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Steady State Free Convection Hydro magnetic Flows of Viscous Fluid with Convective Surface Boundary Condition

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Abstract

The problem of steady state free convection hydro magnetic flow of viscous fluid with convective boundary condition has been studied. The model governing equations are solved by using perturbation method. The results show that, the maximum flow velocity and temperature are recorded at the lower plate by increasing the symmetric wall temperature while opposite phenomenon is observed at the upper plate. the parameters such as ambient temperature parameter magnetic parameter, biot number as well as convective heat transfer parameters has an effects on temperature and velocity.

Keywords: Heat and mass transfer, mixed convection, perturbation method, convective boundaries, vertical channel.

NOMENCLATURE

The different parameters that govern the flow in dimension form are as follows:

Bi1 convective heat transfer parameter at y = 0
Bi2 convective heat transfer parameter at y = 1
B_0: Strength of magnetic field
C: Concentration
Rt ambient temperature parameter
g: Gravitation acceleration
Gr: Grashof Number based on temperature
M: Magnetic parameter
Pr: Prandtl number

1. Introduction

Studies pertaining to fluid flow in a porous pipe or channel have received much attention of several researchers in recent years due to their applications in technological as well as biological flows, with a view to understand some practical phenomena such as transpiration cooling, gaseous diffusion, circulatory system and respiratory system [1] (Particularly, the pulsatile flow in a porous channel is important to understand the process of dialysis of blood in an artificial kidney [2] .Heat transfer to pulsatile flow in a porous channel has been investigated in [3]. In this investigation, the walls were maintained at uniform temperatures and fluid was injected through one wall and removed at opposite wall at the same rate.

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The convective boundary condition is more general and pragmatics, especially with respect to several engineering and industrial process like transpiration cooling process and material drying Makinde *et al.* [4 - 5], and several references therein obtained numerical solutions for the unsteady hydro magnetic generalized coquette flow of a reactive third-grade fluid with asymmetric convective cooling. The Study of MHD Eyring Prandtl fluid with convective boundary conditions in small intestine was carried out in [6]. Optimized analytical solution for oblique flow of a Casson-nanofluid with convective boundary conditions [7]. Discussion of the influence of free convection slip flow of an exothermic fluid in a vertical channel as well as the study of flow of fully developed mixed convection in a vertical channel with chemical reaction by saleh *et al.* [8]. Presentation of a paper to exploit the advantage of obtaining enhanced mass transfer rates in electrochemical processes [9]. Analyzation of heat and mass transfer effects on unsteady free convection flow near a moving vertical plate in porous medium [10].

2. Mathematical Analysis

A number researches is conducted and several papers has been published on MHD convective fluid flow due to linearize and non-linearism Rosseland approximation among of which are pioneer works of Mansur *et al*. [11] as well as Jha *et al.* [12]. However their research works did not consider the flow formation of hydro magnetic free convection in the presence of non linearize Rosseland approximation due to surface boundary condition. And indeed convection is probably the most common boundary condition encountered in practice since most heat transfer surfaces are exposed to an environment at a specified temperature.
A review of the literature shows that to the best of our knowledge not much research has been reported on non-linear thermal approximation MHD flow embedded within vertical walls with convective surface boundary conditions. The present study will therefore extend the works of Jha *et al.* [13] as well as [14] with convective boundary condition within the geometry.
governing equations in dimensional form can be written as

$$\frac{d^2u}{dy^2} - \lambda\frac{du}{dy} - m^2u + Gr\theta = 0 \tag{1}$$

$$\left(1 + \frac{4R}{3}(C_T + \theta)^3\right)\frac{d^2\theta}{dy^2} + 4R(C_T + \theta)^2\left(\frac{d\theta}{dy}\right)^2 - \lambda Pr\frac{d\theta}{dy} = 0 \tag{2}$$

The corresponding boundary condition are

$$u = 0,\ -\frac{d\theta}{dy}\Big|_{y=0} = Bi1(1- \theta(0,t)) \quad \text{at } y=0 \tag{3}$$

$$u = 0,\ -\frac{d\theta}{dy}\Big|_{y=0} = Bi2(\theta(1,t)-rt) \quad \text{at } y=1 \tag{4}$$

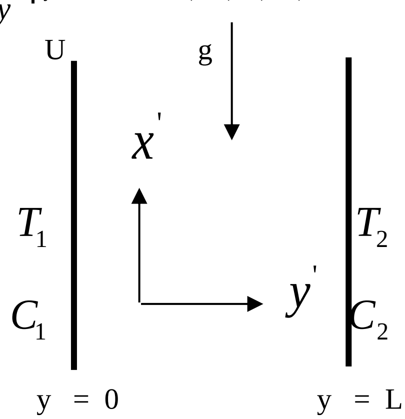

Figure i. Schematic diagram of the physical model and coordinate system

$$\frac{d^2u}{dy^2} - \lambda \frac{du}{dy} - m^2u + G_r\theta = 0 \tag{5}$$

$$\left[1+\frac{4R}{3}(C_T+\theta)^3\right]\frac{d^2\theta}{dy^2} + 4R(C_T+\theta)^2\left(\frac{d\theta}{dy}\right)^2 - \lambda \Pr\frac{d\theta}{dy} = 0 \tag{6}$$

