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Three Error Terms in Modeling Growth Domestic Product in Nigeria

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Abstract

This study investigate three (3) error terms distribution in modeling the Nigerian Growth Domestic Product (GDP). Despite the economy trend in the nation, the study observed the path which the GDP trend on. This research investigate three conditional distribution of the error terms using the principle of parsimony of ARCH (1) and GARCH (1,1) models, with the methodology of unit root, ARCH Effect, Estimation of parameter, models Selection and Forecasting Evaluation using RMSE and MAE. From the results, the return data was stationary, there was presence of ARCH Effect, most of the parameter of the ARCH (1) model and GARCH (1,1) model were significant. In selecting the best fitted models in terms of error term, the AIC and the SIC shows that the GED is the best conditional distribution error term while the GARCH model normal distribution and the GED as the conditional distribution. Forecasting evaluation shows that irrespective of the conditional distribution error term both models are good forecasting model..

Keywords: Error Distribution, ARCH and GARCH, Forecasting Evaluation, Unit root, ARCH Effect

1.0 Introduction

In this research, we survey the ARCH and GARCH parsimonies modeling with special focus on the error term for fitted models to financial return series and this model is to examine with three different distributional assumptions innovations: Gaussian (normal) distribution, Student-t distribution and GED (Generalized Error Distribution). Both the Student-t distribution and the GED have fat tails. The maximum-likelihood approach is used for the parameter estimation.

Estimating any heteroscedastic model implies that we are actually estimating the residual obtained from the mean equation. Therefore, the Nigeria Growth Domestic Product is of vital important in modeling and by evaluating the macroeconomic under different assumption makes that series data to be more robust in term of accurate and precision of what direction or what distribution best fit the series data.

The autoregressive conditional heteroscedasticity (ARCH) model for modeling the changing variance of a time series was first proposed in [1]; he used an ARCH model to study inflation in the United Kingdom under the assumption that the error term followed a normal distribution. An GARCH model with a small number of terms may be more efficient than an ARCH model with many terms under same error term [2]. Empirical studies in recent years have focused on volatility investigation on the pattern of financial assets such as ARCH effect, volatility clustering, and persistence and leverage effect on just one common distribution –*Normal*.

The time series beaviour of daily stock returns of four firms listed in the Nigerian Stock Market from 2nd January, 2002 to 31st December, 2006, using three different models of heteroscedastic processes was investigates [3]. The model adopted was estimated assuming a Gaussian (normal) distribution.

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Stock market volatility of financial return series for three listed equities was examined and modeled on the Ghana Stock Exchange (GSE) [4]. They fitted a GARCH (p, q) model for volatility. GARCH (1, 1), GARCH (1, 2), GARCH (2, 1) and the GARCH (2, 2) models were fitted to residual series of some three equities and where the estimation is tend to follow a normal distribution.

The model built under the skewed-type GARCH models where three assumption of distribution in GARCH model are Normal Distribution, Student-t Distribution, Generalized Error Distribution and there skewed version were compare and fitted to find the best fit [5]. However, in its result shows know a big different between the Skewed Student-t Distribution and Skewed Generalized Error Distribution, one reason of that is both of them have shape and skew parameters using the AIC selection criteria

Behavior of stock returns volatility in both developed and emerging stock markets where he investigates the behavior of stock return volatility of the Nigerian Stock Exchange returns using GARCH (1,1) and the GJR-GARCH(1,1) models assuming the Generalized Error Distribution (GED) was study [6]. The results of GARCH (1,1) model indicate evidence of volatility clustering in the NSE return series. Also, the results of the GJR-GARCH (1,1) model show the existence of leverage effects in the series. Finally, the Generalized Error Distribution (GED) shape test reveals leptokurtic returns distribution.

2. Methodology

Data collection

Data for this study were yearly dataset of the Nigeria Growth Domestic Product traded on the floor of the Nigerian Central Bank of Nigeria (CBN)**.**

2.1 Return series from price

Let $$PS_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \qquad (1)$$

where P_t and P_{t-1} are the present and previous closing prices and PS_t the continuously compounded return series which is the natural logarithm of the simple gross return.

