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Transient Loads Of Structurally Prestressed Non- Uniform Simply Supported Rayleigh Beam With Time-Dependent Boundary Conditions For Moving Force

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Abstract

The transverse vibration of a prismatic non-uniform simply supported Rayleigh beam resting on an elastic foundation and continuously acted upon by concentrated masses moving with non-uniform velocity is studied. A procedure involving Mindlin-Goodman's method to transform, the use of the expression of the Dirac delta function in series form, and the use of Generalized Galakin's method (GGM) is employed to simplify the governing fourth order partial differential equation with singular and variable coefficients. The resulting Galerkin's equations are solved via the use of modified Struble's asymptotic techniques to treat this dynamical beam problems, hence, the analytical solutions for moving force models which is valid for all variants of classical boundary conditions was obtained. We are invariably interested in predicting the response of the dynamical beam model. The proposed analytical procedure is illustrated by examples of some practical engineering interest in which the effects of some important parameters such as boundary conditions, pre-stressed function, mass ratio and elastic foundation are investigated in depth. Resonance phenomenon of the vibrating system is carefully investigated and the condition under which this may occur is clearly scrutinized. The results presented in this paper shows good agreement when compared with that of existing literature.

Keyword: Non-uniform Rayleigh beam, resonance phenomenon, rotatory inertia, concentrated moving loads, dynamic responses.

1. INTRODUTION

The dynamic vibrations of Rayleigh beams resting on elastic foundation to moving concentrated masses is restricted to the case of uniform Beams [1 - 5]. In recent years, such important engineering problems as the vibration of turbines, hulls of ships and bridge girders of variable depths etc involving the theory of vibration of structures of variable cross-section have intensified the need for the study of the vibrations of non-uniform elastic system under the action of moving loads.

Unlike in the [1 - 5], even after the use of Mindlin-Goodman's technique to simplify the governing differential equation of motion, the method of the Generalized Finite Integral transform is inapplicable and we resort to a modification of an approximate method generally referred to as Galerkin's method. This we term Generalized Galerkin's Method (GGM). This method is employed to simplify the governing fourth order partial differential equation with singular and variable coefficients. The resulting Galerkin's equations are solved via the modified Struble's asymptotic techniques already alluded to in [6 - 10]. Thus, this work perhaps, focuses on the dynamic vibrations of structurally prestressed simply supported non-uniform Rayleigh beams resting on elastic foundation and traversed by concentrated moving masses at an arbitrarily prescribed velocity with time-dependent boundary and initial conditions. Effects of some very important beam parameters on the motions of the vibrating systems are investigated.

2. THEORETICAL FORMULATION OF THE GOVERNING EQUATIONS

Considered here is a simply supported non-uniform Rayleigh beam resting on elastic foundation where the beams properties such as the moment of inertia I, and the mass per unit length of the beam μ vary along the span L of the beam. The r^o is the Rotatory inertia, K is the elastic foundation Modulli, x is the spatial coordinate.

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The transverse displacement $U(x,t)$ of the beam when it is under the action of a moving load of mass M which is moving with a non-uniform velocity such that the motion of the contact point of the moving load is described by the function

$$f(t)=\left(x_0+ct+\frac{1}{2}at^2\right) \tag{1.0}$$

where x_0 is the point of application of force $P = Mg$ at the instance $t = 0$, c is the initial velocity and a is the constant acceleration of motion governed by the fourth order partial differential equation given by

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2}{\partial x^2}U(x,t)\right]-N\frac{\partial^2}{\partial x^2}U(x,t)+\mu(x)\frac{\partial^2 U(x,t)}{\partial t^2}-\frac{\partial}{\partial x}\left[\mu(x)r^o\frac{\partial^3 U(x,t)}{\partial x\partial t^2}\right]$$

$$+M\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)\left(\frac{\partial^2}{\partial t^2}+\frac{2c\partial^2}{\partial x\partial t}+\frac{c^2\partial^2}{\partial x^2}\right)U(x,t)+KU(x,t)=Mg\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right) \tag{2.0}$$

where g is the acceleration due to gravity, $I(x)$ is the variable moment of inertia and $\mu(x)$ is the variable mass of the Rayleigh beam per unit area. Next, the example in [7] shall be adopted and $I(x)$ and $\mu(x)$ take the forms:

$$I(x)=I_0\left(1+\sin\frac{\pi x}{L}\right)^3 \quad \text{and} \quad \mu(x)=\mu_0\left(1+\sin\frac{\pi x}{L}\right) \tag{3.0}$$

where I_o and μ_o are constants. The boundary conditions of the above equation (2.0) are taken to be time dependent, thus at each of the boundary points, there are two boundary conditions written as:

$$D_i\left[U(0,t)\right]=f_i(t) \quad i=1,2 \quad \text{and} \quad D_i\left[U(L,t)\right]=f_i(t) \quad i=3,4 \tag{4.0}$$

where D_i are linear homogenous differential operators of order less than or equal to three. The initial conditions of the motion at time $t=0$ are specified by two arbitrary functions thus:

$$U(x,0)=U_0(x) \quad \text{and} \quad \frac{\partial U(x,0)}{\partial t}=\dot{U}_0(x) \tag{5.0}$$

But

$$\left(1+\sin\frac{\pi x}{L}\right)^3=\frac{1}{4}\left[10-6\cos\frac{2\pi x}{L}+15\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right] \tag{6.0}$$

Substituting equations (3.0) to (6.0) into equation (2.00) on simplifications and rearrangements, gives.

$$\frac{EI_0}{4}\frac{\partial^2}{\partial x^2}\left[\left(10-6\cos\frac{2\pi x}{L}+15\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)\frac{\partial^2 U(x,t)}{\partial x^2}\right]-N\frac{\partial^2 U(x,t)}{dx^2}+\mu_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2 U(x,t)}{dt^2}$$

$$-\mu_o r^o\frac{\partial}{\partial t}\left[(1+\sin\frac{\pi x}{L})\frac{\partial^3 U(x,t)}{\partial x^2\partial t}\right]+M\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)\left(\frac{\partial^2}{\partial t^2}+\frac{2(c+at)\partial^2}{\partial x\partial t}+\frac{(c+at)^2\partial^2}{\partial x^2}\right)U(x,t)$$

$$+KU(x,t)=Mg\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right) \tag{7.0}$$

