Transactions of the Nigerian Association of Mathematical Physics
Volume 5,(September and November, 2017), pp33 – 38
© Trans. of NAMP

The Stability of the Rational Interpolation Method in Ordinary Differential Equations at K=6

Ebhohimen F. and Anetor O.

Ambrose Alli University, Ekpoma, Department of Mathematics,
Faculty of Physical Sciences, P.M.B. 14, Edo State

Abstract

In this paperwe designed the stability of the rational interpolation Method in ordinary differential equations. This was achieved by considering the rational interpolation formula for k = 6

$$y_{n+1} = \frac{\sum_{i=0}^{5} p_i X_{n+1}^i}{1+\sum_{i=1}^{6} q_i X_{n+1}^i} \quad \text{and } y_{(x)} = U_{(x)} = \frac{P_0+P_1x+P_2x^2+P^3x^3+p_4x^4+P_5x^5}{1+q_1x+q_2x^2+q_3x^3+q_4x^4+q_5x^5+q_6x^6}$$

Satisfying $U(X_{n+i}) = y_{n+i}$, $i = 0,1,2,3,4,5,6.$

Keywords: Rational, Interpolation, Stability, Derivation and Computation.

1.0 Introduction

Numerical methods for systems of Ordinary Differential Equations (ODEs) have been attracting much attention due to their need in the solution of problems arising from the mathematical formulation of physical situations in chemical kinetics, population models, mechanical oscillations, planetary motions, electrical networks, nuclear reactor control, tunnel switching problems, reversible enzyme kinetic often lead to Initial Value Problems (IVPs) in Ordinary Differential Equations. In this paper research work we shall be concern with the stability of the rational interpolation method for $k = 6$ in ordinary differential equations[1-18].

Some Definitions

Definition 1.A one-step method is said to be **A-STABLE** if when applied to the test equation $y^1= \lambda y$ with $\text{Re}(\lambda) < 0$, it gives,$y_{n+1} = S(\bar{h})y_n$, with the stability function $S(\bar{h})$ satisfying,$|S(\bar{h})| < 1$ for all Re $(\bar{h}) < 0$, $\bar{h} = \lambda h$

Definition 2.A given One-Step Method is said to be **L-Stable** if it is A-Stable and in addition, $\text{Re}(\bar{h}) \xrightarrow{\text{limit}} -\infty, \quad |S(\bar{h})| = 0$

Definition 3. A numerical integrator method is said to be **A(α) – STABLE** for some $\alpha \in [0, \pi/2]$ if the infinite wedge. $S\alpha = \{Ƶ: |\text{Arg}(-Ƶ)|\} < \alpha, Ƶ \neq 0$ i.e. $Ƶ = \bar{h}$ is contained in the region of absolute stability.

Definition 4.A numerical method is said to be A(0) – Stable if it is A(α) – Stable for some $\alpha \in [0,\pi/2]$. $A(\pi/2)$ - Stability ≡ A–stability.

2. The Derivation

The derivation of thestability of the rational interpolation method consists of matching the Taylor series expansion of $y(x_{n+1})$ with the Taylor series of the approximation value of y_{n+1}. At the point $x = x_n$ in the interval $[x_n, x_{n+1}]$, we set $y_n = y(x_n)$, since Our Taylor series about x_n for both $y(x_{n+1})$ and y_{n+1} require the use of $h = x_{n+1} - x_n$, we choose in sufficiently small enough so that x_{n+1} and x_n are very close.

