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Comparing the Ordering Policies of Useful Lifetime Based and Quantity Based, Fixed Lifetime Inventory Models.

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Abstract

The ordering policy of a fixed lifetime inventory system is either based on the quantity of items on hand or the number of useful lifetime remaining on the items on hand. In this work, we enumerate the features of both the quantity based and useful lifetime based models and apply their ordering policies to two fixed lifetime products, egg and bread and compare the performance of the policies.

Keyword: Inventory system, ordering policy, fixed lifetime, quantity based, useful lifetime based.

1. INTRODUTION

The fixed lifetime inventory system is one of the most critical aspect of operations management. This is because of the effect of outdating on products, when not used to meet demand during their useful lifetime in inventory. Over the years, fixed lifetime inventory models have based the decision to reorder new items on the quantity of items on hand, rather than the age of items on hand at the point of reorder.There are many authors in the literature with ordering policies based on the quantity of items on hand. The ordering policy (Q,r) which order Q whenever inventory on hand drops to the reorder point r was considered in [1 - 5]. The ordering policy (s,S) which order up to S when inventory on hand drops to the reorder point s was considered in [6 - 9]. The ordering policy $(S-1,S)$ in which an order is placed for exactly one item each time inventory is depleted by either demand or outdating was considered in [10 -12]. The ordering policy that maintains a minimum volume of inventory, whenever inventory drops to this level a new order is placed, was considered in [13]. As a deviation from these ordering policies, a new fixed lifetime inventory model where the decision to reorder is based on the number of useful lifetime remaining on the items on hand rather than just the quantity of items on hand was developed in [14]. The useful lifetime based model takes the form $(y,m-1)$ interpreted as order y when the useful lifetime remaining on the items on hand is one period. One advantage of this policy over existing policies is that, the policy is not fixed. If the demand is high, the inventory manager can decide to reorder new items with two periods remaining on the items on hand instead of one period. This is very common during festive periods when sales are high. If the demand drops, the inventory manager can reverse back to placing new orders with one useful period remaining on the items on hand.

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Features of Quantity based model.

A fixed lifetime inventory model where the decision to reorder is based on the amount / quantity of items on hand at the point of reorder is called a quantity based model. The features of the model are;

(1) Place new orders whenever quantity on hand drops to a certain point (called the reorder point) determined by the model's assumption.
(2) The ordering policy is fixed or rigid, since items on hand must drop to the reorder point before new orders are can be placed.
(3) The quantity ordered by the model is always fixed e.g. order Q whenever inventory drops to r , order up to S whenever inventory drops to s
(4) Does not consider the demand for items in placing new orders.
(5) Quantity based model exists in many forms. A few examples are (Q,r) , (s,S),$(S-1,S)$

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Features of Useful lifetime based model.

A fixed lifetime inventory model where the decision to reorder is based on the number of useful lifetime remaining on the items on hand at the point of reorder is called a useful lifetime based model. The features of the model are;

(1) Place new orders when the useful lifetime remaining on the items on hand is as stipulated by the model's assumption eg 1 useful lifetime remaining, 2 useful lifetime remaining etc.
(2) The ordering policy is not fixed or rigid, since the number of useful lifetime remaining before placing an order can be changed whenever the need arises.
(3) The quantity ordered can vary from one order to another.
(4) Considers prevailing demand for items in placing new orders
(5) Model exist in only one form, that is $(y, m-1)$ as developed by Izevbizua and Omosigho.

As a way of comparing the ordering policies of these models, we will use the quantity based model of [1] and the useful lifetime based model we developed in [14]. Also, two Benin City based companies (Uwa poultry products and Ilos supermarket) applied theseordering policiesto two fixed lifetime products, egg and bread respectively. The results will be given in this work.

