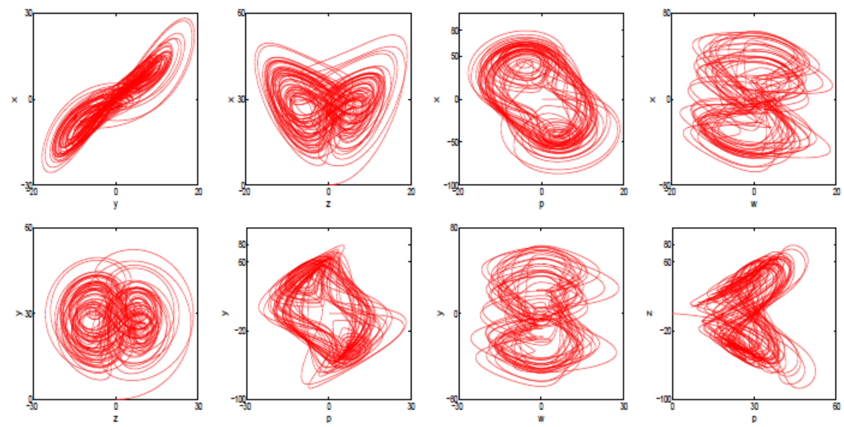

Fig. 1. The phase portraits of the 5D system in different planes x $-y, x-z, x-p, x-w, y-z, y-p, y-w$ and $z-p$ showing possibleplanes in which synchronization could take place for $a=10, b=8/3, c=28, h=2.25, k_1=-0.12, k_1=11.3$ are shown respectively

Comment 1

Definition 1 represents the error dynamics for three(3) indices, three (3) being the number of systems in consideration. We can write this error dynamics as $e_{\alpha\beta\gamma}=\|Cz_{\alpha r}-Ax_{\beta d1}-By_{\gamma d2}\|$ so that the indices are subsets of the n-dimensional n of the systems. For easy identification of the mathematics function, we assume that the maximum number of state space variable is five (5), each denoted by dimensions $1,2,3,4,5=i,j,k,l,m$ for the five (5) dimensional systems in consideration.

Comment 2

By definition 1 and comment 1, it follows that the indices of the error states as in definition 1 are strictly chosen to satisfy the definition $e_{\alpha\beta\gamma}(\alpha=\beta=\gamma)$,where α, β and γ are indices taken from the dimension i,j,k,l,m of the 5D system.

Definition 2 [1]

If the error states in relation to Definition 1 and the comments above are redefined such that for$e_{\alpha\beta\gamma}$any, combination of, or all of the equality signs as described in Comment 2 is changed, different from the dimension of the corresponding response sub-system, in at least one ofthe sub-systems, and$e_{\alpha\beta\gamma}=\|Cz_{\alpha r}-Ax_{\beta d1}-By_{\gamma d2}\|$then, systems (2), (3) and (4) are said to be in multi-switching combination synchronization state if$lim_{t\to\infty}e_{\alpha\beta\gamma}\to 0$.

Comment 3.

(i) The conditions in Definition 2 are generic conditionsthat must be met and these are dependent on the choice of the dimension, as the indices of the error system.

(ii) By implication, for a complete set of the 5D system,we have five 5 sets of 3-indices,α, β and γ, chosenfrom i,j,k,l,m.

(iii) This means that one determining factor for a complete set in (ii) is the arrangement of the dimensions in the three 3 indices of the 5D system.

(iv) Notably, in synchronization, the arrangement of the response systemis kept in order and that the arrangements of the driver systems can be varied for varieties, each driver to be treated on its own merit.

In line with above definitions and comments, bearing in mind that the same number and type of switches exist for the second driver system, we generated all possible arrangements, henceforth referred to as switches, for the 3D, 4D and 5D cases. In brief, for the 3D, there are 27 switches in 5 groups. For the 4D, there are 256 switches in 15 groups and for the 5D, we have 3125 switches with 52 groups. It follows that each of the master systems can be multi-switched in 3125 way coined from the 52 groups. In what follows, we present all the switches below:

3D switch groups:

$1. i=j=k, 2. I=j\neq k, \quad 3. i=k\neq j, \quad 4. i\neq j=k$ and $5. i\neq j\neq k.$

4D switch groups:

$1. i=j=k=l, 2. I=j=k\neq l, \quad 3. i=j=l\neq k, \quad 4. i=j\neq k=l,$
$5. i=j\neq k\neq l, 6. i=l=k\neq j, \quad 7. i=k\neq j=l, \quad 8. i=k\neq j\neq l,$
$9. i=l\neq j=k, 10. i\neq j=k=l, \quad 11. i\neq j=k\neq l, \quad 12. i=l\neq j\neq k,$
$13. i\neq j\neq k=l, 14. i\neq j\neq k\neq l$ and $15. 1\neq j=l\neq k.$