2.2 Unit root test

Stationarity of the return series is one of the major assumptions in financial time series modelling. This assumption can be checked using the unit root, an **Augmented Dickey–Fuller test (ADF)** is a test for a unit root in a time series sample [7]

Let $x_t = \phi_1 x_{t-1}$

$$x_t - x_{t-1} = \phi_1 x_t - x_{t-1}$$

$$\Delta x_t = (\phi_i - 1)x_{t-1}$$

$$\Rightarrow \phi_1 - 1 = 0 \text{ Or } \phi_1 = 1$$

Null hypothesis is $H_0 : \phi_1 = 1$; alternative hypothesis is: $H_1 : \phi_1 < 1$

The Test Statistic (t-ratio): $$= \frac{\phi_1^n - 1}{std(\phi_1)} = \frac{\sum_{t=1}^{S} P_{t-1}e_t}{\sigma^2\sqrt{\sum_{t=1}^{S} P_{t-1}^2}} \qquad (2)$$

Where $$\phi_1 = \frac{\sum_{t=1}^{S} p_{t-1}p_t}{\sum_{t=1}^{S} p_{t-1}^2} \text{ and } \hat{\sigma}^2 = \frac{\sum_{t=1}^{T}(p_t - \hat{\phi}_1 p_{t-1})^2}{S-1}$$

$P_0 = 0$, S is the sample size and ϕ_1 for each factors.

The null hypothesis is rejected if the calculated value of t is greater than t critical value [8]

2.3 Arch Effect

To test for the presence of heteroscedasticity in residuals of Nigerian Insurance stock return series, the Lagrange Multiplier (LM) test for ARCH effects proposed in [1] is applied. The test procedure is performed by first obtaining the residuals e_t from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes; (ARMA) process

2.4 Selection Criteria

[9,10] are the most commonly used model selection criteria

$$\text{AIC} = 2K - 2\ln(L) = 2K + \ln\left(\frac{RSS}{n}\right) \tag{3}$$

[10] is given as

$$SIC = K\log n - 2\ln(LL) = K\log n + \ln\left(\frac{RSS}{n}\right) \tag{4}$$

where k is the number of parameters in the model and L is the maximized value of the likelihood function for the model and $RSS = \sum e^2$ is the residual sum of squares.

2.5 ARCH and GARCH Models

The ARCH (q) model proposed in [1] formulates volatility as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 + r_t \tag{5}$$

where $\alpha_i > 0,$ for i=0, 1, 2… q are the parameters of the models

The GARCH (p, q) model was stated as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 + \beta_1\varepsilon_{t-1}^2 + \ldots + \beta_p\varepsilon_{t-p}^2 + r_t \tag{6}$$

where $\alpha_i > 0$ and $\beta_i > 0$ for all i and j

2.6 Forecasting Evaluation

The most widely used evaluation measures are Mean Square Error (MSE), Mean Absolute Error (MAE) expressed as:

$$MAE = \sum_{t=T+1}^{T+K} \frac{\left|\hat{\sigma}^2{}_t - \sigma^2{}_t\right|}{W} \tag{7}$$

Root Mean Square Error (MSE) given by,

$$\text{RMSE} = \sqrt{\frac{\sum_{t=T+1}^{T+K}\left(\hat{\sigma}^2{}_t - \sigma^2{}_t\right)^2}{W}} \tag{8}$$

Where, W is the number of steps ahead, T is the sample size, $\hat{\sigma}_t$ and σ_t are the square root of the conditional forecasted volatility and the realized volatility respectively.

2.7 Commonly used conditional distribution assumptions

Normal Distribution

Normal distribution is a symmetric distribution with density function:

$$f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}}\exp^{-\left(\frac{x_i-\mu}{2\sigma}\right)}, x > 0, \tag{9}$$

where mean is zero (0) with constant variance, thus $X \sim N(0, \sigma_t^2)$. The so-called standard normal, ε_t has the conditional variance σ_t^2, the joint likelihood of realization ε_t through ε_p is given as

$$L = \prod_{t=1}^{p}\left(\sqrt{\frac{1}{2\pi\sigma_t^2}}\right)\exp\left\{-\frac{\varepsilon_t^2}{2\sigma_t^2}\right\} \tag{10}$$

Student-t Distribution

The conditional log likelihood function of $\underline{\alpha}$ with a pre-specified ν degrees of freedom can be expressed as

$$\ell(\underline{\alpha}) = -\sum_{t=m+1}^{n}\left[\frac{\nu+1}{2}\ell n\left(1+\frac{a_t^2}{(\nu-2)\sigma_t^2}\right)+\frac{1}{2}\ell n(\sigma_t^2)\right]. \tag{11}$$

with $\upsilon > 2$ degrees of freedom

Generalized Error Distribution (GED),

The Generalized Error Distribution (GED) is a symmetric distribution that can be both leptokurtic and platykurtic depending on the degree of freedom r (r>1). The GED includes the Normal distribution as a special case, along with many other distributions, some more fat tailed than the Normal. ε_t Follow a standardized GED, the log likelihood function can be derived as:

$$l_t = -\frac{1}{2}\log\left(\frac{\Gamma\left(\frac{1}{r}\right)^3}{\Gamma\left(\frac{3}{r}\right)\left(\frac{r}{2}\right)^2}\right) - \frac{1}{2}\log\sigma_t^2 - \left(\frac{\Gamma\left(\frac{3}{r}\right)\left(y_t - X_t^{'}\theta\right)^2}{\sigma_t^2\Gamma\left(\frac{1}{r}\right)}\right)^{\frac{r}{2}} \tag{12}$$

r>0. The GED is a normal distribution, if r=2

3.0 Results

Table 1: shows the Descriptive Statistic of GDP

	DESCRIPTIVE
Mean	0.016379
Median	0.017369
Maximum	0.124729
Minimum	-0.086156
Std. Dev.	0.033193
Skewness	-0.142077
Kurtosis	5.263813
Jarque-Bera	16.26748
Probability	0.000293
Sum	1.228422
Sum Sq. Dev.	0.081529
Observations	75

Table 1, shows the descriptive statistic were the return of the mean is positive, that the series is negative skewed and the result of the normality shows that the series is not normally distributed.

Unit root

Table 2: shows the ADF of GDP

Null Hypothesis: GDP has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic – based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.563084	0.0000
Test critical values:	1% level	-3.522887	
	5% level	-2.901779	
	10% level	-2.588280	

Table 2 shows the result of the test for unit root of the GDP which from the evaluation statistic shows that the series is stationary and comparing it with graphical presentation also shows that the series is stationary as display in appendix A below.

Test of ARCH Effect

Table 3: ARCH Effect Test

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	4.402749	Prob. F(2,72)	0.0157
Obs*R-squared	8.172863	Prob. Chi-Square(2)	0.0168

Table 3 shows that result of the test of heteroscedastic model were the probability value is less that 5% Confident interval. Therefore we conclude that there is presence of ARCH Effect and continuous with the heteroscedastic estimation of the models.

Table 4: Parameter Estimation of ARCH (1) and GARCH (1, 1) Models

Models	*Models Selection*	*Conditional Distribution of the Error Terms*		
		Normal	**Student's t**	**GED**
ARCH (1)	Mean Eq			
	C	0.0161**	0.0170**	0.0174**
	Variance Eqn			
	C	0.0016**	0.0014	0.0011**
	A	-0.0865**	-0.0848	-0.0736
GARCH (1, 1)	Mean Eq			
	C	0.0169**	0.0162**	0.0165**
	Variance Eqn			
	C	0.0005**	0.0048**	0.0005**
	A	-0.1234**	-0.1169**	-0.1133**
	B	1.1096**	1.1018**	1.1030*

** at 0.01 and * at 0.05

Table 4 shows the results of the obtained ARCH and GARCH models under different conditional distribution error terms which estimate the parameters of the models and from the results analysis shows that most of the parameter is significant at 1%.

Table 5: Results of Model selection criteria

Models	*Models Selection*	*Conditional Distribution of the Error Terms*		
		Normal	**Student's t**	**GED**
ARCH (1)	AIC	-3.9657	-4.0559	-4.0736
	SIC	-3.8730	-3.9323	-3.9490
GARCH (1, 1)	AIC	-4.1651	-4.1484	-4.1662
	SIC	-4.0415	-3.9937	-4.0117

Table 5 shows the selection criteria under different conditional distribution assumptions of ARCH (1) and GARCH (1, 1) models. Using the AIC and SIC, the ARCH (1) model of the GED is considered the best since the AIC has the smallest value than the others distributions. While the GARCH (1, 1) model selects two distribution, the normal and the GED where the AIC GED and the SIC Normal conditional distribution.

Table 6: Results of Forecasting Evaluation

Models	*Forecasting Evaluation*	*Conditional Distribution of the Error Terms*		
		Normal	**Student's t**	**GED**
ARCH (1)	RMSE	0.0330	0.0330	0.0330
	MAE	0.0230	0.0230	0.0230
GARCH (1, 1)	RMSE	0.0330	0.0330	0.0330
	MAE	0.0230	0.0230	0.0230

Table 6 shows the forecasting evaluation of the Nigeria Growth Domestic Product under different conditional distributions assumption. From the results obtained, thrown more light that the GDP in Nigeria can follow any of these three error distribution in forecasting the Nation Growth Domestic Product (GDP) because the test statistics for both RMSE and MAE yielded the same values.