The following equations are results of matching of the Taylor series of $y(x_{n+1})$ and y_{n+1} for the case $k = 6$ where

$$y_{n+1} = \frac{\sum_{i=0}^{k-1} P_i X_{n+1}^i}{1+\sum_{i=1}^{k} q_i X_{n+1}^i} = \frac{\sum_{i=0}^{5} P_i X_{n+1}^i}{1+\sum_{i=1}^{6} q_i X_{n+1}^i} \quad (1)$$

Correspondence Author: Ebhohimen F., E.mail: ebhohimenfidelis@yahoo.com, Tel.:+2348066282372

$$y_{n+1} = \frac{P_0 + P_1 x_{n+1} + P_2 x_{n+1}^2 + P_3 x_{n+1}^3 + P_4 x_{n+1}^4 + P_5 x_{n+1}^5}{1 + q_1 x_{n+1} + q_2 x_{n+1}^2 + q_3 x_{n+1}^3 + q_4 x_{n+1}^4 + q_5 x_{n+1}^5 + q_6 x_{n+1}^6} \quad (2)$$

where,

$$P_j = \sum_{i=1}^{j} \frac{h^{(j+1-i)}\, y_n^{(j+1-i)}\, q_{i-1}}{(j+1-i)!\,(x_{n+1})^{(j+1-i)}} + y_n q_j, \quad j = 1(1)k-1 \quad (3)$$

when $j = 0$, $i = 1$, from (3) we have,

$P_0 = y_n q_0$ i.e. $q_0 = 1$ $P_0 = y_n$ (4)

when $j = 1$, $i = 1$, (3) becomes.

$$P_1 = \frac{h y_n^{(1)} q_0}{1!\, x_{n+1}} + y_n q_1 \Rightarrow \frac{h y_n^{(1)}}{1!\, x_{n+1}} + y_n q_1$$

$$\frac{h y_n^{(1)}}{1!} = x_{n+1}\left[-p_0 q_1 + p_1\right] \quad (5)$$

when $j = 2$, $i = 1$, (3) becomes

$$P_2 = \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} + \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_1 + y_n q_2$$

$$\frac{h^2 y_n^{(2)}}{2!} = x_{n+1}^2\left[-p_0 q_2 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_1 + p_2\right] \quad (6)$$

when $j = 3, 4, 5, 6, 7, 8, 9, 10, 11$, $i = 1$ we have in (7)

$$\frac{h^3 y_n^{(3)}}{3!} = x_{n+1}^3\left[-p_0 q_3 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_1 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_2 + p_3\right] \quad (7)$$

$$\frac{h^4 y_n^{(4)}}{4!} = x_{n+1}^4\left[-p_0 q_4 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_1 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_2 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_3 + p_4\right] \quad (8)$$

$$\frac{h^5 y_n^{(5)}}{5!} = x_{n+1}^5\left[-p_0 q_5 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_1 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_2 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_3 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_4 + p_5\right] \quad (9)$$

$$\frac{h^6 y_n^{(6)}}{6!} = x_{n+1}^6\left[-p_0 q_6 - \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^5} q_1 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_2 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_3 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_4 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_5 + p_6\right] \quad (10)$$

$$\frac{h^7 y_n^{(7)}}{7!} = x_{n+1}^7\left[-p_0 q_7 - \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^6} q_1 - \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^5} q_2 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_3 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_4 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_5 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_6 + p_7\right] \quad (11)$$

$$\frac{h^8 y_n^{(8)}}{8!} = x_{n+1}^8\left[-p_0 q_8 - \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^7} q_1 - \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^6} q_2 - \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^5} q_3 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_4 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_5 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_6 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_7 + p_8\right] \quad (12)$$

$$\frac{h^9 y_n^{(9)}}{9!} = x_{n+1}^9\left[-p_0 q_9 - \frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^8} q_1 - \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^7} q_2 - \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^6} q_3 - \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^5} q_4 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_5 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_6 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_7 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_8 + p_9\right] \quad (13)$$

$$\frac{h^{10} y_n^{(10)}}{10!} = x_{n+1}^{10}\left[-p_0 q_{10} - \frac{h^9 y_n^{(9)}}{9!\, x_{n+1}^9} q_1 - \frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^8} q_2 - \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^7} q_3 - \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^6} q_4 - \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^5} q_5 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_6 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_7 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_8 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_9 + p_{10}\right] \quad (14)$$

$$\frac{h^{11} y_n^{(11)}}{11!} = x_{n+1}^{11}\left[-p_0 q_{11} - \frac{h^{10} y_n^{(10)}}{10!\, x_{n+1}^{10}} q_1 - \frac{h^9 y_n^{(9)}}{9!\, x_{n+1}^9} q_2 - \frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^8} q_3 - \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^7} q_4 - \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^6} q_5 - \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^5} q_6 - \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^4} q_7 - \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^3} q_8 - \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^2} q_9 - \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_{10} + p_{11}\right] \quad (15)$$