5D switch groups:

$1. i=j=k=l=m, \quad 2. i=j=k=l\neq m, \quad 3. i=j=k=m\neq l,$
$4. i=j=k\neq l=m, 5. i=j=k\neq l\neq m, \quad 6. i=j=l=m\neq k,$
$7. i=j=l\neq k=m, 8. i=j=m\neq k=l, \ 9. i=j\neq k=l=m,$
$10. i=j\neq k=l\neq m, 11. i=j=m\neq k\neq l, \ 12. i=j\neq k\neq l=m,$
$13. i=j\neq k\neq l\neq m, 14. i=j=l\neq k\neq m, \ 15. i=k=l=m\neq j,$
$16. i=k=l\neq j=m, 17. i=k=l\neq j\neq m, \ 18. i=k\neq j=l=m,$

19. $i = k \neq j = l \neq m$, 20. $i = k = m \neq j \neq l$, 21. $i = k \neq j = m \neq l$,
22. $i = k \neq j \neq l = m$, 23. $i = k \neq j \neq l \neq m$, 24. $i = l = m \neq j = k$,
25. $i = l \neq j = k = m$, 26. $i = l \neq j = k \neq m$, 27. $i = m \neq j = k = l$,
28. $i \neq j = k = l = m$, 29. $i \neq j = k = l \neq m$, 30. $i = m \neq j = k \neq l$,
31. $i \neq j = k = m \neq l$, 32. $i \neq j = k \neq l = m$, 33. $i \neq j = k \neq l \neq m$,
34. $i = l = m \neq j \neq k$, 35. $i = l \neq j = m \neq k$, 36. $i = l \neq j \neq k = m$,
37. $i = l \neq j \neq k \neq m$, 38. $i = m \neq j = l \neq k$, 39. $i = m \neq j \neq k = l$,
40. $i \neq j = l = m \neq k$, 41. $i \neq j = l \neq k \neq m$, 42. $i \neq j = m \neq k = l$,
43. $i \neq j \neq k = l = m$, 44. $i \neq j \neq k = l \neq m$, 45. $i \neq j = m \neq k \neq l$,
46. $i \neq j \neq k = m \neq l$, 47. $i = m \neq j \neq k \neq l$, 48. $i \neq j \neq k \neq l = m$,
49. $i \neq j \neq k \neq l \neq m$, 50. $i = k = m \neq j = l$, 51. $i = j \neq k = m \neq l$ *and*
52. $i \neq j = l \neq k = m$.

In this paper, five (5) cases were considered, each having two (2) sets of switches drawn from a particular group. For the first case, we chose groups 1 and 6 (representing groups in first quarter). For case 2, groups 10 and 38 (middle groups); case 3, groups 5 and 36 (about quarter to the ends); case 4, group 5 for both systems and case 5, group 49 for both drivers(making use of all the variables.)

4. Design of controllers for 5-D hyperchaotic systems

Let us redefine the variables of system (1) as follows, $x = x_1, y = x_2, z = x_3, p = x_4, w = x_5$ *and* $x = y_1, y = y_2, z = y_3, p = y_4$ *and* $w = y_5$ for the mastersystems and $x = z_1, y = z_2, z = z_3, p = z_4$ *and* $w = z_5$ for the slave system.Thus, for the five dimensional system defined in (1), the master systems are given by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4, \\ \dot{x}_2 &= cx_1 - x_1x_3 + x_5, \\ \dot{x}_3 &= -bx_3 + x_1x_2, \\ \dot{x}_4 &= -hx_4 - x_1x_3, \\ \dot{x}_5 &= -k_1x_1 - k_2x_2. \end{aligned} \tag{5}$$

and

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_4, \\ \dot{y}_2 &= cy_1 - y_1y_3 + y_5, \\ \dot{y}_3 &= -by_3 + y_1y_2, \\ \dot{y}_4 &= -hy_4 - y_1y_3, \\ \dot{y}_5 &= -k_1y_1 - k_2y_2. \end{aligned} \tag{6}$$

while

$$\begin{aligned} \dot{z}_1 &= a(z_2 - z_1) + z_4 + u_1, \\ \dot{z}_2 &= cz_1 - z_1z_3 + z_5 + u_2, \\ \dot{z}_3 &= -bz_3 + z_1z_2 + u_3, \\ \dot{z}_4 &= -hz_4 - z_1z_3 + u_4, \\ \dot{z}_5 &= -k_1z_1 - k_2z_2 + u_5. \end{aligned} \tag{7}$$

is the slave system, where u_1, u_2, u_3, u_4 *and* u_5 are the set of nonlinear controllers.Based on previous definitions, the switching combinations are chosen as follows:

Case 1:
Group 1: $i = j = k = l = m$, switch (1,1,1,1,1)
Group 6: $i = j = l = m \neq k$, *switch* (2,2,1,2,2)

$$\begin{aligned} &e_{112} = z_1 - x_1 - y_2; e_{212} = z_2 - x_1 - y_2; \\ &e_{311} = z_3 - x_1 - y_1; e_{412} = z_4 - x_1 - y_2; \\ &e_{512} = z_5 - x_1 - y_2. \end{aligned} \tag{8}$$

Case 2:
Group 10: $i = j \neq k = l \neq m$, *switch* (3,3,1,1,5)
group 38: $i = m \neq j = l \neq k$, *switch* (1,3,2,3,1)

$$\begin{aligned} &e_{131} = z_1 - x_3 - y_1; e_{233} = z_2 - x_3 - y_3; \\ &e_{312} = z_3 - x_1 - y_2; e_{413} = z_4 - x_1 - y_3; \\ &e_{551} = z_5 - x_5 - y_1. \end{aligned} \tag{9}$$

Case 3:
Group 5: $i = l \neq k = m \neq j$, *switch* (4,1,2,4,2)
Group 36: $i = j = k \neq l \neq m$, *switch* (4,4,4,1,5)

$$\begin{aligned} &e_{144} = z_1 - x_4 - y_4; e_{214} = z_2 - x_1 - y_4; \\ &e_{324} = z_3 - x_2 - y_4; e_{441} = z_4 - x_4 - y_1; \\ &e_{525} = z_5 - x_2 - y_5. \end{aligned} \tag{10}$$

Case 4:
$Group\ 5: i = j = k = l = m, switch\ (5,5,5,5,5)$
$Group\ 5: i = j = k = l = m, switch\ (5,5,5,5,5)$
$e_{155} = z_1 - x_5 - y_5;\ e_{255} = z_2 - x_5 - y_5;$
$e_{355} = z_3 - x_5 - y_5; e_{455} = z_4 - x_5 - y_5;$
$e_{555} = z_5 - x_5 - y_5.$ (11)