3.0 Operational Simplifications of Equation

In this work, the initial-boundary value problem (7.0) consisting of a non-homogeneous partial differential equation with non-homogeneous boundary conditions is transformed to a non-homogeneous partial differential equation with homogeneous boundary conditions, using the Mindlin-Goodman's method described in [1-5]. In order to solve the above initial-boundary value problem. Thus, we introduce the auxiliary variable $Z(x,t)$ in the form

$$U(x,t)=Z(x,t)+\sum_{i=1}^{4}f_i(t)g_i(x) \tag{8.0}$$

Substituting equation (8.0) into the boundary value problem (7.0) and simplifying, transforms the latter into a boundary value problem in terms of $Z(x,t)$. The displacement influence functions $g_i(x)$ are chosen so as to render the boundary conditions for the boundary value problem in $Z(x,t)$ homogenous. Thus, gives;

$$\frac{EI_0}{4}\left[\left(10-6\cos\frac{2\pi x}{L}+15\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)\frac{\partial^4}{\partial x^4}Z(x,t)+6\frac{\pi}{L}\left(4\sin\frac{2\pi x}{L}+5\cos\frac{\pi x}{L}-\cos\frac{3\pi x}{L}\right)\frac{\partial^3}{\partial x^3}Z(x,t)\right.$$

$$\left.+3\frac{\pi^2}{L^2}\left(8\cos\frac{2\pi x}{L}-5\sin\frac{\pi x}{L}+3\sin\frac{3\pi x}{L}\right)\frac{\partial^2}{\partial x^2}Z(x,t)\right]+\mu_0\left(1+\sin\frac{\pi x}{L}\right)Z_{tt}(x,t)$$

$$-\mu_0 r^o\left[\frac{\partial^2}{\partial x^2}Z_{tt}(x,t)+\sin\frac{\pi x}{L}\frac{\partial^2}{\partial x^2}Z_{tt}(x,t)+\frac{\pi}{L}\cos\frac{\pi x}{L}\frac{\partial^2}{\partial x^2}Z_{tt}(x,t)\right]$$

$$+M\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)\left[Z_{tt}(x,t)+\frac{2c\partial}{\partial x}Z_{tt}(x,t)+\frac{c^2\partial^2}{\partial x^2}Z_{tt}(x,t)\right]+KZ(x,t)-N\frac{\partial^2}{\partial x^2}Z(x,t)$$

$$=Mg\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)-\sum_{i=1}^{\Psi}\left[\frac{EI_0}{4}f_i(t)\left(10-6\cos\frac{2\pi x}{L}+15\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)g_i^{IV}(x)\right.$$

$$\left.+6\frac{\pi}{L}\left(4\sin\frac{2\pi x}{L}+5\cos\frac{\pi x}{L}-\cos\frac{3\pi x}{L}\right)g_i^{III}(x)+3\frac{\pi^2}{L^2}\left(8\cos\frac{2\pi x}{L}-5\sin\frac{\pi x}{L}+3\sin\frac{3\pi x}{L}\right)g_i^{II}(x)\right]$$

$$+\mu_0\ddot{f}_i(t)\left(1+\sin\frac{\pi x}{L}\right)g_i(x)-\mu_0 r^o\ddot{f}_i(t)\left(g_i^{II}(x)+\sin\frac{\pi x}{L}g_i^{II}(x)+\frac{\pi}{L}\cos\frac{\pi x}{L}g_i^{I}(x)\right)$$

$$+M\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)\left(\ddot{f}_i(t)g_i(x)+2c\dot{f}_i(t)g_i^I(x)+c^2f_i(t)g_i^{II}(x)\right)+Kf_i(t)g_i(x)+Nf_i(t)g_i(x)\Big] \quad (9.0)$$

3.1 Method of Solution

Evidently, an exact closed form solution of the above partial differential equation does not exist. The method of separation of variables is inapplicable as difficulties arise in getting separate equations whose functions are functions of a single variable. As a result of these difficulties, one resort to an approximate method commonly called Galerkin's method.

3.2 Galerkin's Method

The Galerkin's method is used to solve equations of the form

$$\Gamma[Z(x,t)]-P(x,t)=0 \quad (10)$$

where Γ is the differential operator.

$Z(x,t)$ is the structural displacement and

$P(x,t)$ is the transverse load acting on the structure

A solution of the form

$$Z_j(x,t)=q_j(t)\phi_j(x) \quad \text{for } j=1,2,3,...,n\cdot \quad (11)$$

is sought when j = 1,2,3, …………..n.

The function $\phi_j(x)$ are chosen to satisfy the approximate boundary conditions. The Galerkin's method requires that the expression (11) be orthogonal to the function $\phi_i(x)$ for $i=1,2,3,...,n$.

Thus $\int_0^l\left[\Gamma\sum_{j=1}^{n}q_j(t)\phi_j(x)-P\right]\phi_i(x)dx=0$ for i =1,2,………,n (12)

This gives us a set of ordinary differential equations in $q_j(t)$ to be solved. These differential equations are called Galerkin's equations.

3.3 Analytical Approximate Solution.

The Galerkin's method requires that the solution of equation (9.0) takes the form

$$Z_n(x,t)=\sum_{m=1}^{n}Y_m(t)V_m(x) \quad (13)$$

where $V_m(x)$ is chosen such that the desired boundary conditions are satisfied.

Equation (13) When substituted into equation (9.0) yields

$$\sum_{m=1}^{n}\left[\frac{EI_o}{4}\left(\left(10-6\cos\frac{2\pi x}{L}+15\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)V_m^{IV}(x)+6\frac{\pi}{L}\left(4\sin\frac{2\pi x}{L}+5\cos\frac{\pi x}{L}-\cos\frac{3\pi x}{L}\right)V_m^{IIII}(x)\right.\right.$$

$$+3\frac{\pi^2}{L^2}\left(8\cos\frac{2\pi x}{L}-5\sin\frac{\pi x}{L}+3\sin\frac{3\pi x}{L}\right)V_m^{II}(x)\Bigg)Y_m(t)-NV_mY_m(t)+\mu_o\left(V_m(x)+\sin\frac{\pi x}{L}V_m(x)\right)\ddot{Y}_m(t)$$

$$-\mu_or^o\left(V_m^{II}(x)+\sin\frac{\pi x}{L}V_m^{II}(x)+\frac{\pi}{L}\cos\frac{\pi x}{L}V_m^{I}(x)\right)\ddot{Y}_m(t)$$

$$+M\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)\left(V_m(x)\ddot{Y}_m(t)+2CV_m^I(x)\dot{Y}_m(t)+C^2V_m^{II}(x)Y_m(t)\right)$$

$$+KV_m(x)Y_m(t)\ \Big]-Mg\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)+\sum_{i=1}^{4}\left[\frac{EI_o}{4}f_i(t)\left(\left(10-6\cos\frac{2\pi x}{L}+15\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)g_i^{IV}(x)\right]\right.$$

$$+6\frac{\pi}{L}\left(4\sin\frac{2\pi x}{L}+5\cos\frac{\pi x}{L}-\cos\frac{3\pi x}{L}\right)g_i^{III}(x)+3\frac{\pi^2}{L^2}\left(8\cos\frac{2\pi x}{L}-5\sin\frac{\pi x}{L}+3\sin\frac{3\pi x}{L}\right)g_i^{II}(x)$$

$$+\ \mu_o\ddot{f}_i(t)\left(1+\sin\frac{\pi x}{L}\right)g_i(x)-\mu_or^o\ddot{f}_i(t)\left(g_i^{II}(x)+\sin\frac{\pi x}{L}g_i^{II}(x)+\frac{\pi}{L}\cos\frac{\pi x}{L}g_i^{I}(x)\right)$$

$$+M\delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)\left(\ddot{f}_i(t)g_i(x)+2C\dot{f}_i(t)g_i^I(x)+C^2f_i(t)g_i^{II}(x)\right)-Nf_i(t)g_i+Kf_i(t)g_i(x)\ \Big]=0 \quad (14)$$

In order to determine $Y_m(t)$, it is required that the expression on the left hand side of equation (13) be orthogonal to the function $V_k(x)$.

