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Single Compound Synchronization Between Three Identical or Different Chaotic Systems

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Abstract

In this paper, we investigate the single compound synchronization of three identical or three non-identical chaotic systems. The proposed scheme employed in this paper is among two drive systems and a single response system. For most synchronization schemes involving two drive systems and a single response system, the combination technique has been given a lot of attention in literature. This work aims at using the compound synchronization technique, which is different from the combination technique, to achieve the coupling of two drive systems. The controllers chosen have been found to be effective from the simulated results.

Keyword: Chaos; compound synchronization ; memristor ; controllers ; chaotic oscillator , nonlinear circuit

1. INTRODUTION

The idea of chaos synchronization between coupled or forced chaotic systems was introduced by Pecora and Carroll [1] and ever since then the subject has attracted the interest of many researchers. The subject of chaos synchronization has various intriguing features [2] and has been applied to secure communication system, neural network, modeling brain activity just to mention a few. Because of the complex nature of dynamical systems, several kinds of chaos synchronization have been investigated, such as anti- synchronization [3], partial synchronization [4], phase synchronization[5], generalized synchronization [6,7], projective synchronization [8,9], multi-switching synchronization[10], combination-combination synchronization[11] compound synchronization[12] etc. Although, many researchers focused on the synchronization between a single master system and a single slave system, recently some researchers have extended the idea of chaos synchronization to more than two systems [10-12]. Researchers have realized that the synchronization of more than two chaotic systems can improve the anti-attack and anti-translated ability in communication system [13]. Despite the fact that researchers have begun synchronizing more than two chaotic systems, there are still more possibilities to be explored. Researchers who have worked on the synchronization of three chaotic systems studied a two drive system and a single response system, and in their work they combined the two drive system by adding, but the synchronization of two drive systems using the compound technique (that is coupling of two chaotic systems by multiplication) is very in literature. Recently, several nonlinear oscillators based Chua's circuit, have been proposed [14,15]. These type of nonlinear chaotic oscillators have been applied in the design of more complicated time series which finds application in secure communication. Motivated by the above discussions, we studied the synchronization of three identical chaotic system (modified Chua's circuit system [16]) and three non-identical chaotic systems. In this case the two drive systems are combined by multiplication and the final result is used to drive the response system in order to achieve synchronization.

2. Single compound synchronization scheme.

In this section, the single compound synchronization scheme is designed for three chaotic system with two drive systems and one response system which can be represented schematically in Fig.1. The two drive systems are given by;

$$\dot{x}_{1d} = f_{1x}(x_{1d}, \cdots), \dot{x}_{2d} = f_{2x}(x_{1d}, \cdots), \dots\ \dot{x}_{nd} = f_{nx}(x_{1d}, \cdots) \quad (1)$$

and

$$\dot{y}_{1d} = f_{1y}(y_{1d}, \cdots), y_{2d} = f_{2y}(y_{1d}, \cdots), \dots\ y_{nd} = f_{ny}(y_{1d}, \cdots) \quad (2)$$

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and the controlled response system is given by

$$\dot{z}_{1r} = g_{1z}(z_{1r}, \cdots) + U_1, \dot{z}_{2r} = g_{2z}(z_{1r}, \cdots) + U_2, \ldots\ \dot{z}_{nr} = g_{nz}(z_{1r}, \cdots) + U_3 \quad (3)$$

where $x_{id}, y_{jd}, z_{kr}(i, j, k = 1,2,3, \ldots n) \in R^n$ are state space vectors of the systems,
f_{id}, f_{jd}, g_{kr}: $R^n \rightarrow R^n$
are three continuous vector functions composed of linear and nonlinear components, and $U_i(i = 1,2,3, \ldots n)$: $R^n \rightarrow R^n$ is a nonlinear control function. We have used the indices d and r to represent drive and response system respectively.