This lead to a generalized equation

$$\frac{h^m y_n^{(m)}}{m!} = x_{n+1}^m \Bigg[-y_n q_i - \frac{h^{m-v}\, y_n^{(m-v)}}{(m-v)!\, x_{n+1}^{m-v}} q_{i-(m-1)} - \frac{h^{m-v+1}\, y_n^{(m-v+1)}}{(m-v+1)!\, x_{n+1}^{m-v+1}} q_{i-(m-2)} - \frac{h^{m-v+2}\, y_n^{(m-v+2)}}{(m-v+2)!\, x_{n+1}^{m-v+2}} q_{i-(m-3)}$$

$$- \frac{h^{m v+3}\, y_n^{(m-v+3)}}{(m-v+3)!\, x_{n+1}^{m-v+3}} q_{i-(m-4)} - \frac{h^{m-v+4}\, y_n^{(m-v+4)}}{(m-v+4)!\, x_{n+1}^{m-v+4}} q_{i-(m-5)} - \frac{h^{m-v+5}\, y_n^{(m-v+5)}}{(m-v+5)!\, x_{n+1}^{m-v+5}} q_{i-(m-6)} - \qquad (16)$$

$$\frac{h^{m-v+6}\, y_n^{(m-v+6)}}{(m-v+6)!\, x_{n+1}^{m-v+6}} q_{i-(m-7)} - \ldots\ldots\ldots\ldots - \frac{h^{m-v+r}\, y_n^{(m-v+r)}}{(m-v+r)!\, x_{n+1}^{m-v+r}} q_{i-(m-t)} + p_i \Bigg]$$

Hence equation (15) gives our general derivation method. And where $P_0=y_n$, v=1, m = i=1, 2, 3, . . . t =1, 2, 3, . . . and r =1, 2,3 . . . As the parameters of p_1, p_2, p_3, p_4, p_5, q_1, q_2, q_3, q_4, q_5 and q_6 cannot be easily obtained using equations (6) – (15) We therefore adopt the method of writing one result as a combination of others as was used by [6]. By writing (15)as a combination of equation (6) – (14) we get,

$$\frac{h^{10} y_n^{(10)}}{10!\, x_{n+1}^{10}} q_1 + \frac{h^9 y_n^{(9)}}{9!\, x_{n+1}^{9}} q_2 + \frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^{8}} q_3 + \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^{7}} q_4 + \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^{6}} q_5 + \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^{5}} q_6 = -\frac{h^{11} y_n^{(11)}}{11!\, x_{n+1}^{11}} \qquad (17)$$

$$\frac{h^9 y_n^{(9)}}{9!\, x_{n+1}^{9}} q_1 + \frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^{8}} q_2 + \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^{7}} q_3 + \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^{6}} q_4 + \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^{5}} q_5 + \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^{4}} q_6 = -\frac{h^{10} y_n^{(10)}}{10!\, x_{n+1}^{10}} \qquad (18)$$

$$\frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^{8}} q_1 + \frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^{7}} q_2 + \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^{6}} q_3 + \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^{5}} q_4 + \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^{4}} q_5 + \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^{3}} q_6 = -\frac{h^9 y_n^{(9)}}{9!\, x_{n+1}^{9}} \qquad (19)$$