Thus,

$$\sum_{m=1}^{n}\left[\left[H_i(m,k)+H_2(m,k)-r^0\left(H_3(m,k)+H_4(m,k)+\frac{\pi}{L}H_5(m,k)\right)\right]\ddot{Y}_m(t)\right.$$

$$+\left\{\frac{EI_o}{4}\left([10H_6(m,k)+15H_7(m,k)-6H_8(m,k)-H_9(m,k)]\right.\right.$$

$$+6\frac{\pi}{L}[4H_{10}(m,k)+15H_{11}(m,k)-H_{12}(m,k)]+3\frac{\pi^2}{L^2}[8H_{13}(m,k)+15H_4(m,k)+3H_{14}(m,k)]\ )$$

$$-\frac{N}{\mu_0}H_3(m,k)\ \}Y_m(t)+\frac{K}{\mu_0}H_i(m,k)\ \}Y_m(t)+\frac{M}{\mu_o}\left[H_{15}(m,k)\ddot{Y}_m(t)+2cH_{16}(m,k)\dot{Y}_m(t)+c^2H_{17}(m,k)Y_m(t)\right]\Big]$$
$$-\frac{Mg}{\mu_0}V_k(ct)+\left[G_a(t)-G_b(t)+G_c(t)\ -G_d(t)+G_e(t)+G_f(t)-G_g(t)+G_h(t)-G_i(t)\right.$$
$$\left.+G_j(t)+G_k(t)+G_l(t)-G_m(t)-G_N(t)-G_o(t)\ +G_p(t)+G_q(t)+G_R(t)+G_s(t)+G_T(t)\right]=0 \qquad (15)$$

where

$$H_i(m,k)=\int_0^L V_m(x)V_k(x)dx, \qquad H_3(m,k)=\int_o^L V_m^{II}(x)V_k(x)dx$$

$$H_2(m,k)=\int_0^L \sin\frac{\pi x}{L}V_m(x)V_k(x)dx,, \qquad H_4(m,k)=\int_0^L \sin\frac{\pi x}{L}V_m^{II}(x)V_k(x)dx$$

$$H_5(m,k)=\int_0^L \cos\frac{\pi x}{L}V_m^{I}(x)V_k(x)dx, \qquad H_6(m,k)=\int_0^L V_m^{IV}(x)V_k(xdx)..$$

$$.H_7(m,k)=\int_0^L \sin\frac{\pi x}{L}V_m^{IV}(x)V_k(x)dx, \qquad H_8(m,k)=\int_0^L \cos\frac{2\pi x}{L}V_m^{IV}(x)V_k(x)dx$$

$$.H_9(m,k)=\int_0^L \sin\frac{3\pi x}{L}V_m^{IV}(x)V_k(x)dx, \qquad H_{10}(m,k)=\int_0^L \sin\frac{2\pi x}{L}V_m^{III}(x)V_k(x)dx$$

$$.H_{11}(m,k)=\int_0^L \cos\frac{\pi x}{L}V_m^{III}(x)V_k(x)dx, \qquad H_{12}(m,k)=\int_0^L \cos\frac{3\pi x}{L}V_m^{III}(x)V_k(x)dx$$

$$.H_{13}(m,k)=\int_0^L \cos\frac{2\pi x}{L}V_m^{II}(x)V_k(x)dx, \qquad H_{14}(m,k)=\int_0^L \sin\frac{3\pi x}{L}V_m^{II}(x)V_k(x)dx$$

$$H_{15}(m,k)=\int_0^L \delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)V_m(x)V_k(x)dx \qquad H_{16}(m,k)=\int_0^L \delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)V_m^{I}(x)V_k(x)dx$$

$$H_{17}(m,k)=\int_0^L \delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)V_m^{II}(x)V_k(x)dx \qquad (16)$$

Furthermore,

$$G_a(t)=10\frac{EI_o}{4\mu_o}\sum_{i=1}^{4} f_i\int_0^L g_i^{IV}(x)V_k(x)dx, \qquad G_b(t)=\frac{6EI_o}{4\mu_o}\sum_{i=1}^{4} f_i(t)\int_0^L \cos\frac{2\pi x}{L}g_i^{IV}(x)V_k(x)dx$$

$$G_c(t)=\frac{15EI_o}{4\mu_o}\sum_{i=1}^{4} f_i(t)\int_0^L \sin\frac{\pi x}{L}g_i^{IV}(x)V_k(x)dx, \quad G_d(t)=\frac{EI_o}{4\mu_o}\sum_{i=1}^{4} f_i(t)\int_0^L \sin\frac{3\pi x}{L}g_i^{IV}(x)V_k(x)dx,$$

$$G_e(t)=\frac{24EI_o}{4\mu_o}\frac{\pi}{L}\sum_{i=1}^{4} f_i(t)\int_0^L \sin\frac{2\pi x}{L}g_i^{III}(x)V_k(x)dx$$

$$G_f(t)=\frac{30EI_o}{4\mu_o}\frac{\pi}{L}\sum_{i=1}^{4} f_i(t)\int_0^L \cos\frac{\pi x}{L}g_i^{III}(x)V_k(x)dx$$

$$G_g(t)=\frac{6EI_o}{4\mu_o}\frac{\pi}{L}\sum_{i=1}^{4} f_i(t)\int_0^L \cos\frac{3\pi x}{L}g_i^{III}(x)V_k(x)dx$$

$$G_h(t)=\frac{24EI_o}{4\mu_o}\frac{\pi^2}{L^2}\sum_{i=1}^{4} f_i(t)\int_0^L \cos\frac{2\pi x}{L}g_i^{II}(x)V_k(x)dx$$

$$G_i(t)=\frac{15EI_o}{4\mu_o}\frac{\pi^2}{L^2}\sum_{i=1}^{4} f_i(t)\int_0^L \sin\frac{\pi x}{L}g_i^{II}(x)V_k(x)dx$$

$$G_j(t)=\frac{9EI_o}{4\mu_o}\frac{\pi^2}{L^2}\sum_{i=1}^{4} f_i(t)\int_0^L \sin\frac{3\pi x}{L}g_i^{II}(x)V_k(x)dx, \qquad G_k(t)=\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L g_i(x)V_k(x)dx$$

$$G_l(t)=\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L \sin\frac{\pi x}{L}g_i(x)V_k(x)dx, \qquad G_m(t)=r^o\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L g_i^{II}(x)V_k(x)dx$$

$$G_N(t)=r^o\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L \sin\frac{\pi x}{L}g_i^{II}(x)V_k(x)dx,$$

$$G_p(t)=\frac{M}{\mu_0}\sum_{i=1}^{4}\ddot{f}_i(t)\int_0^L \delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)g_i(x)V_k(x)dx$$

$$G_q(t)=\frac{2(c+at)M}{\mu_0}\sum_{i=1}^{4}\dot{f}_i(t)\int_0^L \delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)g^{I}{}_i(x)V_k(x)dx$$

$$G_R(t)=\frac{2(c+at)^2M}{\mu_0}\sum_{i=1}^{4}f_i(t)\int_0^L \delta\left(x-\left(x_0+ct+\frac{1}{2}at^2\right)\right)g_i^{II}(x)V_k(x)dx$$

$$G_s(t)=\frac{K}{\mu_o}\sum_{i=1}^{4} f_i(t)\int_0^L g_i(x)V_k(x)dx, \qquad G_T(t)=\frac{N}{\mu_o}\sum_{i=1}^{4} f_i(t)\int_0^L g_I(x)V_k(x)dx \qquad (17)$$