A: Stationarity plot of GDP

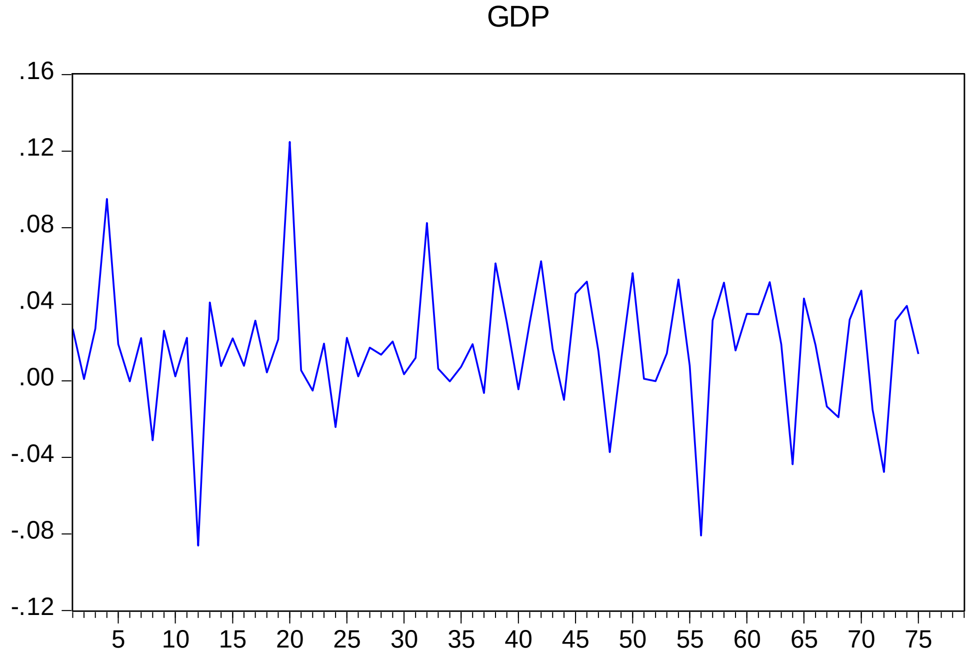

Figure 1: Stationary plot

4.0 Conclusion

The descriptive statistic where the return of the mean is positive, shows that the series is negative skewed and the result of the normality shows that the series is not normally distributed and the series is stationary and also the series shows evidence of ARCH Effect. The estimation of the ARCH and GARCH parameters are all significant at 5%CI. The selection criteria are under different conditional distribution assumptions of ARCH (1) and GARCH (1, 1) models. Using the AIC and SIC, the ARCH (1) model of the GED is considered the best since the AIC has the smallest value than the other distributions. While the GARCH (1, 1) model selects two distributions, the normal and the GED, where the AIC is GED and the SIC Normal conditional distribution. The results obtained, throws more light that the GDP in Nigeria can follow any of these three error distribution in forecasting the Nation Growth Domestic Product (GDP) because the test statistics for both RMSE and MAE yielded the same values.

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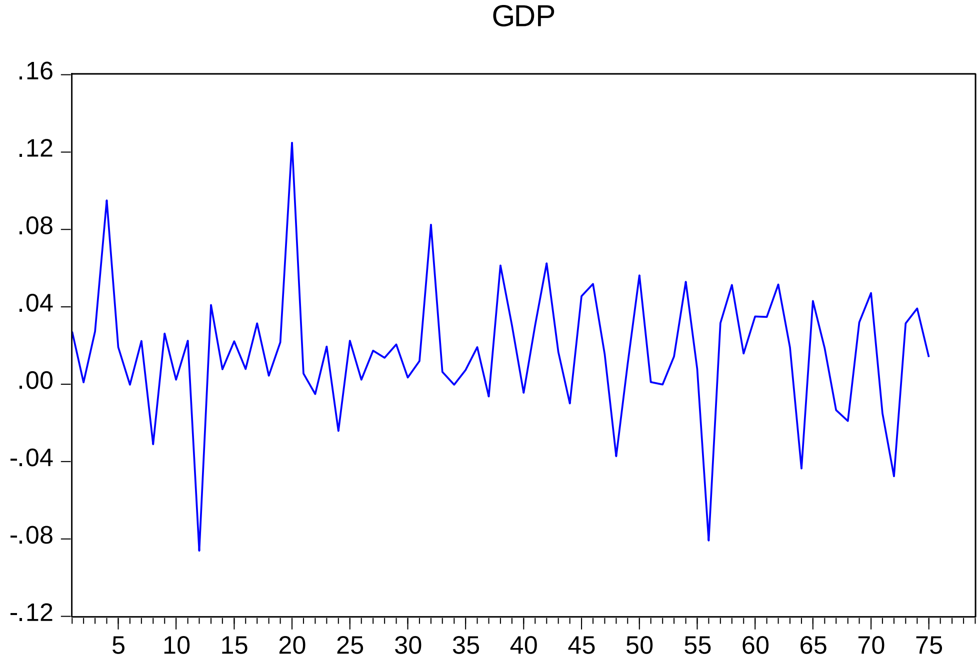

Figure 2: Stationary plot

5.0 References

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