Chiu's model (quantity based)

Chiu considered a continuous review perishable inventory model by approximating the expected shortage quantity, the expected outdate quantity and the expected on hand inventory. He developed a (Q, r) ordering policy where Q is the ordered quantity and r the reorder point. We used simple minimization method to obtain an expression for Q, the order quantity.The model is shown here. Details can be seen in [1]

Notation

$m = lifetime\ of\ product$
$L = order\ leadtime$
$c = replenishment\ \text{cost}\ per\ u\text{int}$
$k = fixed\ ordering\ \text{cost}$
$p = shortage\ \text{cost}$
$w = outdate\ \text{cost}$
$h = holding\ \text{cost}$
$d_m = demand\ during\ lifetime\ of\ product$
$d_L = demand\ during\ leadtime$
$Q = ordered\ quantity$
$r = reorder\ po\text{int}$
$D = mean\ of\ demand\ distribution$
$ER = outdate\ quantity$
$ES = shortage\ quantity$
$ET = cycle\ time$
$OH = on\ hand\ inventory$
$c_c = total\ \text{cost}$

The total cost function for the model was given as

$$EAC(Q,r) = \{(k + cQ + pES + wER)/ET\} + hOH \qquad (1)$$

$where$

$$ES = \int_r^{\infty} (d_L - r) f_L(d_L) d_L \qquad (2)$$

$$ER = \int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) d_{m+L} - \int_0^{r} (r - d_{m+L}) f_{m+L}(d_{m+L}) d_{m+L} \qquad (3)$$

$$ET = \frac{Q + ES - ER}{D} \qquad (4)$$

$$OH = r - (DL) + \frac{Q}{2} \qquad (5)$$

so that (3.1) becomes

$$EAC(Q,r) = \{k + cQ + p\int_{r}^{\infty}(d_L - r)f_L(d_L)d_L + w\{\int_{0}^{r+Q}(r + Q - d_{m+L})f_{m+L}(d_{m+L}) - \int_{0}^{r}(r - d_{m+L})f_{m+L}(d_{m+L})d_{m+L}\}\}/ET + h(r - DL + \frac{Q}{2}) \quad (6)$$

To implement the model, we obtain the partial derivative of equation (6) with respect to the ordered quantity using MATHEMATICA 8 and equate to zero. The resulting equation yields (7).

$$\frac{hD^2 f_{m+l}Q^6}{36} + (\frac{Dwf_{m+l}}{6} - \frac{w(f_{m+l})^2}{6D})Q^5 - \frac{hDf_{m+l}Q^4}{3} + (2\frac{wf_{m+l}}{2} - \frac{wf_{m+l}}{3} + \frac{2cf_{m+l}}{2})Q^3 + (h + \frac{k}{2D}f_{m+l})Q^2 - 2k = 0 \quad (7)$$

We use equation (7) to generate the order quantity

Izevbizua and Omosigho's model (useful lifetime based)

Details of the derivation of the cost components for our model are contained in [14]. We will only state the required equations here.

The shortage, outdate, and holding costs are as shown in equations (8), (9) and (10) .

$$Expected \quad (unsatisfied \ demand) = \int_{x+y}^{\infty}(t - (x + y))f^*(t)dt \quad (8)$$

$$Expected \quad (outdate) \ = \int_{0}^{x}(x - t)f(t)dt \quad (9)$$

$$Holding \ \text{cost} = h\int_{0}^{x+y}(x + y - t)f^*(t)dt \quad (10)$$

There is a fixed ordering cost k per unit ordered, so that our ordering cost is ky .

The total cost function is

$$C(x,y) = \min_{y \geq 0}\{ky + h\int_{0}^{x+y}(x + y - t)f^*(t)dt + v\int_{x+y}^{\infty}(t - (x + y))f^*(t)dt + \theta\int_{0}^{y}(y - t)f^*(t)dt\} \quad (11)$$

$where$

$f^*(t) = \ distribution \ of \ total \ demand \ in \ periods \ 1,2, \ . \ . \ . \ m.. \ that \ is$

$f^*(t) \ is \ distribution \ of \ t = \sum_{i=1}^{m} d_i$

m = lifetime of the product. m is a positive integer

$d_i = demand \ in \ period \ i$

$t = \sum_{i=1}^{m} d_i, \ total \ demand \ with \ distribution \ f^*(t)$

x represent the quantity of products with one useful period remaining in them. That is $x = y - \sum_{i=1}^{m-1} d_i$

y = new products ordered/entering into inventory with age zero

θ = outdate cost per unit

v = shortage cost per unit

h = holding cost per unit

k = fixed ordering cost per unit.