At this juncture, a solution valid for all cases of classical boundary conditions is sought. Consequently, $V_m(x)$ is chosen as the beam function given as

$$V_m(x)=\sin\frac{\lambda_m x}{L}+A_m\cos\frac{\lambda_m x}{L}+B_m\sinh\frac{\lambda_m x}{L}+C_m\cosh\frac{\lambda_m x}{L}$$

Thus, (18)

$$V_k(x) = \sin\frac{\lambda_k x}{L} + A_k\cos\frac{\lambda_k x}{L} + B_k\sinh\frac{\lambda_k x}{L} + C_k\cosh\frac{\lambda_k x}{L}$$

Consequently,

$$V_k\left(x_0 + ct + \frac{1}{2}at^2\right) = \sin\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} + A_k\cos\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} + B_k\sinh\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} + C_k\cosh\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} \tag{19}$$

In order to evaluate the evolving integrals $I_1,......I_{144}, H_1(m,k),.......H_{18}(m,k), H_{17}(m,n,k)...etc$, (20)

Use is made of the property of the Dirac Delta function as an even function to express it in Fourier cosine series namely:

$$\delta\left[x - \left(x_0 + ct + \frac{1}{2}at^2\right)\right] = \frac{1}{L} + \frac{2}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right)\cos\frac{n\pi x}{L} \tag{21}$$

Thus, in view of (18), using equation (19) in equation (15), after some simplification and rearrangements yields.

$$\sum_{m=1}^{n}\Big[\alpha_0(m,k)\ddot{Y}_m(t) + \alpha_1(m,k)Y_m(t) + \varepsilon\Big(\Big[H_1(m,k) + 2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right)H_{1A}(m,n,k)\Big]\ddot{Y}_m(t)$$
$$+ 2c\Big[H_{18}(m,k) + 2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right)H_{18A}(m,n,k)\Big]\dot{Y}_m(t)$$
$$+ c^2\Big[H_3(m,k) + 2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right)H_{3A}(m,n,k)\Big]Y_m(t)\Big)\Big]$$
$$= \frac{Mg}{\mu_0}\left[\begin{array}{l}\sin\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} + A_k\cos\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} + B_k\sinh\frac{\lambda k\left(x_0 + ct + \frac{1}{2}at^2\right)}{L} + \\ + C_k\cosh\lambda_k\frac{\left(x_0 + ct + \frac{1}{2}at^2\right)}{L}\end{array}\right]$$
$$-\big[G_a(t) - G_b(t) + G_c(t) - G_d(t) + G_e(t) + G_f(t) - G_g(t) + G_h(t) - G_i(t) +$$
$$G_j(t) + G_k(t) + G_l(t) - G_m(t) - G_n(t) - G_o(t) + G_p(t) + G_q(t) + G_r(t) + G_s(t) + G_T(t)\big] \tag{22}$$

where

$$\in = \frac{mL}{\mu_0} \tag{23}$$

and

$$\alpha_0(m,k) = \left[H_1(m,k) + H_2(m,k) - r^0\left(H_3(m,k) + H_4(m,k) + \frac{\pi}{L}H_5(m,k)\right)\right]$$
$$\alpha_1(m,k) = \frac{EI_0}{4\mu_0}\Big[\big[10H_6(m,k) + 15H_7(m,k) - 6H_8(m,k) - H_9(m,k)\big] - \frac{N}{\mu_0}H_3(m,k) + \frac{K}{\mu_0}H_1(m,k)$$
$$+ 6\frac{\pi}{L}\big[4H_{10}(m,k) + 5H_{11}(m,k) - H_{12}(m,k)\big] + \frac{3\pi^2}{L^2}\big[8H_{13}(m,k) - 5H_4(m,k) + 3H_{14}(m,k)\big]\Big] \tag{24}$$

Equation (22) is the transformed equation governing the problem of non-uniform Rayleigh beam resting on a constant elastic foundation and transverse by a moving load. This second order differential equation is valid for all variants of the classical boundary conditions. In what follows, we shall consider boundary conditions such as simply supported boundary conditions as illustrative example.

3.4 Simply-Supported Boundary Conditions.

The deflection and bending moment at $x = 0$ and $x = L$ vanish for a non-uniform Rayleigh beam having simple supports at both ends.

$$Z(0,t) = 0 = Z(L,t), \qquad \frac{\partial^2 Z(0,t)}{\partial x^2} = 0 = \frac{\partial^2 Z(L,t)}{\partial x^2} \tag{25}$$

also, for normal modes

$$V_m(0) = 0 = V_m(L), \qquad \frac{d^2V_m(0)}{dx^2} = 0 = \frac{d^2V_m(L)}{dx^2} \tag{26}$$

Similarly

$$V_k(0) = 0 = V_k(L), \qquad \frac{d^2V_k(0)}{dx^2} = 0 = \frac{d^2V_k(L)}{dx^2} \tag{27}$$

Thus, it can be shown that

$$A_m = B_m = C_m = A_k = B_k = C_k = 0 \tag{28}$$

with the frequency equation

$$\sin\lambda_m = \sin\lambda_k = 0 \tag{29}$$

which implies that

$$\lambda_m = m\pi \text{ and } \lambda_k = k\pi\ . \tag{30}$$

Substituting, equations (25) to (30) into equation (24), one obtains

$$\sum_{n=1}\Bigg[\Bigg(\Big(I_1+I_{17}+r^0\frac{m^2\pi^2}{L^2}\Big(I_1+I_{17}-\frac{L}{m}I_{33}\Big)\Big)\ddot{Y}_m(t)$$

$$+\Bigg\{\frac{EI_0}{4\mu_0}\Bigg(\frac{m^4\pi^4}{L^4}\big[10I_1+5I_{17}-6I_{49}-I_{65}\big]$$

$$+\frac{m^3\pi^4}{L^4}\big[-24I_{81}+30I_{33}+6I_{97}\big]+\frac{m^2\pi^4}{L^4}\big[-24I_{49}+15I_{17}-9I_{65}\big]\Bigg)+\frac{K}{\mu_0}I_1\Bigg\}Y_m(t)$$

$$+\varepsilon\Bigg(\Bigg[I_1+2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\Big(x_0+ct+\frac{1}{2}at^2\Big)I_{113}\Bigg]\ddot{Y}_m(t)+\frac{2(c+at)m\pi}{L}\Bigg[I_5+2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\Big(x_0+ct+\frac{1}{2}at^2\Big)I_{129}\Bigg]\dot{Y}_m(t)$$