Case 5:
$Group\ 49: i \neq j \neq k \neq l \neq m., switch\ (1,2,3,4,5)$
$Group\ 49: i \neq j \neq k \neq l \neq m. switch\ (5,4,3,2,1)$
$e_{115} = z_1 - x_1 - y_5;\ e_{224} = z_2 - x_2 - y_4;$
$e_{333} = z_3 - x_3 - y_3; e_{442} = z_4 - x_4 - y_2;$
$e_{551} = z_5 - x_5 - y_1.$ (12)
Using the backstepping method of synchronization as presented in [1], we consider case 1 with the appropriate notations. Differentiating the error variables of (8), we have
$\dot{e}_{112} = a(e_{212} - e_{112}) + e_{412} + x_1 + y_2 - a(x_2 - x_1) - x_4 - y_1(c - y_3) - y_5 + u_1\dot{e}_{212}$
$= ae_{212} + ce_{311} - y_3 e_{311} + (c - z_3)(e_{112} + x_1 + y_2) - a(x_2 - z_2 + y_2) +$
$(z_3 - x_1)(y_3 - c) + z_5 - x_4 - y_5 + u_2,$
$\dot{e}_{311} = -ae_{311} - bz_3 + 2e_{112}e_{212} - z_1(e_{112} + e_{212} + z_1) - a(x_2 + y_2 - z_3) -$
$x_4 - y_4 + u_3,$
$\dot{e}_{412} = -e_{412}(h + 1) - (x_1 + y_2)(h + e_{311} + x_1 + y_1) - e_{112}(e_{311} + x_1 + y_1) -$
$a(x_2 - y_2 - z_4) - (z_3 - x_1 - e_{311})(c - y_3) - x_4 - y_5 + u_4,$
$\dot{e}_{512} = -ae_{512} - k_1(e_{112} + x_1 + y_2) - k_2(e_{212} + x_1 + y_2) - a(x_2 - z_5 + y_2) -$
$(z_3 - x_1 - e_{311})(c - y_3)x_4 - y_5 + u_5.$ (13)
With error dynamics (3.110), if appropriate $u_1, u_2, u_3, u_4\ and\ u_5$ are chosen suchthat equilibrium $(0, 0, 0, 0, 0)$ of the error system is stable and unchanged thenstabilization would be realized leading to stable synchronisation of the system.
If $\eta_1 = e_{112}$, its time derivative is $\dot{e}_{112}$and write the first part of (13)as$\dot{\eta}_1 = a(e_{212} - \eta_1) + e_{412} + x_1 + y_2 - a(x_2 - x_1) - x_4 - y_1(c - y_3) -$
$y_5 + u_1,$ (14)
We can stabilize (14) using the Lyapunov function
$v_1 = \frac{1}{2}\eta_1^2$ (15)
By substituting for $\dot{\eta}_1$ in the derivative of (3.112), choosing $e_{212} = \alpha_1(\eta_1) = 0$ as avirtual controller and $u_1 = -e_{412} - x_1(1 + a) - y_2 + ax_2 + x_4 + y_1(c - y_3) + y_5 + k\eta_1,$
$\dot{v}_1 = -(a - k)\eta_1^2 \leq 0.$ (16)
Thus, $\dot{v}_1$ is negative definite if $k \leq 0$ and a takes on positive value showing that thesubsystem $(\dot{\eta}_1)$ is asymptotically stable. Since the error between e_{212} and $\alpha_1(\eta_1)$is estimative as $\eta_2 = e_{212}$ and its derivative is written as $\dot{\eta}_2 = \dot{e}_{212}$, the $(\dot{\eta}_1, \dot{\eta}_2)$ subsystems as
$\dot{\eta}_1 = -\eta_1(a - k) + a\eta_2,$
$\dot{\eta}_2 = a\eta_2 + ce_{311} - y_3 e_{311} + (c - z_3)(\eta_1 + x_1 + y_2) - a(x_2 - z_2 + y_2)$
$+ (z_3 - x_1)(y_3 - c) + z_5 - x_4 - y_5 + u_2$ (17)
We stabilize (17) by choosing the second Lyapunov function given as
$v_2 = v_1 + \frac{1}{2}\eta_2^2$ (18)
By substituting for $\dot{\eta}_2$ in the derivative of (18) choosing$e_{311} = \alpha_2(\eta_2) = 0$ asa virtual controller and choosing$u_2 = -2a\eta_2 + y_3 e_{311} - (c - z_3)(\eta_1 + x_1 + y_2) + a(x_2 - z_2 + y_2) - (z_3 - x_1)(y_3 - c) - z_5 + x_4 + y_5 + k\eta_2$
We have
$\dot{v}_2 = -(a - k)(\eta_1^2 21 + \eta_2^2) \leq 0.$ (19)
Thus, $\dot{v}_2$ is negative definite $if\ k \leq 0$ and a takes on positive value showing thatthe subsystem $(\dot{\eta}_1, \eta_2)$ is asymptotically stable. Since the error between e_{311} and $\alpha_2\ (\eta_2)$ is estimative as
$\eta_3 = e_{311}$and its derivative is written as $\dot{\eta}_3 = \dot{e}_{311}$, the (η_1, η_2, η_3) subsystem is
$\dot{\eta}_1 = -\eta_1(a - k) + a\eta_2,$
$\dot{\eta}_2 = -\eta_2(a - k) - c\eta_3,$
$\dot{\eta}_3 = -ae_{311} - b\, z_3 + 2e_{112}e_{212} - z_1\,(e_{112} + e_{212} + z_1) -$
$a(x_2 + y_2 - z_3) - x_4 - y_4 + u_3.$ (20)
We can stabilize (20) by choosing the third Lyapunov function given as
$v_3 = v_2 + \frac{1}{2}\eta_3^2$ (21)
By substituting for $\dot{\eta}_3$ in the derivative of (21) choosing $\eta_1 = \alpha_3(\eta_3) = 0$ as avirtualcontroller and $u_3 = b\, z_3 - z_1(\eta_1 - \eta_2 + z_1) + a(x_2 - y_2 - z_3) + x_4 + y_4 + k\eta_3$,to have