Definition 1
If there exist three constant matrices A, B, C, ϵ R^n and C $\neq$0, such that

$$\lim_{t\rightarrow\infty}\|e\| = \lim_{t\rightarrow\infty}\|Ax_{id}By_{jd} - Cz_{kr}\| = 0 \quad (4)$$

Where $\|\cdot\|$ is the matrix norm and A, B, C are scaling matrices, then systems (1), (2) and (3) are said to achieve single compound synchronization.

Remark 1
The constant matrices A,B and C are called the scaling matrices, which are capable of extending the functional diagonal matrices of state variables x,y and z respectively.

3. Single compound synchronization of three identical chaotic systems.

In this section, we can realize single compound synchronization among three identical modified Chua's circuit systems. The two drive modified Chua's systems are given as;

$$\begin{cases} \dot{x}_1 = p_1\left(x_2 - \frac{2}{7}x_1^3 + \frac{1}{7}x_1\right) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -q_1x_2 \end{cases} \quad (5)$$

$$\begin{cases} \dot{y}_1 = p_2\left(y_2 - \frac{2}{7}y_1^3 + \frac{1}{7}y_1\right) \\ \dot{y}_2 = y_1 - y_2 + y_3 \\ \dot{y}_3 = -q_2y_2 \end{cases} \quad (6)$$

the drive systems (5) and (6) and the response system (7) exhibit a chaotic attractor at $p_1=p_2=p_3=10$ and $q_1=q_2=q_3=100/7$ as shown in Fig. 2, while the chaotic attractor of the two drive systems is shown in Fig. 3. The third modified Chua's system which serves as the response system is given as

$$\begin{cases} \dot{z}_1 = p_3\left(z_2 - \frac{2}{7}z_1^3 + \frac{1}{7}z_1\right) + U_1 \\ \dot{z}_2 = z_1 - z_2 + z_3 + U_2 \\ \dot{z}_3 = -q_3z_2 + U_3 \end{cases} \quad (7)$$

where U_1, U_2 and U_3 are the control parameters to be designed. In this work, we assume A= diag(a_1 a_2 a_3), B= diag(b_1 b_2 b_3) and C= diag(c_1 c_2 c_3) for the synchronization scheme.
The error systems is as follows:

$$\begin{cases} e_1 = a_1x_1b_1y_1 - c_1z_1 \\ e_2 = a_2x_2b_2y_2 - c_2z_2 \\ e_3 = a_3x_3b_3y_3 - c_3z_3 \end{cases} \quad (8)$$

The error dynamics of Eqn. (8) is given as

$$\begin{cases} \dot{e}_1 = \emptyset_1 - c_1p_1\left(z_2 - \frac{2}{7}z_1^3 + \frac{1}{7}z_1\right) - c_1U_1 \\ \dot{e}_2 = \emptyset_2 - c_2(z_1 - z_2 + z_3) - c_2U_2 \\ \dot{e}_3 = \emptyset_3 + c_3q_3z_2 - c_3U_3 \end{cases} \quad (9)$$

where

$$\begin{cases} \emptyset_1 = a_1p_1\left(x_2 - \frac{2}{7}x_1^3 + \frac{1}{7}x_1\right)b_1y_1 + a_1x_1b_1p_2\left(y_2 - \frac{2}{7}y_1^3 + \frac{1}{7}y_1\right) \\ \emptyset_2 = a_2(x_1 - x_2 + x_3)b_2y_2 + a_2x_2b_2(y_1 - y_2 + y_3) \\ \emptyset_3 = -a_3q_1x_2b_3y_3 - a_3x_3b_3q_2y_2 \end{cases} \quad (10)$$