$c_0 = total \quad \text{cost}$

To find the optimal order quantity, we minimize (6) with respect to the order quantity.

$$(\frac{h}{2} + \frac{v}{2} + \frac{\theta}{2})f^*y^2 + (hx + vx)f^*y + k + \frac{hx^2 f^*}{2} - v + \frac{vx^2 f^*}{2} = 0 \quad (12)$$

Equation (12) is used for obtaining the order quantity for our model.

Empirical Examples:

In this section, the useful lifetime ordering policy was first applied to egg and bread inventories and thereafter the ordering policy of quantity based model was applied to the same order distribution. We compared the outdate quantities and the total cost for both models using equations (6) and (11).

Example 1
Product: bread
Useful lifetime: 4 days
Source: Illos supermarket, Benin City.

The ordering policy of the useful lifetime based model was applied by Illos supermarket to bread inventory. New orders were placed with one useful lifetime remaining on the items on hand. The daily demand, outdates, order and quantity of items in an order reordered in Table 1.

Table1: Inventory on hand ,demand and outdates, using the useful lifetime model. With the useful lifetime remaining before order as one period.

Day	Order(s)	On hand inventory	demand	Outdates
1	1	50	15	-
2	1	35	10	-
3	1,2	25 5,30	20	-
4	1,2	5,30	16(5:11)	-
5	2	19	8	-
6	2,3	11 2,35	9	-
7	2,3	2,35	14(2:12)	-
8	3	23	10	-
9	3,4	13 8,60	5	-
10	3,4	8,60	28(8:20)	-
11	4	40	27	-
12	4,5	13 3,32	10	-
13	4,5	3,32	12(3/9)	-
14	5	23	8	-
15	5,6	15 10,40	5	-
16	5,6	10,40	23(10/13)	-
17	6	27	10	-
18	6,7	17 8,35	9	-
19	6,7	8,35	6(6/0)	2
20	7	35	15	-
21	7,8	20 4,40	16	-
22	7,8	4,40	22(4/18)	-
23	8	22	12	-
24	8,9	10 0,50	15(10/5)	-
25	8,9	0,45	19(0/19)	-
26	9	26	10	-
27	9,10	16 3,42	13	-
28	9,10	3,42	20(3/17)	-
29	10	25	14	-
30	10/11	11 0,45	12(11/1)	-
31	10/11	0,44	21	-
32	11	23	18	-
33	11/12	5 70,	15	-

Observe that after 12 orders , we have 2 outdates. Next, we obtain the total cost for each order in example 1 using our model (11) and Chiu's model (6). We used the exponential distribution in our computation.

Table2:Comparing total cost for example1 for our model and Chiu. m=4, h=1,v=1, $\theta = 0.5$

Order	k	y	x	D	C_c	C_o
1	120	50	5	50	7405	6272
2		30	2	30	4744	3856
3		35	8	35	5448	4473
4		60	3	60	8724	7473
5		32	10	32	5042	4120
6		40	8	38	6068	5052
7		35	4	35	5374	4462
8		40	0	40	5980	5060
9		50	8	50	7395	6281
10		42	4	42	6585	5306
11		45	5	45	6691	5669
12		70	10	70	10162	8699
13		65	8	65	9433	8088
14		60	9	60	8764	7490
15		55	5	55	8024	6874
16		71	12	69	10339	8796
17		80	15	78	11675	9883
18		45	3	45	6667	5665
19		50	3	50	7328	6268
20		65	4	65	9364	8077

Table2 shows that our total cost per order are lower than that of Chiu. The reason for this is shown in Table3 where we see the effect of placing new orders when inventory drops to 5 with respect to example 1. That is applying the quantity based model to the bread inventory.