$$-\frac{(c+at)^2m^2\pi^2}{L^2}\Bigg[I_1+2\sum_{n=1}^{\infty}\cos\frac{n\pi\Big(x_0+ct+\frac{1}{2}at^2\Big)}{L}I_{113}\Bigg]Y_m(t)\Bigg)\Bigg]=\frac{Mg}{\mu_0}\sin\frac{k\pi}{L}\Big(x_0+ct+\frac{1}{2}at^2\Big)-\big[G_a(t)-G_b(t)+$$

$$G_c(t)-G_d(t)+G_e(t)+G_f(t)-G_g(t)G_h(t)-G_i(t)+G_j(t)+G_k(t)+G_l(t)-G_m(t)-G_n(t)-G_o(t)$$

$$G_p(t)+G_q(t)+G_r(t)+G_s(t)+G_T(t)\big]=0 \quad (31)$$

where the integrals ($I_1,\dots\dots I_{133}$) when solved gives

$$I_1=\begin{cases}0 & \text{for } k\neq m\\ \frac{L}{2} & \text{for } k=m\end{cases} \quad (32)$$

$$I_5=\begin{cases}\frac{-2kL}{\pi(m^2-k^2)}, & \text{if } k\pm m=\text{odd}\\ 0, & \text{if } k\pm m=\text{even}\end{cases} \quad (33)$$

$$I_{17}=\begin{cases}\frac{-4mLk}{\pi[1-(m-k)^2][1-(m+k)^2]}, & \text{if } k\pm m=\text{odd}\\ 0, & \text{if } k\pm m=\text{even}\end{cases} \quad (34)$$

$$I_{33}=\begin{cases}\frac{-2kL[1+m^2-k^2]}{\pi[1-(m+k)^2][1-(m-k)^2]}, & \text{if } k\pm m=\text{odd}\\ 0, & \text{if } k\pm m=\text{even}\end{cases} \quad (35)$$

$$I_{49}=\begin{cases}\frac{-mL}{4}, & \text{if } k\pm m=2\\ 0, & \text{if } k\pm m\neq 2\end{cases} \quad (36)$$

$$I_{65}=\begin{cases}\frac{-12mkL}{\pi[9-(m-k)^2][9-(m+k)^2]}, & \text{if } k\pm m=\text{odd}\\ 0, & \text{if } k\pm m=\text{even}\end{cases} \quad (37)$$

$$I_{81}=\begin{cases}\frac{kL}{4}, & \text{if } m\pm k=2\\ 0, & \text{if } m\pm k\neq 2\end{cases} \quad (38)$$

$$I_{97}=\begin{cases}\frac{-2kL[9+m^2-k^2]}{\pi[9-(m+k)^2][9-(m-k)^2]}, & \text{if } k\pm m \text{ odd}\\ 0, & \text{if } k\pm m \text{ even}\end{cases} \quad (39)$$

$$I_{113}=\frac{L}{4}\Big|_{n+m=k \text{ or } n=k-m}-\frac{L}{4}\Big|_{n-m=k \text{ or } n=k+m} \quad (40)$$

and

$$I_{129}=\frac{-2kL[n^2+m^2-k^2]}{\pi[n^2-(m+k)^2][n^2-(m-k)^2]} \quad (41)$$

Consequently,

$$2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\Big(x_0+ct+\frac{1}{2}at^2\Big)I_{133}=2L\sin\frac{n\pi}{L}\Big(x_0+ct+\frac{1}{2}at^2\Big)\sin\frac{m\pi}{L}\Big(x_0+ct+\frac{1}{2}at^2\Big) \quad (42)$$

In view of equations (32) to (42)Simplifying and rearranging equation (31) yields

$$\sum_{m=1}^{\infty}\alpha_0^*(m,k)\ddot{Y}_m(t)+\alpha_1^*(m,k)Y_m(t)+\varepsilon\left[Q_1(t)\ddot{Y}_m(t)+Q_2(t)\dot{Y}_m(t)+Q_3(t)Y_m(t)\right]$$
$$=\frac{Mg}{\mu_0}\sin\frac{k\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)-\left[G_a(t)-G_b(t)+G_c(t)-G_d(t)+G_e(t)+G_f(t)-G_g(t)+G_h(t)-G_i(t)\right.$$
$$\left.+G_j(t)+G_k(t)+G_l(t)-G_m(t)-G_n(t)-G_o(t)+G_p(t)+G_q(t)+G_r(t)+G_s(t)+G_T(t)\right]=0 \tag{43}$$

where $$\alpha_0^*(m,k)=\left(I_1+I_{17}+r^0\frac{m^2\pi^2}{L^2}\left(I_1+I_{17}-\frac{1}{m}I_{33}\right)\right) \tag{44}$$

$$\alpha_1^*(m,k)=\frac{EI_0}{4\mu_0}\left[\frac{m^4\pi^2}{L^4}\left(5L+\frac{3}{2}mL\frac{-60mL}{\pi\left[1-(m-k)^2\right]\left[1-(m+k)^2\right]}+\frac{12mkL}{\pi\left[9-(m-k)^2\right]\left[9-(m+k)^2\right]}\right)\right.$$
$$\frac{-m^3\pi^4}{L^4}\left(6kL+\frac{12kL\left(9+m^2-l^2\right)}{\pi\left[9-(m-k)^2\right]\left[9-(m+k)^2\right]}+\frac{60kL\left(1+m^2-k^2\right)}{\pi\left[1-(m-k)^2\right]\left[1-(m+k)^2\right]}\right)$$
$$+\frac{m^2\pi^4}{L^4}\left(6mL-\frac{60mkl}{\pi\left[1-(m-k)^2\right]\left[1-(m+k)^2\right]}+\frac{108mkl}{\pi\left[9-(m-k)^2\right]\left[9-(m+k)^2\right]}\right)$$
$$\left.+\frac{KL}{2\mu_0}+\frac{Nm^2\pi^2}{2\mu_0 L}\right] \tag{45}$$

$$Q_1(k,m,t)=\frac{L}{2}\left(1+4\sum\sin\frac{k\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)\sin\frac{m\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)\right) \tag{46}$$

$$Q_2(k,m,n,t)=-4(c+at)mk\left(\frac{1}{m^2-k^2}+\frac{2\sum_{n=1}^{\infty}\left(n^2+m^2-k^2\right)\cos\frac{n\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)}{\pi\left[n^2-(m-k)^2\right]\left[n^2-(m+k)^2\right]}\right) \tag{47}$$

and $$Q_3(k,m,t)=-\frac{(c+at)^2m^2\pi^2}{L^2}Q_1(k,m,t) \tag{48}$$

At this juncture, it is pertinent to obtain the particular functions $g_i(x)$ that ensure zeros of the right hand sides of the boundary conditions for simply supported beam. thus,

$$g_1(x)=1-\frac{x}{L},\quad g_2(x)=-\frac{L}{3}x+\frac{x^2}{L}-\frac{1}{6L}x^3,\quad g_3(x)=\frac{x}{L}\ \text{and}\ g_4(x)=-\frac{L}{6}x \tag{49}$$