Theorem 1 If the control laws are chosen as follows:

$$\begin{cases} U_1 = \frac{1}{c_1}\emptyset_1 - p_3\left(z_2 - \frac{2}{7}z_1^3 + \frac{1}{7}z_1\right) + \frac{1}{c_1}(a_1x_1b_1y_1 - c_1z_1) + \frac{1}{c_1}(a_2x_2b_2y_2 - c_2z_2) \\ U_2 = \frac{1}{c_2}\emptyset_2 - (z_1 - z_2 + z_3) + \frac{1}{c_2}(a_2x_2b_2y_2 - c_2z_2) - \frac{1}{c_2}(a_1x_1b_1y_1 - c_1z_1) - \frac{q_1}{c_2}(a_3x_3b_3y_3 - c_3z_3) \\ U_3 = \frac{1}{c_3}\emptyset_3 + q_3z_2 + \frac{1}{c_3}(a_3x_3b_3y_3 - c_3z_3) + \frac{q_1}{c_3}(a_2x_2b_2y_2 - c_2z_2) \end{cases} \quad (11)$$

then the drive systems (5) and (6) will achieve single compound synchronization with the response system (7).
Proof: If we choose a Lyapunov function as follows:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (12)$$

then

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \quad (13)$$

Putting (9) into (13)

$$\dot{V} = e_1(\emptyset_1 - c_1\dot{z}_1 - c_1U_1) + e_2(\emptyset_2 - c_2\dot{z}_2 - c_2U_2) + e_3(\emptyset_3 - c_3\dot{z}_3 - c_3U_3) \quad (14)$$

Putting (11) into (14)
$\dot{V} = e_1(-e_1 - e_2) + e_2(e_1 - e_2 + q_2 e_3) + e_3(-q_2 e_3 - e_3)$
$\dot{V} = (-e_1^2 - e_2^2 - e_3^2)$
Since $\dot{V} \leq 0$ as $t \to \infty$ then it is negative definite and according to the Lyapunov theorem, e_i tends to zero (i=1,2,3), and this means that the drive systems (5) and (6) will achieve single compound synchronization with the response system (7).
Corollary 1 Suppose $a_1 = a_2 = a_3 = 0$, $b_1 = b_2 = b_3 = 0$, and $c_1 = c_2 = c_3 = 1$ if the controllers are chosen as follow:

$$\begin{cases} U_1 = -p_3\left(z_2 - \frac{2}{7}z_1^3 + \frac{1}{7}z_1\right) - z_1 - z_2 \\ U_2 = -(z_1 - z_2 + z_3) - z_1 - z_2 + z_3 \\ U_3 = +q_3 z_2 - z_3 - z_2 \end{cases} \quad (15)$$

then the equilibrium point (0,0,0) of the response system is asymptotically stable.
Numerical simulations are presented in order to show the effectiveness of the controllers. The fourth-order Runge-Kutta method with time step size 0.001 is used. During simulation we assume $a_1 = a_2 = a_3 = 1, b_1 = b_2 = b_3 = 1, c_1 = c_2 = c_3 = 1$ and the initial states for the drive systems are $(x_1, x_2, x_3) = (0.5, 0.01, 2)$,$(y_1, y_2, y_3) = (0.5, 0.01, 2)$ and $(z_1, z_2, z_3) = (0.1, 0.03, 0.5)$. The corresponding numerical results are shown in Figs. 4-7. Fig. 4 shows the time response of the synchronization error $e_i(i = 1,2,3)$, which implies that drive systems (5) and (6) have achieved single compound synchronization with the response system (7). Figs. 5-7 are the time response of states $x_1 y_1, z_1$, $x_2 y_2, z_2$ and $x_3 y_3, z_3$.