$\dot{v}_3 = -(a - k)(\eta_1^2 + \eta_2^2 + \eta_3^2) \leq 0,$ (22)
Thus, $\dot{v}_3$ is negative definite if $k \leq 0$ and a takes on positive value showing thatthe subsystem ($\dot{\eta}_1$, η_2, η_3) is asymptotically stable. Let $\eta_4 = e_{412}$ and its derivative$\dot{e}_{412}$. and the $\eta_1, \eta_2, \eta_3, \eta_4$) subsystem is
$\dot{\eta}_1 = -\eta_1(a - k) + a\eta_2,$
$\dot{\eta}_2 = -\eta_2(a - k) - c\eta_3,$
$\dot{\eta}_3 = -\eta_3(a - k) + 2\eta_1\eta_2,$
$\dot{\eta}_4 = -\eta_4(h + 1) - (x_1 + y_2)(h + \eta_3 + x_1 + y_1) - \eta_1(\eta_3 + x_1 + y_1) -$
$a(x_2 - y_2 - z_4) - (z_3 - x_1 - \eta_3)(c - y_3) - x_4 - y_5 + u_4.$ (23)
We can stabilize (23) by defining the fourth Lyapunov function given as
$v_4 = v_3 + \frac{1}{2}\eta_4^2$ (24)
By substituting for $\dot{\eta}_4$ in the derivative of (24) and choosing$u_4 = (x_1 + y_2)(h + \eta_3 + x_1 + y_1) - \eta_1(\eta_3 + x_1 + y_1) - a(x_2 - y_2 - z_4) - y_3(z_3 - x_1 - \eta_3) + x_4 + y_4,$
Write have
$\dot{v}_4 = -(a - k)(\eta_1^2 + \eta_2^2 + \eta_3^2) - (h + 1 - k)\eta_4^2 \leq 0,$ (25)
Thus, $\dot{v}_4$ is negative definite if $k \leq 0, a$ and h take on positive values showingthat the subsystem ($\dot{\eta}_1$,η_2, η_3, η_4) is asymptotically stable. Let $\eta_5 = e_{512}$ and itsderivative be$\dot{e}_{512}$, the whole system is
$\dot{\eta}_1 = \eta_1(a - k) + a\eta_2,$
$\dot{\eta}_2 = -\eta_2(a - k) - c\eta_3,$
$\dot{\eta}_3 = -\eta_3(a - k) + 2\eta_1\eta_2,$
$\dot{\eta}_4 = -\eta_4(h + 1 - k),$ (26)
$\dot{\eta}_5 = -a\eta_5 - k_1(\eta_1 - x_1 + y_2) - k_2(\eta_2 + x_1 + y_2) - a(x_2 - z_5 + y_2)$
$- (z_3 - x_1 - \eta_3)(c - y_3) - x_4 - y_5 + u_5.$ (27)
We can stabilize (26) by defining the fifth Lyapunov function given as
$v_5 = v_4 + \frac{1}{2}\eta_5^2$ (28)
By substituting for $\dot{\eta}_5$ in the derivative of (3.125) and choosing$u_5 = k_1(\eta_1 + x_1 + y_2) + k_2(\eta_2 + x_1 + y_2) + (x_2 - z_5 + y_2) + z_3 - x_1 - \eta_3)(c - y_3) + x_4 + y_5 + k\eta_5,$
$\dot{v}_5 = -(a - k)(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_5^2) - (h + 1 - k)\eta_4^2 \leq 0,$ (29)
Thus, $\dot{v}_5$ is negative definite if $k \leq 0, a$ and h take on positive values. The wholesystem is expressed as
$\dot{\eta}_1 = \eta_1(a - k) + a\eta_2,$
$\dot{\eta}_2 = -\eta_2(a - k) - c\eta_3,$
$\dot{\eta}_3 = -\eta_3(a - k) + 2\eta_1\eta_2,$
$\dot{\eta}_4 = -\eta_4(h + 1 - k),$