$$\frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^{7}} q_1 + \frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^{6}} q_2 + \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^{5}} q_3 + \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^{4}} q_4 + \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^{3}} q_5 + \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^{2}} q_6 = -\frac{h^8 y_n^{(8)}}{8!\, x_{n+1}^{8}} \qquad (20)$$

$$\frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^{6}} q_1 + \frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^{5}} q_2 + \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^{4}} q_3 + \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^{3}} q_4 + \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^{2}} q_5 + \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_6 = -\frac{h^7 y_n^{(7)}}{7!\, x_{n+1}^{7}} \qquad (21)$$

$$\frac{h^5 y_n^{(5)}}{5!\, x_{n+1}^{5}} q_1 + \frac{h^4 y_n^{(4)}}{4!\, x_{n+1}^{4}} q_2 + \frac{h^3 y_n^{(3)}}{3!\, x_{n+1}^{3}} q_3 + \frac{h^2 y_n^{(2)}}{2!\, x_{n+1}^{2}} q_4 + \frac{h y_n^{(1)}}{1!\, x_{n+1}} q_5 + y_n q_6 = -\frac{h^6 y_n^{(6)}}{6!\, x_{n+1}^{6}} \qquad (22)$$

For each positive integer m, the expression $\frac{h^m y_n^m}{m!\, x_{n+1}^m}$ is a real number, for this reason equations (17) – (22) represent simultaneous Linear Equation (SLE) in q_1, q_2, q_3, q_4, q_5 and q_6. In the course of using the integrator 1.3 of higher values of k, the use of matrix equation gives rise to clearer solutions and make clearer the investigation of stability properties of the integrator, hence we put the simultaneous linear equation in matrix form as shown below:

$$\begin{bmatrix} \frac{h^{10} y_n^{(10)}}{10!x_{n+1}^{10}} & \frac{h^9 y_n^{(9)}}{9!x_{n+1}^{9}} & \frac{h^8 y_n^{(8)}}{8!x_{n+1}^{8}} & \frac{h^7 y_n^{(7)}}{7!x_{n+1}^{7}} & \frac{h^6 y_n^{(6)}}{6!x_{n+1}^{6}} & \frac{h^5 y_n^{(5)}}{5!x_{n+1}^{5}} \\ \frac{h^9 y_n^{(9)}}{9!x_{n+1}^{9}} & \frac{h^8 y_n^{(8)}}{8!x_{n+1}^{8}} & \frac{h^7 y_n^{(7)}}{7!x_{n+1}^{7}} & \frac{h^6 y_n^{(6)}}{6!x_{n+1}^{6}} & \frac{h^5 y_n^{(5)}}{5!x_{n+1}^{5}} & \frac{h^4 y_n^{(4)}}{4!x_{n+1}^{4}} \\ \frac{h^8 y_n^{(8)}}{8!x_{n+1}^{8}} & \frac{h^7 y_n^{(7)}}{7!x_{n+1}^{7}} & \frac{h^6 y_n^{(6)}}{6!x_{n+1}^{6}} & \frac{h^5 y_n^{(5)}}{5!x_{n+1}^{5}} & \frac{h^4 y_n^{(4)}}{4!x_{n+1}^{4}} & \frac{h^3 y_n^{(3)}}{3!x_{n+1}^{3}} \\ \frac{h^7 y_n^{(7)}}{7!x_{n+1}^{7}} & \frac{h^6 y_n^{(6)}}{6!x_{n+1}^{6}} & \frac{h^5 y_n^{(5)}}{5!x_{n+1}^{5}} & \frac{h^4 y_n^{(4)}}{4!x_{n+1}^{4}} & \frac{h^3 y_n^{(3)}}{3!x_{n+1}^{3}} & \frac{h^2 y_n^{(2)}}{2!x_{n+1}^{2}} \\ \frac{h^6 y_n^{(6)}}{6!x_{n+1}^{6}} & \frac{h^5 y_n^{(5)}}{5!x_{n+1}^{5}} & \frac{h^4 y_n^{(4)}}{4!x_{n+1}^{4}} & \frac{h^3 y_n^{(3)}}{3!x_{n+1}^{3}} & \frac{h^2 y_n^{(2)}}{2!x_{n+1}^{2}} & \frac{h y_n^{(1)}}{1!x_{n+1}} \\ \frac{h^5 y_n^{(5)}}{5!x_{n+1}^{5}} & \frac{h^4 y_n^{(4)}}{4!x_{n+1}^{4}} & \frac{h^3 y_n^{(3)}}{3!x_{n+1}^{3}} & \frac{h^2 y_n^{(2)}}{2!x_{n+1}^{2}} & \frac{h y_n^{(1)}}{1!x_{n+1}} & y_n \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} -\frac{h^{11} y_n^{(11)}}{11!x_{n+1}^{11}} \\ -\frac{h^{10} y_n^{(10)}}{10!x_{n+1}^{10}} \\ -\frac{h^9 y_n^{(9)}}{9!x_{n+1}^{9}} \\ -\frac{h^8 y_n^{(8)}}{8!x_{n+1}^{8}} \\ -\frac{h^7 y_n^{(7)}}{7!x_{n+1}^{7}} \\ -\frac{h^6 y_n^{(6)}}{6!x_{n+1}^{6}} \end{bmatrix} \qquad (23)$$