6. Synchronization between three different chaotic system

We have also applied the single synchronization technique among three different chaotic systems namely the Rossler system, and the Chen system which are taken as the drive systems and the Tigan system which is taken as the response system. The two drive systems are as follows:

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = x_1 + a_1 x_2 \\ \dot{x}_3 = b_1 + x_3(x_1 - c_1) \end{cases} \quad (16)$$

$$\begin{cases} \dot{y}_1 = a_2(y_2 - y_1) \\ \dot{y}_2 = (c_2 - a_2)y_1 - y_1 y_3 + c_2 y_2 \\ \dot{y}_3 = y_1 y_2 - b_2 y_3 \end{cases} \quad (17)$$

while the response system is as follows:

$$\begin{cases} \dot{z}_1 = a_3(z_2 - z_1) + U_1 \\ \dot{z}_2 = (c_3 - a_3)z_1 - a_3 z_1 z_3 + U_2 \\ \dot{z}_3 = z_1 z_2 - b_3 z_3 + U_3 \end{cases} \quad (18)$$

For chaotic attractor to occur, the Rossler system must have its parameters at $a_1 = 0.2, b_1 = 0.2, c_1 = 5.7$, and for Chen system $a_2 = 35, b_2 = 3, c_2 = 28$ while for the Tigan system $a_3 = 2.1, b_3 = 0.6, c_3 = 30$. For convenience of our discussion, we take A= diag($\alpha_1\ \alpha_2\ \alpha_3$), B= diag($\beta_1\ \beta_2\ \beta_3$) and C= diag($\gamma_1\ \gamma_2\ \gamma_3$) for the synchronization scheme.
We have the error system as

$$\begin{cases} e_1 = \alpha_1 x_1 \beta_1 y_1 - \gamma_1 z_1 \\ e_2 = \alpha_2 x_2 \beta_2 y_2 - \gamma_2 z_2 \\ e_3 = \alpha_3 x_3 \beta_3 y_3 - \gamma_3 z_3 \end{cases} \quad (19)$$

The error dynamics of (19) is

$$\begin{cases} \dot{e}_1 = \emptyset_1 - \gamma_1 a_3(z_2 - z_1) - \gamma_1 U_1 \\ \dot{e}_2 = \emptyset_2 - \gamma_2\left((c_3 - a_3)z_1 - a_3 z_1 z_3\right) - \gamma_2 U_2 \\ \dot{e}_3 = \emptyset_3 + -\gamma_3(z_1 z_2 - b_3 z_3) - \gamma_3 U_3 \end{cases} \quad (20)$$

Theorem 2 If the control laws are chosen as follows:

$$\begin{cases} U_1 = \frac{1}{\gamma_1}\emptyset_1 - a_3(z_2 - z_1) + \frac{1}{\gamma_1}(a_1 x_1 b_1 y_1 - \gamma_1 z_1) + \frac{a_1}{\gamma_1}(a_2 x_2 b_2 y_2 - \gamma_2 z_2) \\ U_2 = \frac{1}{c_2}\emptyset_2 - \left((c_3 - a_3)z_1 - a_3 z_1 z_3\right) + \frac{1}{\gamma_2}(a_2 x_2 b_2 y_2 - \gamma_2 z_2) - \frac{a_1}{\gamma_2}(a_1 x_1 b_1 y_1 - \gamma_1 z_1) - \frac{b_1}{\gamma_2}(a_3 x_3 b_3 y_3 - \gamma_3 z_3) \\ U_3 = \frac{1}{c_3}\emptyset_3 - (z_1 z_2 - b_3 z_3) + \frac{1}{\gamma_3}(a_3 x_3 b_3 y_3 - \gamma_3 z_3) + \frac{b_1}{\gamma_3}(a_2 x_2 b_2 y_2 - \gamma_2 z_2) \end{cases} \quad (21)$$

then the drive systems (16) and (17) will achieve single compound synchronization with the response system (18).
Proof: If we choose a Lyapunov function as follows:

$$V(e_1, e_2, e_3) = \tfrac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (22)$$

then

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \quad (23)$$

Putting (20) into (23)

$$\dot{V} = e_1(\emptyset_1 - \gamma_1\dot{z}_1 - \gamma_1 U_1) + e_2(\emptyset_2 - \gamma_2\dot{z}_2 - \gamma_2 U_2) + e_3(\emptyset_3 - \gamma_3\dot{z}_3 - \gamma_3 U_3) \quad (24)$$