Table 3: Effect of outdating on quantity based model, placing neworder when on hand inventory drops to 5

Period	Order(s)	On hand inventory	Demand	Outdates
1	1	50	15	-
2		35	10	-
3		25	15	-
4		10	5	-
5	1,2	5 30	11	5
6		19	8	-
7	2,3	11 60	9(6/3)	-
8		2 60	20	-
9		42	10	-
10		32	10	-
11		22	12	-
12	3,4	10 32	10	10
13		22	8	-
14		14	8	-
15	4,5	6 40	5(1/4)	-
16		1 40	13	-
17		28	10	-
18		18	9	-
19	5,6	9 35	6(4/2)	-
20		3 35	15	3
21	6,7	20 50	16(15/1)	-
22		4 50	15	-
23		39	9	-
24		30	10	-
25		20	15	-
26		5		5

From table 3, the number of outdates after 7 orders is 23 as against 2 from 12 orders in Table 1. The high number of outdates is the reason for the higher cost associated with the total cost of quantitybased model.

Example 2:

Product: egg
Useful Lifetime: 5weeks
Previous outdate rate: 10 creates per order.
Previous Quantity per order: 100 creates of eggs
Source: Uwa poultry products, Benin City.

In example 2, the ordering policy of the useful lifetime model was applied to egg inventory. New orders were placed with one useful period remaining on the items on hand. Table4 shows the daily demand, outdates, order and quantity of items in an order. Also, we looked at the effect of continuously ordering 100 creates of egg instead of allowing demand determine the quantity order. Table 6 shows the number of outdates.

Table 4: on hand inventory , daily demand and outdates for the egg inventory

Date	week	Order	On hand inv	demand	Outdates
24-30/6	1	1	84	24	-
1-7/7	2	1	60	16	-
8-14/7	3	1	44	10	-
15-21/7	4	1	34	12	-
22-28/7	5	1,2	22,75	25	-
29-4/8	6	2	72	17	-
5-11/8	7	2	55	15	-
12-18/8	8	2	40	16	-
19-25/8	9	2	24	10	-
26-1/9	10	2,3	14,90	13	1
2-8/9	11	3	90	20	-
9-15/9	12	3	70	21	-
16-22/9	13	3	49	9	-
23-29/9	14	3	40	23	-
30-6/10	15	3,4	17,80	23	-
7-13/10	16	4	74	14	-
14-20/10	17	4	60	16	-
21-27/10	18	4	44	15	-
28-3/11	19	4	29	9	-
4-10/11	20	4,5	20,100	21	-
11-17/11	21	5	99	30	-
18-24/11	22	5	69	10	-
25-1/12	23	5	59	14	-
2-8/12	24	5	45	29	-

9-15/12	25	5,6	16,150	38	-
16-22/12	26	6	128	40	-
23-29/12	27	6	88	35	-
30-5/12	28	6	53	15	-
6-12/1	29	6	38	15	-
13-19/1	30	6,7	23,60	20	3
20-26/1	31	7	60	12	-
27-2/2	32	7	48	14	-
3-9/2	33	7	34	19	-
10-16/2	34	7	15	10	-
17-23/2	35	7,8	5,60	13	-

We observe from Table 4 that the number of outdates is 4 creates of eggs after 7 orders. This is an improvement on the company's outdate record. Again, we compute the total cost for the inventory using both models.

Table5: total cost per order for the egg inventory. m=5,h=1, v=20, $\theta = 10$

Order	k	y	x	D	C_c	C_o
1	500	84	22	84	45324	43075
2		75	14	74	40442	38553
3		90	17	90	48367	46066
4		80	20	80	43180	41068
5		100	16	100	53616	51069
6		150	23	147	80623	76110
7		60	5	60	32500	31033
8		80	12	80	43012	41052
9		90	15	90	48320	46062
10		70	11	70	38781	36046

Again the cost are lower with the useful lifetime model. As stated above, the quantity based model order a fixed quantity whenever a new order is placed. We look at the effect of ordering 100 creates of eggs continuously, given the same demand.

