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Existence, Enumeration and Structure Of The Combinatorial Algorithms For *Communication Problems*

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Abstract

In this paper, we address the problem of existence, enumeration and structure of the combinatorial elements in telephone number plan and Internet Addresses for communication subscribers. We propose a novel approach for this type of communication problems based on the generation algorithm produces for a list of all the combinatorial structures of a particular type in a certain order, such as lexicographic or minimal change order. However, in enumeration, we compute the number of different elements such that, the communication problem of large numbers of subscribers which has resulted to insecurity, interruption and congestion in the system be given an adequate look.

Keywords: *k- inclusion, k-non-inclusion, r-arrangement, combinatorial structures, telephone number plan, Internet Address plan*

1.0 Introduction

Combinatorial generation has a long and distinguished history. In fact, the exhaustive listing of combinatorial elements was one of the first nontrivial problems to be tackled by computer [1]. This paper briefly reviews the relevant literature and current state of knowledge pertaining to the proposed communication problem. It also describes the basic terminologies that are helpful in analyzing and understanding the combinatorial algorithms.

Combinatorial algorithms scrutinize, examine and investigate combinatorial elements such as permutations, combinations, set partitions, numerical partitions, binary trees and graphs. They determine solutions to the fundamental questions byexamining the existence, enumeration and structure of the combinatorial element. Informally, combinatorial algorithms are classified into three categories: search, generation, and enumeration. The search algorithm, as the name specifies searches for a specific element in a combinatorial class. The generation algorithm produces a list of all the combinatorial structures of a particular type in a certain order, such as lexicographic or minimal change order. However, in enumeration, we compute the number of different elements of a certain type. Every generation algorithm is essentially an enumeration algorithm, since each element is counted as it is generated.

2. Ordering in Combinatorial Algorithms

Combinatorial generation algorithms produce the list of elements in some specific order. There are two types of orderings that are mostly used: lexicographic order, and minimal change order [2, 3].

2.1 Lexicographic Order

The lexicographic order is a natural way of listing the combinatorial element. Formally, it can be defined as an increasing numeric order; i.e. $a_1 < a_2 < a_3 < \cdots < a_n < b_1 < b_2 < b_3 < \cdots < b_m$, if
1. for some k,$a_k < b_k$and $a_i = b_i \, for \;\; \mathrm{i} \; = \; 1, \; 2, \; \ldots, \; \mathrm{k} - 1$, or
2. $n < m$and$a_i = b_i \, for \;\; \mathrm{i} \; = \; 1, \; 2, \; \cdots \; , \; \mathrm{n}$
where< denote the ordering of the symbols of the alphabets [2, 3]. For permutations, lexicographic order is simply the alphabetic order for lists of symbols. For example, the permutations of {a,b,c} in lexicographic order are abc, acb, bac, bca, cab, and cba. In the lexicographic ordering of subsets, two subsets are ordered by their smallest elements. For example, the

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subsets of$\{a, b, c\}$in lexicographic order are { }, {a}, {a, b}, {a, b, c}, {a, c}, {b}, {b, c}, {c}. The lexicographic order is sometimes called dictionary order. It is easy to comprehend, and proves quite handy in different situations. Many fastest known combinatorial algorithms list the elements in lexicographical order; they exploit the underlying lexicographic structure of the element to achieve efficiency.

2.2 Minimal Change Order

The minimal change order, is an ordering in which consecutive elements differ slightly in some pre-specified pattern. Lists of combinatorial elements in minimal change order are often called gray codes. Savage has conducted a comprehensive survey on the combinatorial gray codes [4]. A classic example of gray codes is binary reflected codes. In this method, we generate all binary numbers of particular bit- length such that consecutive numbers differ by only one bit. The listL(i) of binary reflected codes, where i is 1,2,3 bit numbers is as follows:

L(1) = 0 = 1

L(2) = 00 = 01 = 11 = 10

L(3) = 000 = 001 = 011 = 010 = 110 = 111 = 101 = 100

Variable length codes can provide better compression than fixed length as can be seen on the tables below.

a	B	R	a	c	a	d	a	b	r	A	I
1100001	1100010	1110010	1100001	1100011	1100001	1100100	1100001	1100010	1110010	1100001	1111111

a	b	R	a	C	a	d	a	b	r	a	i
000	001	100	000	010	000	011	000	001	100	000	111

a|b|r|a|c|a|d|a|b|r|a|i
01111100101001000111111001011

Every type defines a variable length code and the best of such variable length code for a given message is called the Huffman code named after David Huffman in 1950.Gray codes have various applications and are deployed in areas such as image processing [5], data compression [6], circuit testing (Robinson and Cohn 1981), and combinatorial games and puzzles [7].