$\dot{\eta}_5 = -\eta_5(a - k).$ (30)
For other switches 2, 3, 4 and 5, the controllers were also obtained following the above procedures, and are presented in equations (31), (32), (33) and (34) respectively.
$u_1 = -a(y_3 - y_2) - e_{413} - x_1(1 - x_2) - y_3 - bx_1 + y_3 + k\eta_1,$
$u_2 = -(x_3 + y_1)(c - e_{312} - x_1 - y_2) - e_{413} - x_1(1 - x_2) + y_1y_2 - y_3 - bz_2 + k\eta_2 + \eta_1(x_1 + y_2),$
$u_3 = b(x_1 + y_2) - (e_{131} + x_3 + y_1)(e_{233} + x_3 + y_3) - a(z_4 - y_3 - x_2) + x_4 + y_5 + (z_1 - x_3 - e_{131})(c - z_4 + x_1) + \eta_4(z_1 - x_3) + k\eta_3,$
$u_4 = h(x_1 + y_3) + (x_1 + y_2)(e_{131} + x_3 + y_1) + a(x_2 - z_3 + y_2) + x_4 - b(z_4 - x_1) + y_1(z_3 - x_1) + x_3e_{312} + k\eta_4,$
$u_5 = k_1(e_{131} + x_3 + y_1 - x_1) + k_2(e_{233} + x_3$
$+y_3 - x_2) + a(y_2 - z_5 + x_5) + y_4 + k\eta_5.$ (31)
$u_1 = -e_{441}(1 - y_3) - a(x_1 - x_4) - hz_1 - x_4 - y_1 + x_3e_{214} - x_3(z_2 - y_4) - y_3(z_4 - x_4) + k\eta_1,$
$u_2 = -(x_4 + y_4)(c - e_{324} - x_2 - y_4) - e_{525} - x_2 - y_5 - a(z_2 - y_4 - x_2) - x_4 - h(z_2 - x_1) - y_1y_3 + \eta_1(x_2 + y_4) + k\eta_2,$
$u_3 = b(x_2 + y_4) - (e_{144} + x_4 + y_4)(e_{214} + x_1 + y_4) + x_1(c - x_3) + x_5 - hy_4 - y_3(z_4 - x_4) + y_3e_{441} + k\eta_3,$
$u_4 = h(x_4 + y_1) + (e_{144} + x_4 + y_4)(e_{324} + x_2 + y_4) - hx_1 - x_1x_3 + a(y_2 - y_1) + y_4 + k\eta_4,$
$u_5 = -\eta_5(1 - k) + k_1(z_1 - y_1) + k_2(z_2 - y_2) + cx_1 - x_1x_3 + x_5 + k\eta_5.$ (32)
$u_1 = -e_{455} - x_5 - y_5 - k_1(x_1 + y_1) - k_2(x_2 + y_2) + k\eta_1, u_2 = -(x_5 + y_5)(c - e_{355} - x_5 - y_5) - e_{555} - z_2 - k_1(x_1 + y_1) - k_2(x_2 + y_2) + \eta_1(x_5 + y_5) + k\eta_2,$
$u_3 = -(x_5 + y_5)(e_{255} + x_5 + y_5 - b) - k_1(x_1 + y_1) - k_2(x_2 + y_2) - \eta_1(x_5 + y_5) + k\eta_3,$
$u_4 = hz_5 + (e_{155} + x_5 + y_5)(e_{355} + x_5 + y_5) - k_1(x_1 + y_1) - k_2(x_2 + y_2) + k\eta_4,$
$u_5 = -\eta_5(1 - k) + k_1(z_1 - x_1 - y_1) + k_2(z_2 - x_2 - y_2) + k\eta_5.$ (33)
$u_1 = -a(y_4 - y_5) - e_{442}(1 - k_2) - y_2 - k_1y_1 - k_2(z_4 - x_4) + k\eta_1,$
$u_2 = -cy_1 + z_3(x_1 + y_5) - x_3(z_1 - y_5) - e_{551} - y_1 - h(z_2 - x_2) - y_1y_3 + \eta_1(z_3 + x_3) + k\eta_2,$
$u_3 = -z_1z_2 + x_1x_2 + y_2(z_5 - x_5) - y_2e_{551} + k\eta_3,$
$u_4 = hy_2 + z_1z_3 - x_1x_3 + y_1(c - z_3 + x_3) + y_5 + y_1e_{333} + k\eta_4,$
$u_5 = -k_1(e_{115} + y_5) + k_2(e_{224} + y_4) + a(y_2 - z_5 + x_5) + y_4 + k\eta_5.$ (34)