To solve for q_1, q_2, q_3, q_4, q_5 and q_6 we use the matrix equation (23) p_1, p_2, p_3, p_4, and p_5 can be obtained from the equations (4) – (9).That is,

$$P_1 = \frac{h\, y_n^{(1)}\, q_0}{1!\, x_{n+1}} + y_n\, q_1 \tag{24}$$

$$P_2 = \frac{h^2\, y_n^{(2)}}{2!\, x_{n+1}^2} + \frac{h\, y_n^{(1)}}{1!\, x_{n+1}}\, q_1 + y_n q_2 \tag{25}$$

$$P_3 = \frac{h^3\, y_n^{(3)}}{3!\, x_{n+1}^3} + \frac{h^2\, y_n^{(2)}}{2!\, x_{n+1}^2}\, q_1 + \frac{h\, y_n^{(1)}}{1!\, x_{n+1}} q_2 + y_n q_3 \tag{26}$$

$$P_4 = \frac{h^4\, y_n^{(4)}}{4!\, x_{n+1}^4} + \frac{h^3\, y_n^{(3)}}{3!\, x_{n+1}^3} q_1 + \frac{h^2\, y_n^{(2)}}{2!\, x_{n+1}^2}\, q_2 + \frac{h\, y_n^{(1)}}{1!\, x_{n+1}} q_3 + y_n q_4 \tag{27}$$

$$P_5 = \frac{h^5\, y_n^{(5)}}{5!\, x_{n+1}^5} + \frac{h^4\, y_n^{(4)}}{4!\, x_{n+1}^4} q_1 + \frac{h^3\, y_n^{(3)}}{3!\, x_{n+1}^3} q_2 + \frac{h^2\, y_n^{(2)}}{2!\, x_{n+1}^2}\, q_3 + \frac{h\, y_n^{(1)}}{1!\, x_{n+1}} q_4 + y_n q_5 \tag{28}$$

By adopting equations (24) – (28) produces the values of p_1, p_2, p_3, p_4 and p_5 when the values of q_1, q_2, q_3, q_4 and q_5 are substituted into it.