As stated in the [7-10], it is only necessary to compute those of the $g_i(x)$ for which the corresponding $f_i(t)$ do not vanish. Thus, we need only $g_i(x)$ and $g_3(x)$ for our boundary displacement functions $f_1(t)$ and $f_3(t)$ defined in [7-10]

In view of equations (49). $G_a(t)=G_b(t)=G_c(t)=G_d(t)=G_e(t)=G_f(t)=G_g(t)=0$

and $G_h(t)=G_I(t)=G_j(t)=G_m(t)=G_n(t)=G_r(t)=G_T(t)=0$ (50)

while,

$$G_k(t)=\ddot{f}_1(t)N_1+\left(\ddot{f}_3(t)-\ddot{f}_1(t)\right)\frac{1}{L}N_2 \tag{51}$$

$$G_l(t)=\ddot{f}_1(t)N_3+\left(\ddot{f}_3(t)-\ddot{f}_1(t)\right)\frac{1}{L}N_4 \tag{52}$$

$$G_0(t)=\frac{r^0N_5}{L}\left(\ddot{f}_3(t)-\ddot{f}_1(t)\right) \tag{53}$$

$$G_p(t)=\begin{bmatrix}\frac{M}{\mu_0}\ddot{f}_1(t)\sin\frac{k\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)+\\ +\frac{M}{L^2\mu_0}\left(N_2+2\sum_{n=1}^{\infty}\cos\frac{n\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)N_6\right)\left(\ddot{f}_3(t)-\ddot{f}_1(t)\right)\end{bmatrix} \tag{54}$$

$$G_q(t)=\frac{2(c+at)m}{L\mu_0}\sin\frac{k\pi}{L}\left(x_0+ct+\frac{1}{2}at^2\right)\left(\ddot{f}_3(t)-\ddot{f}_1(t)\right) \tag{55}$$

$$G_5(t)=\left[\frac{k}{\mu_0}f_1(t)N_1+\frac{k}{L\mu_0}\left(f_3(t)-f_1(t)\right)\right] \tag{56}$$

Solving the evolving integrals ($N_{1,\ldots\ldots}N_5$) in equations (51) to (56) thus: $N_1=\begin{cases}\frac{2L}{k\pi}, & \text{if } (k\pm1) \text{ is even}\\ 0, & \text{if } (k\pm1) \text{ is odd}\end{cases}$ (57)

$$N_2=\frac{L^2}{k\pi}(-1)^{k+1} \tag{58}$$

$$N_3=\begin{cases}0, & \text{if } k\pm1\\ \frac{L}{2}, & \text{if } k=1\end{cases} \tag{59}$$

$$N_4 = \begin{cases} 0\,, & \text{if } (1 \pm k) \text{ even} \\ \dfrac{-4kl^2}{\pi^2(1-k^2)^2}\,, & \text{if } (1 \pm k) \text{ odd} \end{cases} \tag{60}$$

$$N_5 = \begin{cases} 0\,, & \text{if } (1 \pm k) \text{ even} \\ \dfrac{2Lk}{\pi(k^2-1)}\,, & \text{if } (1 \pm k) \text{ odd} \end{cases} \tag{61}$$

$$N_6 = \begin{cases} \dfrac{-l^2}{2\pi(k^2-n^2)}\left[(k-n)(-1)^{k+n} + (k+n)(-1)^{k-n}\right], & \text{if } k \neq n \\ 0\,, & \text{if } k = n \end{cases} \tag{62}$$

substituting (49 to 62) into (43), simplified an arranged gives:

$$\sum_{m=1}^{n}\left[\alpha_0^*(m,k)\ddot{Y}_m(t) + \alpha_1^*(m,k)Y_m(t) + \varepsilon\left[Q_1(t)\ddot{Y}_m(t) + Q_2(t)\dot{Y}_m(t)Q_3(t)Y_m(t)\right]\right]$$

$$= \frac{Mg}{\mu_0}\sin\frac{k\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right) - \left[F_1(t) + F_2(t) + F_3(t) + F_4(t)\right] - \varepsilon L\left[F_5(t) + F_6(t) + F_7(t) + F_8(t)\right] \tag{63}$$

Where $F_1(t) = \left(\ddot{f}_1(t) + (-1)^{k+1}\ddot{f}_3(t)\right)\dfrac{L}{k\pi}$ (64)

$$F_2(t) = \frac{L}{2}\ddot{f}_1(t) \tag{65}$$

$$F_3(t) = \left(f_1(t) + (-1)^{k+1}f_3(t)\right)\frac{l}{\mu_0\pi} \tag{66}$$

$$F_4(t) = \left(\ddot{f}_3(t) - \ddot{f}_1(t)\right)\left(\frac{2r^0k}{\pi(1-k^2)} - \frac{4kl}{\pi^2(1-k^2)^2}\right) \tag{67}$$

$$F_5(t) = \ddot{f}_1(t)\sin\frac{k\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right) \tag{68}$$

$$F_6(t) = \left(\ddot{f}_3(t) - \ddot{f}_1(t)\right)\frac{(-1)^{k+1}}{k\pi} \tag{69}$$

$$F_7(t) = \left(\ddot{f}_3(t) - \ddot{f}_1(t)\right)\sum_{n=1}^{\infty}\left\{\frac{(k-n)(-1)^{k+n} + (k+n)(-1)^{k-n}}{\pi(k^2-n^2)}\right\}\cos\frac{n\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right) \tag{70}$$

$$F_8(t) = \left(\dot{f}_3(t) - \dot{f}_1(t)\right)\frac{2(c+at)}{L}\sin\frac{k\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right) \tag{71}$$

Equation (63) represents the transformed equation of the non-uniform Rayleigh beam simply-supported at both ends and having boundary and initial conditions which are time dependent.

In order to solve equation (63), two cases are involved, namely

1) Moving force problem,
2) Moving mass problem. This case 2 because of the level of difficulties posed may not be considered in this paper but in subsequent one.

3.5 Simply Supported Non-Uniform Rayleigh Beam Traversed by Moving Force

In this instance, the inertial effect of the moving mass M is neglected and we set $\in$ to zero in equation (63) to obtain

$$\sum_{m=1}^{n}\left[\ddot{Y}_m(t) + \gamma_{mf}^2 Y_m(t)\right] = A_0\sin\frac{k\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right) - \frac{\left[F_1(t) + F_2(t) + F_3(t) + F_4(t)\right]}{\alpha_0^*(m,k)} \tag{72}$$

where $\gamma_{mf}^2 = \dfrac{\alpha_1^*(m,k)}{\alpha_0^*(m,k)}$ (73)

and $A_0 = \dfrac{mg}{\mu_0\alpha_0^*(m,k)}$ (74)

we consider a simply–supported beam, one of whose end $x = 0$, (say) is subjected to a sine-wave (undamped) transient displacement, starting from rest and the other end, $x = l$ is subjected to a damped sine wave transient displacement starting from rest. Consequently, we have:

$f_1(t) = BSin\Omega t$ and $f_3(t) = Ae^{-\beta t}Sin\Omega t$ (75)

Where: $A, B, \Omega, and\beta$ are as defined earlier.