5. Numerical Simulations

Here we present our numerical simulation in order to verify the effectiveness of the controllers u1, u2, u3, u4 and u5 for case 1 above as well as the controllers for other cases presented in (31) - (34). We used the fourth-order Runge - Kutta algorithm. We maintained that our interest is to achievemulti switching combination synchronization of the 5D hyperchaotic system. The system parameters are chosen as $a = 10, b = 8/3, c = 28, h = 2.25, k_1 = -0.12, k_2 = 11.3$when the initial conditions were $x_1 = 0.1, y_1 = 0.1, z_1 = 0.1, p_1 = 0.1, w_1 = 0.1, x_2 = 0.5, y_2 = 0.01, z_2 = 0.8, p_2 = 0.7$ *and* $w_2 = 0.5$. The step size was maintained at H = 0.005. For all the cases, the controllers u_i $(i = 1,2, \ldots 5)$ were activated at $t \geq 10$. The result for multi-switchingcombination synchronized states $e_{112}, e_{212}, e_{311}, e_{412}$ *and* e_{512} for case 1 is shown in Figure (2). For other cases 2,3, 4 and 5, the results are as shown in Figures (3), (4), (5) and (6), respectively. In all cases of the multi-switching

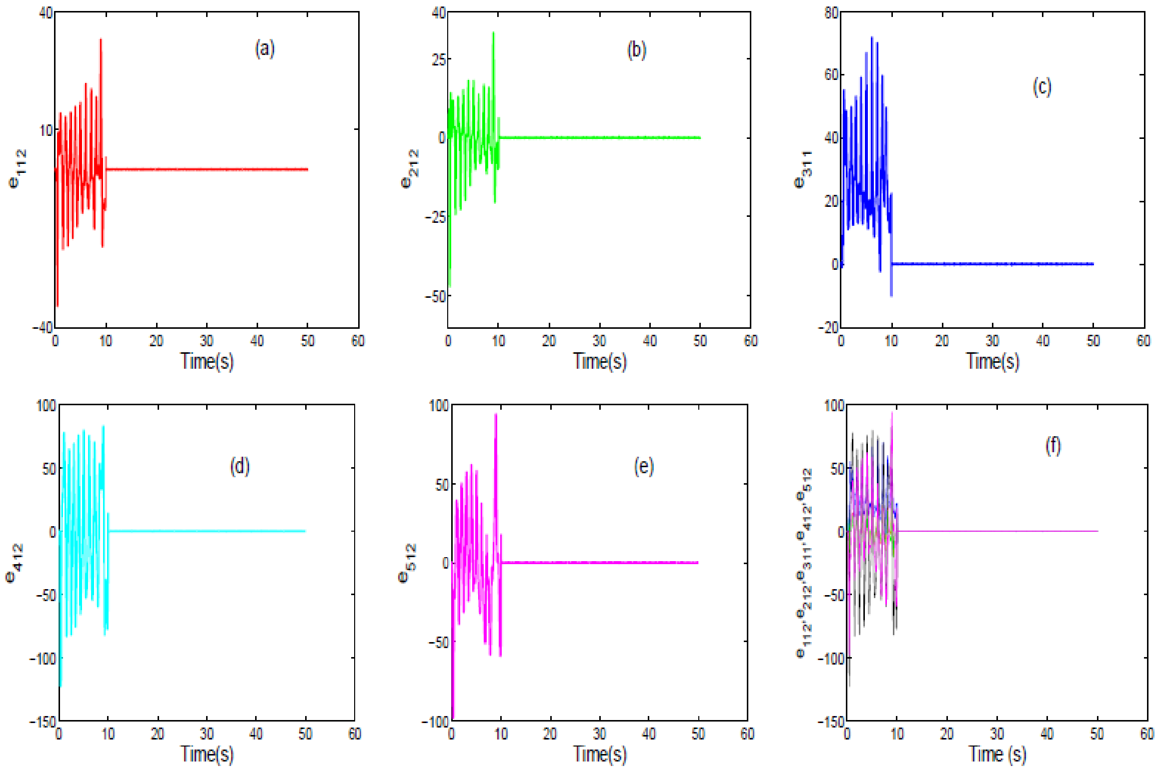

Fig. 2. Multi switched combination synchronization case 1 for states $e_{112}, e_{212}, e_{311}, e_{412}$ *and* e_{512}, when t was activated at $t \geq 10$. Thesub-plot (f) is the combined state $e_{112}, = red, e_{212}, = green, e_{311}, = blue, e_{412}, = cyan$ and e_{512}, = magenta when $a = 10, b = 8/3, c = 28, d = 2.25, p = -0.12, k = 11.3$ and the initial conditions were $x_1 = 0.1, y_1 = 0.1, z_1 = 0.1, p_1 = 0.1, w_1 = 0.1, x_2 = 0.5, y_2 = 0.01, z_2 = 0.8, p_2 = 0.7$ *and* $w2 = 0.5, H = 0.005$ at $t \geq 10$

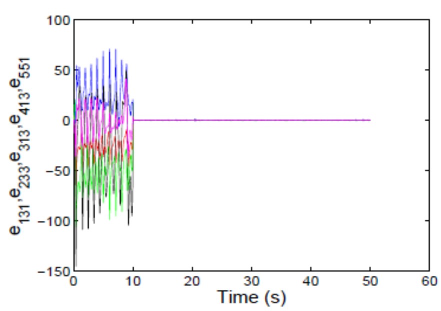

Fig. 3. Multi switched combination synchronization case 2 for states $e_{131}, = red, e_{233}, = blue, e_{312}, = green, e_{413}, = cyan$ and e_{551}, = magenta when $a = 10, b = 8/3, c = 28, d = 2.25, p = -0.12, k = 11.3$ and the initial conditions were $x_1 = 0.1, y_1 = 0.1, z_1 = 0.1, p_1 = 0.1, w_1 = 0.1, x_2 = 0.5, y_2 = 0.01, z_2 = 0.8, p_2 = 0.7$ *and* $w2 = 0.5, H = 0.005$ at $t \geq 10$