3. Stability of the Rational Interpolation Method

In this paper, we shall investigate the stability properties of the rational interpolation method for k = 6.

$$y_{n+1} = \frac{\sum_{i=0}^{5} P_i X_{n+1}^{i}}{1 + \sum_{i=1}^{6} q_i X_{n+1}^{i}} \tag{29}$$

However, the use of matrix equation (5) will give us investigative advantage to obtain results easily.

$$\begin{bmatrix}
\frac{\bar{h}^{10}}{10!\, x_{n+1}^{10}} & \frac{\bar{h}^{9}}{9!\, x_{n+1}^{9}} & \frac{\bar{h}^{8}}{8!\, x_{n+1}^{8}} & \frac{\bar{h}^{7}}{7!\, x_{n+1}^{7}} & \frac{\bar{h}^{6}}{6!\, x_{n+1}^{6}} & \frac{\bar{h}^{5}}{5!\, x_{n+1}^{5}} \\
\frac{\bar{h}^{9}}{9!\, x_{n+1}^{9}} & \frac{\bar{h}^{8}}{8!\, x_{n+1}^{8}} & \frac{\bar{h}^{7}}{7!\, x_{n+1}^{7}} & \frac{\bar{h}^{6}}{6!\, x_{n+1}^{6}} & \frac{\bar{h}^{5}}{5!\, x_{n+1}^{5}} & \frac{\bar{h}^{4}}{4!\, x_{n+1}^{4}} \\
\frac{\bar{h}^{8}}{8!\, x_{n+1}^{8}} & \frac{\bar{h}^{7}}{7!\, x_{n+1}^{7}} & \frac{\bar{h}^{6}}{6!\, x_{n+1}^{6}} & \frac{\bar{h}^{5}}{5!\, x_{n+1}^{5}} & \frac{\bar{h}^{4}}{4!\, x_{n+1}^{4}} & \frac{\bar{h}^{3}}{3!\, x_{n+1}^{3}} \\
\frac{\bar{h}^{7}}{7!\, x_{n+1}^{7}} & \frac{\bar{h}^{6}}{6!\, x_{n+1}^{6}} & \frac{\bar{h}^{5}}{5!\, x_{n+1}^{5}} & \frac{\bar{h}^{4}}{4!\, x_{n+1}^{4}} & \frac{\bar{h}^{3}}{3!\, x_{n+1}^{3}} & \frac{\bar{h}^{2}}{2!\, x_{n+1}^{2}} \\
\frac{\bar{h}^{6}}{6!\, x_{n+1}^{6}} & \frac{\bar{h}^{5}}{5!\, x_{n+1}^{5}} & \frac{\bar{h}^{4}}{4!\, x_{n+1}^{4}} & \frac{\bar{h}^{3}}{3!\, x_{n+1}^{3}} & \frac{\bar{h}^{2}}{2!\, x_{n+1}^{2}} & \frac{\bar{h}}{1!\, x_{n+1}} \\
\frac{\bar{h}^{5}}{5!\, x_{n+1}^{5}} & \frac{\bar{h}^{4}}{4!\, x_{n+1}^{4}} & \frac{\bar{h}^{3}}{3!\, x_{n+1}^{3}} & \frac{\bar{h}^{2}}{2!\, x_{n+1}^{2}} & \frac{\bar{h}}{1!\, x_{n+1}} & 1
\end{bmatrix}
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}
=
\begin{bmatrix}
\frac{\bar{h}^{11}}{11!\, x_{n+1}^{11}} \\
\frac{\bar{h}^{10}}{10!\, x_{n+1}^{10}} \\
\frac{\bar{h}^{9}}{9!\, x_{n+1}^{9}} \\
\frac{\bar{h}^{8}}{8!\, x_{n+1}^{8}} \\
\frac{\bar{h}^{7}}{7!\, x_{n+1}^{7}} \\
\frac{\bar{h}^{6}}{6!\, x_{n+1}^{6}}
\end{bmatrix} \tag{30}$$