2.3 Revolving Door Order

This is a minimal change ordering in which two consecutive elements have distance two. That is, they differ in exactly two positions [7]. The terminology is based on the revolving door that connects two rooms and is used to swap two persons between the rooms.

3.1 The telephone number plan [8]

The North American numbering plan (NANP) specifies the format of telephone numbers in the U.S., Canada, and many other parts of North America. A telephone number in this plan consists of 10 digits, which are split into a three-digit area code, a three-digit office code, and a four-digit station code. Because of signaling considerations, there are certain restrictions on some of these digits. To specify the allowable format, let X denote a digit that can take any of the values 0 through 9, let N denote a digit that can take any of the values 2 through 9, and let Y denote a digit that must be a 0 or a 1. Two numbering plans, which will be called the old plan, and the new plan, will be discussed. (The old plan, in use in the 1960s, has been replaced by the new plan, but the recent rapid growth in demand for new numbers for mobile phones and devices will eventually make even this new plan obsolete. In this plan, the letters used to represent digits follow the conventions of the North American Numbering Plan.) As will be shown, the new plan allows the use of more numbers. In the old plan, the formats of the area code, office code, and station code are NYX, NNX, and XXXX, respectively, so that telephone numbers had the form NYX-NNX-XXXX. In the new plan, the formats of these codes are NXX, NXX, and XXXX, respectively, so that telephone numbers have the form NXX-NXX-XXXX.

3.2 Counting Internet Addresses [2,8]

In the Internet, which is made up of interconnected physical networks of computers, each computer (or more precisely, each network connection of a computer) is assigned an Internet address.

In Version 4 of the Internet Protocol (IPv4), now in use, an address is a string of 32 bits. It begins with a network number (netid). The netid is followed by a host number (hostid), which identifies a computer as a member of a particular network. Three forms of addresses are used, with different numbers of bits used for netids and hostids.

Class A addresses, used for the largest networks, consist of 0, followed by a 7-bit netid and a 24-bit hostid.

Class B addresses, used for medium-sized networks, consist of 10, followed by a 14-bit netid and a 16-bit hostid.

Class C addresses, used for the smallest networks, consist of 110, followed by a 21-bit netid and an 8-bit hostid.

There are several restrictions on addresses because of special uses: 1111111 is not available as the netid of a Class A network, and the hostids consisting of all 0s and all 1s are not available for use in any network. A computer on the Internet has either a Class A, a Class B, or a Class C address. (Besides Class A, B, and C addresses, there are also Class D addresses, reserved for use in multi casting when multiple computers are addressed at a single time, consisting of 1110 followed by 28 bits, and Class E addresses, reserved for future use, consisting of 11110 followed by 27 bits. Neither Class D nor Class E addresses are assigned as the IPv4 address of a computer on the Internet.) (Limitations on the number of Class A and Class B netids have made IPv4 addressing inadequate; IPv6, a new version of IP, uses 128-bit addresses to solve this problem.)

We shall consider various arrangements of the elements in a set X of cardinality n and a subset K of special interest of cardinality k of X to apply mathematical formulae [11, 12, 13, 14, 15] that gives the exact number for the telephone number plan and Internet Addresses for communication of subscribers. If n is small (2,3,or 4) it is easy to exhaustively list and count all the possible outcomes for either the inclusion or non-inclusion cases. In addition to the conditions on either the inclusion case or the non-inclusion case, we require that the k-elements of the subset K, should be either together or not together in each r-arrangement for n distinct elements of the set X. This application of r-arrangements differ in different areas.