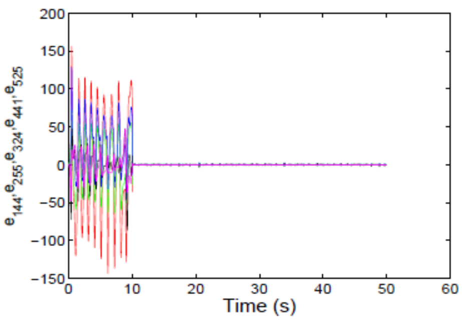

Fig. 4. Multi switched combination synchronization case 3 for states ($e_{144} = red, e_{214} = blue, e_{324} = green, e_{441} = cyan\ and\ e_{525} = magenta\ when\ a = 10, b = 8/3, c = 28, d = 2.25, p = -0.12, k = 11.3$ and the initial conditions were $x_1 = 0.1, y_1 = 0.1, z_1 = 0.1, p_1 = 0.1, w_1 = 0.1, x_2 = 0.5, y_2 = 0.01, z_2 = 0.8, p_2 = 0.7\ and\ w2 = 0.5, H = 0.005$ at $t \geq 10$

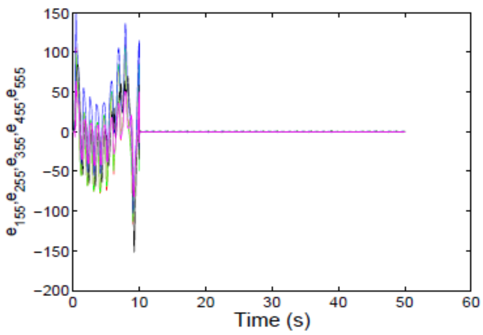

Fig. 5. Multi switched combination synchronization case 4 for states ($e_{155} = red, e_{255} = blue, e_{355} = green, e_{455} = cyan\ and\ e_{555} = magentawhen\ a = 10, b = 8/3, c = 28, d = 2.25, p = -0.12, k = 11.3$ and the initial conditions were $x_1 = 0.1, y_1 = 0.1, z_1 = 0.1, p_1 = 0.1, w_1 = 0.1, x_2 = 0.5, y_2 = 0.01, z_2 = 0.8, p_2 = 0.7\ and\ w2 = 0.5, H = 0.005$ at $t \geq 10$

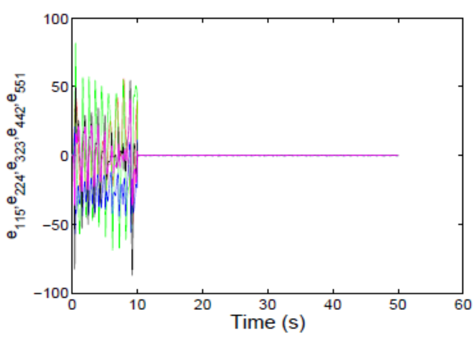

Fig. 6. Multi switched combination synchronization case 5 for states $e_{115} = red, e_{224} = blue, e_{333} = green, e_{442} = cyan\ and\ e_{551} = magentawhen\ a = 10, b = 8/3, c = 28, d = 2.25, p = -0.12, k = 11.3$ and the initial conditions were $x_1 = 0.1, y_1 = 0.1, z_1 = 0.1, p_1 = 0.1, w_1 = 0.1, x_2 = 0.5, y_2 = 0.01, z_2 = 0.8, p_2 = 0.7\ and\ w2 = 0.5, H = 0.005$ at $t \geq 10$

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6. Concluding Remarks

Conclusively, in this paper, we have examined and analysed multi-switching combination synchronization of a 5D hyperchaotic system. We extended the usual master-slave synchronization scheme for low dimensional chaotic systems to study the synchronization of this higher order systems, we provided various multi switches for the design of the controllers and performed the synchronization for combined drivers of the system. We identified 3125 possibleswitches belonging to 52 groupings out of which we used 7 groups including 2 special groups for the purpose of illustration. The synchronization in all the cases were examined and successfully confirmed by numerical simulations.By implication, signal information can be hidden, stored transmitted via any or both of the drivers simultaneously, in split or at different time interval information can be locked up in any of the states in each of the cases, with at least five different switch codes. Such information can be transferred, communicated and retrieved by applying the control inputs for each or all the dynamical states and respective switches. This would further enhance the security of information considering not only the hyperchaotic status of the system in consideration, but also the multiple switches that must be unlocked to retrieve the information and the unpredictable nature of the combined drivers in which the information are stored.

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