When the integrator (29) is applied to $y^1 = \lambda y$ the equation (25) becomes:

where $\bar{h} = \lambda h$, the application of Gaussian elimination to (2) yields,

$$\begin{bmatrix} 1 & \frac{10X_{n+1}}{\bar{h}} & \frac{90X_{n+1}^{2}}{\bar{h}^{2}} & \frac{720X_{n+1}^{3}}{\bar{h}^{3}} & \frac{5040X_{n+1}^{4}}{\bar{h}^{4}} & \frac{302405X_{n+1}^{5}}{\bar{h}^{5}} \\ 0 & 1 & \frac{18X_{n+1}}{\bar{h}} & \frac{216X_{n+1}^{2}}{\bar{h}^{2}} & \frac{5016X_{n+1}^{3}}{\bar{h}^{3}} & \frac{15120X_{n+1}^{4}}{\bar{h}^{4}} \\ 0 & 0 & 1 & \frac{24X_{n+1}}{\bar{h}} & \frac{336X_{n+1}^{2}}{\bar{h}^{2}} & \frac{3360X_{n+1}^{3}}{\bar{h}^{3}} \\ 0 & 0 & 0 & 1 & \frac{28X_{n+1}}{\bar{h}} & \frac{420X_{n+1}^{2}}{\bar{h}^{2}} \\ 0 & 0 & 0 & 0 & 1 & \frac{30X_{n+1}}{\bar{h}} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} -\frac{10!\bar{h}}{11!X_{n+1}} \\ -\frac{9!\bar{h}^{2}}{11!X_{n+1}^{2}} \\ -\frac{8!\bar{h}^{3}}{11!X_{n+1}^{3}} \\ -\frac{7!\bar{h}^{4}}{11!X_{n+1}^{4}} \\ -\frac{6!\bar{h}^{5}}{11!X_{n+1}^{5}} \\ -\frac{5!\bar{h}^{6}}{11!X_{n+1}^{6}} \end{bmatrix} \quad (31)$$

4. The Computational Analysis ofthe Stability oftheRational Interpolation Method.

The computational analysis of the stability of the rational interpolation method for k=6 is stated in Tables 1 and 2.

5. Values of the Denominator

Equation (31) gives the values of the denominator (q) i.e. $\sum_{i=1}^{6} q_i X_{n+1}^{i}$ shown in Table 1.

Table 1: The stability function

$q_1(\bar{h})$	$q_2(\bar{h})$	$q_3(\bar{h})$	$q_4(\bar{h})$	$q_5(\bar{h})$	$q_6(\bar{h})$
$-\frac{3628200\bar{h}}{11!X_{n+1}}$	$-\frac{13426560\bar{h}^{2}}{11!X_{n+1}^{2}}$	$-\frac{2177280\bar{h}^{3}}{11!X_{n+1}^{3}}$	$-\frac{166320\bar{h}^{4}}{11!X_{n+1}^{4}}$	$-\frac{4320\bar{h}^{5}}{11!X_{n+1}^{5}}$	$-\frac{120\bar{h}^{6}}{11!X_{n+1}^{6}}$

6. Values of the Numerator

Substituting the values of q in table 1 above into equations (24) –(28), gives the values of numerator (p)i.e $\sum_{i=0}^{5} P_i X_{n+1}^{i}$ shown in Table 2.

Table 2: The stability function

$p_1(\bar{h})$	$p_2(\bar{h})$	$p_3(\bar{h})$	$p_4(\bar{h})$	$p_5(\bar{h})$
$\frac{36288600\bar{h}}{11!X_{n+1}}$	$\frac{29756760\bar{h}^{2}}{11!X_{n+1}^{2}}$	$\frac{16088060\bar{h}^{3}}{11!X_{n+1}^{3}}$	$\frac{5670820\bar{h}^{4}}{11!X_{n+1}^{4}}$	$\frac{1492585\bar{h}^{5}}{11!X_{n+1}^{5}}$