Putting (21) into (24)
$\dot{V} = e_1(-e_1 - a_1 e_2) + e_2(a_1 e_1 - e_2 + b_1 e_3) + e_3(-b_1 e_2 - e_3)$
$\dot{V} = (-e_1^2 - e_2^2 - e_3^2)$
Since $\dot{V} \leq 0$ as $t \to \infty$ then it is negative definite and according to the Lyapunov theorem, e_i tends to zero (i=1,2,3), and this means that the drive systems (16) and (17) will achieve single compound synchronization with the response system (18).

Corollary 2 Suppose $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\beta_1 = \beta_2 = \beta_3 = 0$, and $\gamma_1 = \gamma_2 = \gamma = 1$ if the controllers are chosen as follow:

$$\begin{cases} U_1 = -a_3(z_2 - z_1) - z_1 - a_1 z_2 \\ U_2 = -\left((c_3 - a_3)z_1 - a_3 z_1 z_3\right) + a_1 z_1 - z_2 + b_1 z_3 \\ U_3 = -(z_1 z_2 - b_3 z_3) - z_3 - b_1 z_2 \end{cases} \tag{25}$$

then the equilibrium point (0,0,0) of the response system (18) is asymptotically stable.

Numerical simulations are presented in order to show the effectiveness of the controllers. The fourth-order Runge-Kutta method with time step size 0.001 is used. During simulation we assume $\alpha_1 = \alpha_2 = \alpha_3 = 1, \beta_1 = \beta_2 = \beta_3 = 1, \gamma_1 = \gamma_2 = \gamma = 1$ and the initial states for the drive systems are $(x_1, x_2, x_3) = (0, 0.01, 0)$,$(y_1, y_2, y_3) = (0, 0.01, 0)$ and $(z_1, z_2, z_3) = (1, -0.4, 3)$. The corresponding numerical results are shown in Figs. 8-11. Fig. 8 shows the time response of the synchronization error $e_i (i = 1,2,3)$, which implies that drive systems (16) and (17) have achieved single compound synchronization with the response system (18). Figs. 9-11 are the time response of states $x_1 y_1, z_1$, $x_2 y_2, z_2$ and $x_3 y_3, z_3$.

5. Conclusion

In conclusion, we investigate the synchronization among three identical chaotic systems and non-identical chaotic systems. Our two drive systems are compounded instead of combination thereby creating room for more anti-attack ability when secure communication is involved. The controllers designed in this work are found to be effective as can be seen from the Figs4-11. Therefore this result can be useful in a broad range of application in telecommunication.

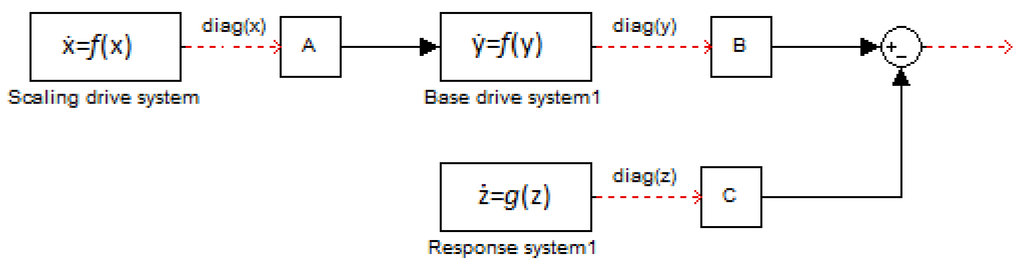

Fig.1 A schematic for single compound synchronization scheme

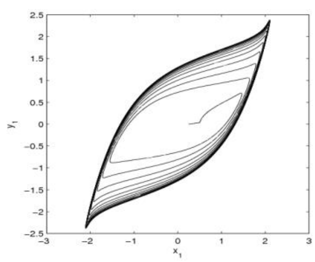

Fig.2 Chaotic attractor of the modified Chua's circuit system.