3.3 Global System for Mobile Communication

As at May 2005, Nigeria with an estimated population of 139,661,303 [6] had more than 9 million Global System for Mobile Communication (GSM) subscribers, making the country one of the fastest growing GSM markets in the world ([1],[10]). In 2017 GSM in Nigeria has four major companies - MTN Nigeria Communications Limited, Airtel Networks Limited, Emerging Market Telecommunications Services Limited (Etisalat) and Globacom Limited. The limited facility vis-a-vis the large number of subscribers has resulted in insecurity, interruption and congestion in the system [6]. Many people are attracted to GSM because of its mobility features. GSM is now a means of livelihood for many people as more individuals are engaged in phone-related businesses. Another thing that attracts many subscribers is the marketing strategies of the operators and competition to get many subscribers, even though their infrastructures cannot sustain them. Some operators complain that they pay an exorbitant fee to obtain licenses for operation in Nigeria, so they have to get as many subscribers as possible for them to recover their money. All of these factors have led to insecurity, interruption and congestion on the Nigerian GSM network ([1], [10]).

The works of the following authors [11, 12, 13, 14, 15] address permutation and combination cases for the following scenarios; k-separable inclusion, k-separable non-inclusion, k-inseparable inclusion, k-inseparable non- inclusion and k-separable non- inclusion and m- non- inclusion on a line. They results shall play a very important role inthe GSM scenario and several other related cases, we relate the so called k-inclusion and k-non-inclusion condition to the GSM and similar problems as follows.

4.0 Applications

Suppose that K is a special subset of interest of set X and the GSM network also classifies the lines into hot lines, priority 1 lines, priority 2 lines, priority 3 lines and other lines. How many eleven digit unique identity lines can be generated from the string (set) X= {a, b, c, d, e, f, g, h, i, j, k, l, m} without repetition if;

i. There are no strings of special interest.
ii. K = {b, d, g, h, k} must be included and always not together in each arrangement is considered hot lines.
iii. K = {b, d, g, h, k} are separately and partially contained in each arrangement is considered priority 1 lines.
iv. K = {b, d, g, h, k} must be included and always together in each arrangement is considered priority2 lines.
v. K = {b, d, g, h, k} are always together and partially contained in each arrangement is considered priority 3 lines.

Solution

i. There are no strings of special interest.
We have that n = 13 r = 11
$P_{(13,11)} = \frac{13!}{2!} = 3{,}113{,}510{,}400$number of lines

ii. K = {b, d, g, h, k} must be included and always not together in each arrangement is considered hot lines.
We have that n = 13 r = 11 k = 5

$$P_{si(n,r,k)} = \frac{(n-k)!}{(n-r)!}\frac{(r-k+1)!}{(r-2k+1)!} = 50{,}803{,}200\text{number of lines}$$

iii. K = {b, d, g, h, k} are separately and partially contained in each arrangement is considered priority 1 lines.
We have that n = 13 , r = 11, k = 5

$$P_{sni(n,r,k)} = \sum_{i-3}^{4} \frac{(n-k)!}{(i)!(n-k-r+i)!}\frac{(r-i+1)!}{(r-2i+1)!}\frac{(k)!}{(k-i)!} = 1{,}557{,}964{,}800\text{number of lines}$$

iv. K = {b, d, g, h, k} must be included and always together in each arrangement is considered priority 2 lines. We have that n = 13 , r = 11, k = 5

$P_{isi(n,r,k)} = \frac{(n-k)!}{(n-r)!}\frac{(r-k+1)!k!}{(r-k)!} = 16{,}934{,}400$number of lines

v. K = {b, d, g, h, k} are always together and partially contained in each arrangement is considered priority 3 lines. We have that n = 13 , r = 11, k = 5

$P_{isni(n,r,k)} = \sum_{i-3}^{4} \frac{(n-k)!}{(n-k-r+i)!}\frac{(r-i+1)!}{(r-i)!}\frac{(k)!}{(k-i)!} = 48{,}988{,}800$number of lines

The GSM network should consider hot lines, priority 1 lines, priority 2 lines and priority 3 lines before other lines to solve insecurity, interruption and congestion problems for her V.I.P. lines on the Nigerian GSM network.

Conclusion

This paper briefly reviews the combinatorial structures and current state of knowledge pertaining to the proposed communication problem. In enumeration, we have computed the number of lines for different subscribers such that, the communication problem of large numbers of subscribers which has resulted to insecurity, interruption and congestion is now subdivided into hot lines, priority 1 lines, priority 2 lines and priority 3 lines. Thus, with this classifications, the subscribers can now enjoy better services and less insecurity, interruption and congestion problems posed to the communication network in general.

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