Table 6: Effect of continuously ordering 100 creates of eggs

Week	Order	On hand inventory	demand	Outdates
1	1	100	24	-
2	1	76	16	-
3	1	60	10	-
4	1	50	12	-
5	1,2	38,100	25	13
6	2	100	17	-
7	2	83	15	-

8	2	68	16	-
9	2	52	10	-
10	2,3	42,100	13	29
11	3	100	20	-
12	3	80	21	-
13	3	59	9	-
14	3	50	23	-
15	3,4	27,100	23	4
16	4	100	14	-
17	4	86	16	-
18	4	70	15	-
19	4	55	9	-
20	4,5	46,100	21	25
21	5	100	30	-
22	5	70	10	-
23	5	60	14	-
24	5	46	29	-
25	5,6	17,100	38	-
26	6	79	40	-
27	6	39	35	-
28	6	4	15	-
29	6	0	15	-
30	6,7	0,100	20	-
31	7	54	12	-
32	7	42	14	-
33	7	28	19	-
34	7	9	10	-
35	7,8	0,100	13	-

The large number of outdates was due to the fixed reorder quantity, an assumption we have relaxed in our model. Fixing the reordered quantity at 100 crates, as previously done by the company yields an outdate quantity of 71 crates over the period.

Conclusion

We have looked at practical application of ordering polices based on quantity of items on hand and based on useful lifetime remaining on items in inventory. Using records provided by egg and bread vendors in Benin City, we observe that the number of items outdating in the quantity based models are higher than that of the useful lifetime based model. This in turn makes the total cost per order for the quantity based model higher than the total cost for the useful lifetime model. Also, to constantly order a fixed quantity without regards to prevailing demand increasesthe number of outdates.

References

[1] Chiu, H.N. (1994); An Approximation to the Continuous review Inventory Model with Perishable Items and Lead Times. European Journal to Operational Research.Vol. 87. pp. 93-108.

[2] Bookbinder .J .H and Cakanyildirim. M (1999); Random lead times and expected orders in (Q, r) inventory systems. European journal of operational research .vol 155, pp300-313.

[3] Mohammad, H.A and Manuel, D.R (2007); An efficient heuristic optimization algorithm for a two – echelon (R, Q) inventory system. International journal of production economics vol 109,pp195-213.

[4] Hariga, M. (2010); A single-item continuous review inventory problem with space restriction.International journal of production economics.Vol 128, pp 153-158.

[5] Siriruk, P (2012); The optimal ordering policy for a perishable inventory system. Proceedings of the world congress on engineering and computer science.Vol II.

[6] Nahmias, S (1978); The fixed-charge perishable inventory problem. Journal of Operations Research.Vol 26, (3) pp464-481.

[7] Hollier et al (1995); Continuous review (s, S) policies for inventory systems incorporating cutoff transaction size. International journal of production res. vol 33 (10), pp 2855-2865.

[8] Liu, L. and Lian, Z. (1999); (s,S) Continuous Review Models for Products with Fixed Lifetime. Operations Research, Vol. 47(1). pp. 150-158.

[9] Silver, E.A, Bischak, D.P and Kok, T (2012); Determining the reorder point and the order-up-to level to satisfy two constraints in a periodic review system under negative binomial demand.

[10] Schmidt, C. P. and Nahmias, S. (1985); (S-1, S) Policies for Perishable Inventory. Management Science.Vol. 31(6). pp. 719-728.

[11] Perry, D. and Posner, M.J.M. (1998); An (-1, S) Inventory System with Fixed Shelf life and Constant lead times. Operations Research Vol. 46(3), pp.65-71.

[12] Olsson, F. and Tydesjo .P (2010): Inventory Problems with Perishable items: Fixed Lifetimes and Backlogging. European Journal of Operational Research.Vol. 202.pp .131-137.

[13] Shen, Z, Dessouky, M and Ordonez, F. (2012); Perishable inventory management system with a minimum volume constraint.Journal of the Operational Research Society.Vol 62, pp 2063-2082.

[14] Izevbizua, O and Omosigho, S.E (2016); The management of inventory system of products with fixed lifetime. unpublishedPh.D thesis submitted to postgraduate school University of Benin, Benin City.