$f_1(t)$, $f_3(t)$, $g_1(x)$ and $g_3(x)$ were substituted into the initial conditions one obtains: $Z(x,0) = 0$ and

$$\frac{\partial Z(x,0)}{\partial t} = -\Omega \tag{76}$$

It is also necessary to note that

$$\dot{f}_1(t) = \Omega Cos\Omega t, \quad \dot{f}_3(t) = e^{-\beta t}(\Omega Cos\Omega t - \beta Sin\Omega t)$$

$$\ddot{f}_1(t) = -\Omega^2 Sin\Omega t, \quad \dot{f}_3(t) = e^{-\beta t}(\Omega Cos\Omega t - \beta Sin\Omega t) \tag{77}$$

substituting equations (75) to (77) into (72) after simplifications and rearrangements, one obtains

$$\sum_{m=1}^{n}\left[\ddot{Y}_m(t) + \gamma_{mf}^2 Y_m(t)\right] = A_0 \sin\frac{k\pi}{L}\left(x_0 + ct + \frac{1}{2}at^2\right) + C F_9 Sin\Omega t + C F_{10} Cos\Omega t + C F_{11} e^{-\beta t} Sin\Omega t \tag{78}$$

where

$$C_{F9} = \frac{1}{\alpha_0^x(m,k)}\left(\frac{\Omega^2 L}{k\pi} - \frac{1}{\mu_o \pi} - \frac{2\Omega^2 k}{\pi(1-k^2)}\left(r^0 - \frac{2L}{\pi(1-k^2)}\right)\right) \tag{79}$$

$$C_{F10} = \frac{1}{\alpha_0^x(m,k)}\left(\frac{2\beta\Omega L}{k\pi} + \frac{4\beta\Omega k}{\pi(1-k^2)}\left(r^0 - \frac{2L}{\pi(1-k^2)}\right)\right) \tag{80}$$

and

$$C_{F11} = \frac{-1}{\alpha_0^x(m,k)}\left((-1)^{k+1}\frac{(\beta^2 - \Omega^2)L}{k\pi} + (-1)^{k+1}\frac{L}{\mu_0 \pi} + \frac{2k(\beta^2 - \Omega^2)}{\pi(1-k^2)}\left(r^0 - \frac{2L}{\pi(1-k^2)}\right)\right) \tag{81}$$

Clearly, equation (78) is solved Using the same procedure and argument in [7-10], it is straight forward to show that

$$Y_m(t) = \frac{1}{\gamma_{mf}}\left[\frac{A_0\gamma_{mf}}{\gamma^2{}_{mf} - \left(\frac{k\pi(c+at)}{L\gamma_{mf}}\right)^2}\left(\left(x_0 + ct + \frac{1}{2}at^2\right) - \frac{k\pi(c+at)}{L\gamma_{mf}}\sin\gamma_{mf}t\right)\right.$$
$$+ \frac{C_{F9}\gamma_{mf}}{2(\gamma_{mf}^2 - \Omega^2)}\left(Sin\Omega t - \frac{\Omega}{\gamma_{mf}}Sin\gamma_{mf}t\right) + \frac{C_{F10}\gamma_{mf}}{(\gamma_{mf}^2 - \Omega^2)}(Cos\Omega t - Cos\gamma_{mf}t)$$
$$+ \frac{2C_{F11}\gamma_{mf}(\gamma_{mf}^2 - \Omega^2 + \beta^2)}{2\left[(\gamma_{mf}^2 + \Omega^2 + \beta^2)^2 - 4\gamma_{mf}^2\Omega^2\right]}e^{-\beta t}Sin\Omega t + \frac{4C_{F11}\gamma_{mf}\Omega\beta}{2\left[(\gamma_{mf}^2 + \Omega^2 + \beta^2)^2 - 4\gamma_{mf}^2\Omega^2\right]}e^{-\beta t}\cos\Omega t$$
$$- \frac{2C_{F11}\Omega(\gamma_{mf}^2 - \Omega^2 - \beta^2)}{2\left[(\gamma_{mf}^2 + \Omega^2 + \beta^2)^2 - 4\gamma_{mf}^2\Omega^2\right]}Sin\gamma_{mf}t + \frac{-4C_{F11}\gamma_{mf}\Omega\beta}{2\left[(\gamma_{mf}^2 + \Omega^2 + \beta^2)^2 - 4\gamma_{mf}^2\Omega^2\right]}Cos\gamma_{mf}$$
$$\left. + \frac{\Omega L}{k\pi}\left(1 + (-1)^{k+1}\right)Sin\gamma_{mf}t\right] \tag{82}$$

and on inversion yields

$$Z(x,t) = \sum_{m=1}^{n}\left[\frac{A_0}{(\gamma_{mf}^2 - Z_0^2)}\left(SinZ_0 t - \frac{Z_0}{\gamma_{mf}}Sin\gamma_{mf}t\right)\right.$$
$$+ \frac{C_{F9}}{2(\gamma_{mf}^2 - \Omega^2)}\left(Cos\Omega t - \frac{\Omega}{\gamma_{mf}}Sin\gamma_{mf}t\right) + \frac{C_{F10}}{(\gamma_{mf}^2 - \Omega^2)}(Cos\Omega t - Cos\gamma_{mf}t)$$
$$+ \frac{C_{F11}Q_2}{Q_0}e^{-\beta t}Sin\Omega t + \frac{2C_{F11}\Omega\beta}{Q_0}e^{-\beta t}Cos\gamma_{mf}t$$
$$\left. + \left(\frac{\Omega L}{K\pi}\left(1 + (-1)^{k+1}\right) - \frac{C_{F11}\Omega Q_3}{Q_0}\right)\frac{1}{\gamma_{mf}}Sin\gamma_{mf}t - \frac{2C_{F11}\Omega\beta}{Q_0}Cos\gamma_{mf}t\right]Sin\frac{m\pi x}{L} \tag{83}$$

where $Q_0 = \left[(\gamma_{mf}^2 + \Omega^2 + \beta^2)^2 - 4\gamma_{mf}^2\Omega^2\right]$, $Q_1 = (\gamma_{mf}^2 + \Omega^2 + \beta^2)$ (84)

$Q_2 = (\gamma_{mf}^2 - \Omega^2 + \beta^2)$, $Q_3 = (\gamma_{mf}^2 - \Omega^2 - \beta^2)$, $Z_0 = \frac{k\pi c}{L}$ (85)

But $U(x,t) = Z(x,t) + \sum_{i=1}^{4} f_1(t) g_i(x)$

Consequently, $U(x,t) = Z(x,t) + Sin\Omega t + (e^{-\beta t} - 1)\frac{x}{L}Sin\Omega t.$ (86)

where $Z(x,t)$ is as given in equation (83).

Equation (86) is the dynamic response of the non- uniform Rayleigh beam to moving force whose two simply–supported edges undergo displacements which vary with time.