Finally, if we apply the results in tables 1 and 2 to the integrator (9). We obtain the stability root given by

$$S(\bar{h}) = \frac{P_0 + P_1x_{n+1} + P_2x_{n+1}^{2} + P_3x_{n+1}^{3} + P_4x_{n+1}^{4} + P_5x_{n+1}^{5}}{1 + q_1x_{n+1} + q_2x_{n+1}^{2} + q_3x_{n+1}^{3} + q_4x_{n+1}^{4} + q_5x_{n+1}^{5} + q_6x_{n+1}^{6}}$$

$$S(\bar{h}) = \frac{3991680 + 36288600\bar{h} + 29756760\bar{h}^{2} + 1608806\bar{h}^{3} + 567020\bar{h}^{4} + 1492585\bar{h}^{5}}{3991680 - 3628200\bar{h} + 13426560\bar{h}^{2} - 2177280\bar{h}^{3} + 166320\bar{h}^{4} - 4320\bar{h}^{5} + 120\bar{h}^{6}} \quad (32)$$

7. Conclusion

In conclusion, from the result of the analysis of the stability above,we discover that by direct proof and definitions equation (32) is A-stable and L-stable. For L-stable the Region of the Absolute Stability (RAS) together with the point of encroachment into the right – half of the complex plane.

References

[1] Aashikpelokhai U.S.U (1991): “A Class of non-linear one- step rational integrator”, Pon PublishersLimited Agbor, Delta State.

[2] Aashikpelokhai U.S.U (2000): “A variable order numerical integration based on rational interpolants”. Journalof the Nigerian Mathematical Society Vol. 19, pg. 27-38.

[3] Aashikpelokhai U.S.U and Momodu I.B.A (2006): “A high order rational integrator for stiff systems” Journal of Nigeria Annuals of Natural Sciences pg. 122 – 143, vol. 4(1).

[4] Abhulimen .C.E., Otunta F.O.(2007): “A family of two-step exponentially fitted multiderivative methods for the Numerical Integration of stiff initial value problems in Ordinary Differential Equations”. Numerical Mathematics (1JNM);2(1):1-121.

[5] Corless R.M (2001): “A new view of the computational complexity of IVP for ODE”, Ontario Research Centre for Computer Algebra, University of Ontario. Cryer C.W (1973): “A class of high stable method: A-stable method”. BIT 13, 153 – 159.

[6] Fatunla, S.O. (1978); “An implicit Two-part numerical integration Formula for linear and non-linear stiff system of ODE”, Mathematics of Computation 32,1-11.

[7] Fatunla,S.O.(1980): “Numerical integrators for stiff and highly oscillatory differential equation”. Mathematics of Computation 34,373-390.

[8] Fatunla,S.O. (1988); “Numerical methods for initial value problems in Ordinary Differential Equations”. Academic Press, IN INC USA.

[9] Hall G. and Watt J.M. (1976): “Modern Numerical Methods for ODEs.Claredon Press,Oxford.

[10] Lambert J.D and Shaw B. (1965): “On the Numerical Solution of y' = f(x,y) by a class of formulae based on Rational Approximation”. Mathematics of Computation19:456-462

[11] Lambert J.D. (1976) “Two unconventional class of methods for stiff System”. (Ed-Willoughby,R.A.) Wildbald, W. Germany, 171-186.

[12] Lambert J.D. (1978) “Numerical Methods for Ordinary Differential Systems, John Wiley and Son, Chichester”.

[13] Lambert J.D. (1973); “Computational Methods in ODE’s John Wiley of sons, New York.

[14] Lambert J.D. (2000) “Numerical Methods for Ordinary Differential Systems, John Wiley and Son, Chichester”.

[15] Niekerk,V.F.D. (1985): “Non-linear and one-step methods of IVP”.Computers and mathematics with application, 3,367-371

[16] Nunn D. and Huang A.(2005): “QUAM-A novel algorithm, for the numerical integration of stiff ordinary Differential equations”.

[17] Norelli,F. (1985): “High-Accuracy explicit Spectral Solver for Stiff,non-Stiff and highly Oscillatory Differential Systems’’. ENEA-Departments reatitori, veloci, ceratroricercheenerglaCasacci 1-62

[18] Vigo-Angular, Martin Jv(2006):Exponential fitting BDF algorithms Explicit and Implicit 0-stable methods. Computation and Applied Mathematics;192-100-113