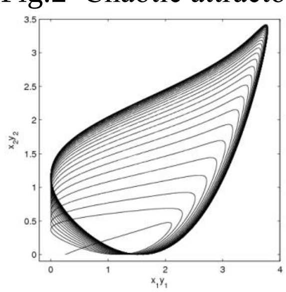

Fig.3 Chaotic attractor of the two drive modified Chua's circuit systems.

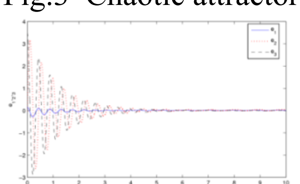

Fig. 4 Synchronization errors between drive systems (5), (6) and response system (7).

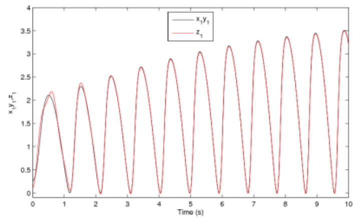

Fig. 5 Response for states x_1y_1 and z_1 between the drive systems (5), (6) and response system (7).

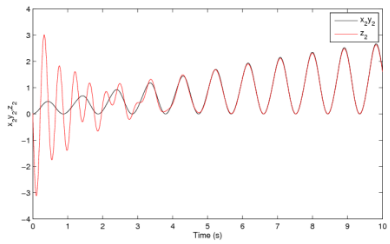

Fig. 6 Response for states x_2y_2 and z_2 between the drive systems (5), (6) and response system (7).

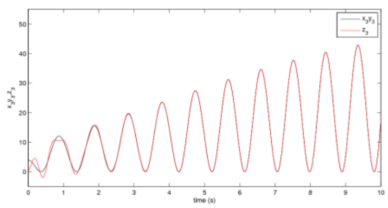

Fig. 7 Response for states x_3y_3 and z_3 between the drive systems (5), (6) and response system (7).

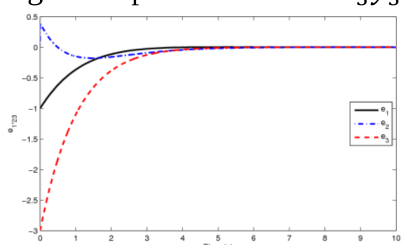

Fig. 8 Synchronization errors between drive systems (16), (17) and response system (18).

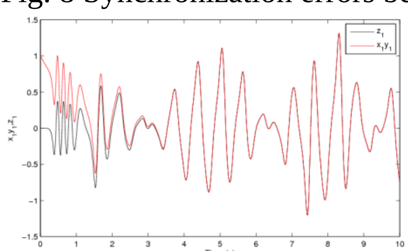

Fig. 9 Response for states x_1y_1 and z_1 between the drive systems (16), (17) and response system (18).

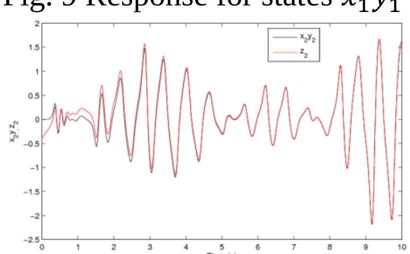

Fig. 10 Response for states x_2y_2 and z_2 between the drive systems (16), (17) and response system (18).

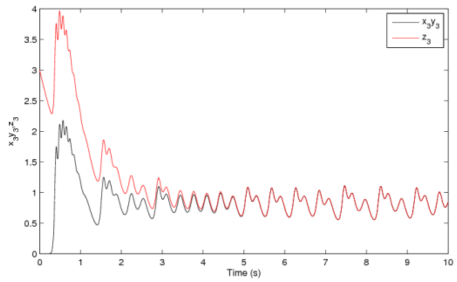

Fig. 11 Response for states x_3y_3 and z_3 between the drive systems (16), (17) and response system (18).

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