4.0 Discussion of the Analytical Solution

If the undamped system such as this is studied, it is desirable to examine the response amplitude of the dynamical system which may grow without bound. This is termed resonance when it occurs. Equation (83) clearly shows that the simply supported elastic Rayleigh beams transverse by a moving force will be in state of resonance whenever

$$\gamma_{mf} = \frac{m\pi u}{L} \tag{87}$$

this implies,

$$\gamma_{mf} = \frac{\frac{m\pi u}{L}}{1-\lambda\left(\frac{u^2 m^2 \pi^2}{L\gamma_{mf}^2}+L\right)\frac{1}{4\alpha_0^*(m,k)}} \quad (88)$$

5.0 Numerical Calculation and Discussion of the Results.

To illustrate the analysis proposed in this work by considering a non-homogenous beam of modulus of elasticity $E = 3.1\times10^{10}$ N/m^2, the moment of inertia I =2.87698×10^{-3} m^4,

the beam span L = 150m and the mass per unit length of the beam μ=2758.291 Kg/m. The values of foundation moduli are varied between 0N/m^3 and 400000N/m^3, the values of axial force N is varied between 0 N and 4·0×10^8 N.

The traverse deflections of the non-uniform Rayleigh beam are calculated and plotted against time for various values of rotatory inertia r^o, axial force N and foundation stiffness K.

Fig.1, displays the transverse displacement response to a moving force of simply supported non- uniform Rayleigh beam for various values of foundation modulli K and for fixed value of rotatory inertia r^o, and axial force N. The graph shows that the response amplitude of the beam decreases as the values of the foundation modulli K increases. Fig 5.2 also shows the deflection profile due to moving force of a simply supported non-uniform Rayleigh beam for fixed value of foundation modulli K and axial force N and for various values of rotatory inertia r^o.

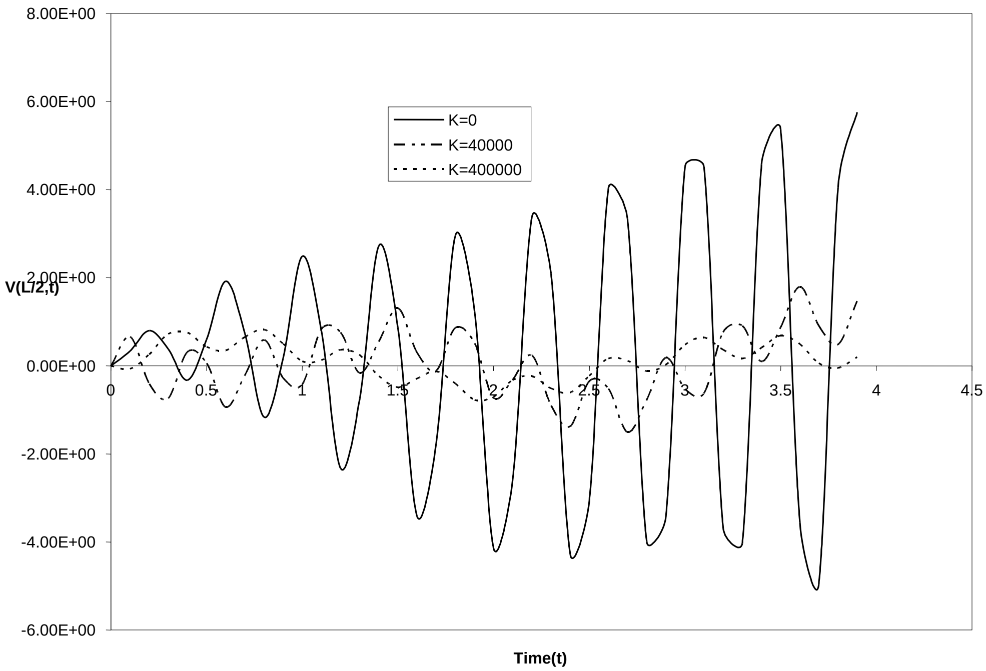

Fig 1. Deflection profile of a simply supported non-uniform Rayleigh beam under moving force for various values of foundation modulli K and for fixed value of axial force N and rotator inertia r(1)

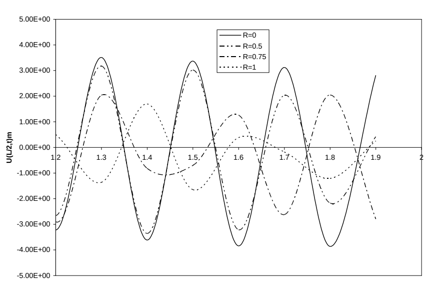

Fig 2: Deflection profile of the simply surported non-uniform Rayleigh beam under a moving force for various values of rotatory inertia and for fixed value of foundation K = 40000

Fig 2: Deflection profile of the simply supported non-uniform Rayleigh beam under a moving force for various values of rotatory inertia and for fixed value of foundation modulus K (40000)

The graph shows that the response amplitude of the beam decreases as the values of the rotatory inertia correction factor r^o are increased. Furthermore, fig.3 shows the deflection profile of simply supported Non-Uniform Rayleigh beam under the action of moving force for various values of axial force N and fixed value of rotatory inertia r^o and foundation modulus K. The graph shows that as the value of axial force N increases the displacement response of the beam decreases.

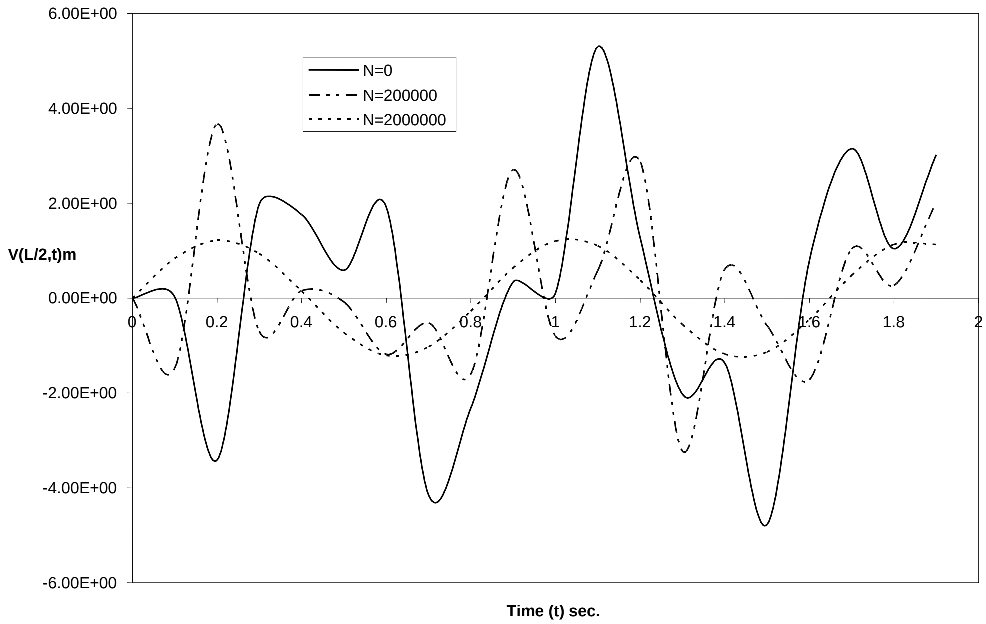

Fig.3.Deflection profile of simply supported Non-Uniform Rayleigh beam under the action of moving force for various values of axial force N and for fixed value of rotatory inertia r(1) and for fixed value of foundation modulus K(40000).

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