The convective boundary condition

$$U = 0,\ -\frac{d\theta}{dy} = Bi1(1-\theta) \text{ at } y = 0 \tag{7}$$

$$U = 0,\ -\frac{d\theta}{dy} = Bi2(\theta - rt) \text{ at } y = 1 \tag{8}$$

3.0 Method Of Solution

Using Perturbation technique we write/Assumed

$$U(y) = \sum_{i=0}^{n} R^i\,\theta \tag{9}$$

$$U(y) = \sum_{i=0}^{n} R^i\,\theta \tag{10}$$

Substituting the assumed solution into eqn(9) and (10) we have

$$\frac{d^2u_0}{dy^2} - \lambda\frac{du_0}{dy} - m^2u_0 = -G_r\Theta_0 \tag{11}$$

$$\frac{d^2u_1}{dy^2} - \lambda\frac{du_1}{dy} - m^2u_1 = -G_r\Theta_1 \tag{12}$$

With the following Boundary condition

$U = 0,$ at $y = 0$

$$U_0 = 0 \Rightarrow U_1 = 0 \tag{13}$$

At $y = 1$ $U = 0,$

$$U_0 = 0 \quad U_1 = 0 \tag{14}$$

New boundary conditions are

$$U = 0, \quad U_1 = 0 \quad \text{At} \quad y = 0 \tag{15}$$

$$U = 0, \quad U_1 = 0 \quad \text{At} \quad y = 1 \tag{16}$$

Substituting the perturbation assumed solution into the temperature eqn i.e. (10) we have

$$\frac{d^2\Theta_0}{dy^2} - \lambda\Pr\frac{d\Theta_0}{dy} = 0 \tag{17}$$

At y = 0

$$-\frac{d\Theta}{dy}\Big|_{y=0} = Bi1(1-\Theta) \tag{18}$$

With the boundary condition in (18)

$$-\frac{d\theta}{dy} = Bi1(1-\theta) \text{ at } y = 0 \tag{19}$$

$$-\frac{d\theta}{dy} = Bi2(\theta - rt) \text{ at } y = 1 \tag{20}$$

The New boundary condition s are

$$-\frac{d\theta_0}{dy} = Bi1(1-\theta_0) \text{ at } y = 0, \quad -\frac{d\theta_1}{dy} = -Bi1\theta_1 \quad \text{At} \quad y = 0 \tag{21}$$

$-\dfrac{d\theta_0}{dy} = Bi2(\theta - rt)$, at $y = 1$ $-\dfrac{d\theta_1}{dy} = -Bi2\theta_1$ At $y = 1$ (22)

$-\dfrac{d\theta_1}{dy} = -Bi1\theta_1$ At y=0 (23)

$-\dfrac{d\theta_1}{dy} = Bi2\theta_1$ At y=1 (24)

Solving for Θ_0 in eqn(17)

$\frac{d^2\Theta_0}{dy^2} - \lambda \Pr \frac{d\Theta_0}{dy} = 0$

By applying integrating factor

Let m= $\frac{d\Theta_0}{dy}$ (25)

IF=$e^{-\int \lambda Pr dy} = e^{-\lambda Pr \int dy} = e^{-\lambda Pr y}$ (26)

$\theta_0 = \dfrac{k1e^{\lambda \Pr y}}{\lambda \Pr} + k2$ (27)

Using the new boundary condition

$-\dfrac{d\theta_0}{dy} = Bi1(1-\theta_0)$ at $y = 0$, (28)

Let $\begin{matrix} \phi_1 = Bi1 - \lambda \Pr \\ \phi_2 = (-\lambda \Pr - Bi2)e^{\lambda \Pr} \end{matrix}$

The constant K1 and K2 are

K1=$\dfrac{(\phi_1\, Bi1 + \phi_1\, Bi2)\,\lambda \Pr\, Bi1 - (\phi_2\, Bi1 + \phi_1\, Bi2rt)}{\phi_1\,\phi_2\, Bi1 + \phi^2 Bi2}$ (29)

K2=$\dfrac{+\phi_2 Bi1 + \phi_1 Bi2rt}{\phi_2 Bi1 + \phi_1 Bi2}$ (30)

Solving U0 using the following

$\frac{d^2u_0}{dy^2} - \lambda\ \frac{du_0}{dy} - m^2 u_0 = -G_r\Theta_0$ (31)

$U_0 = A1e^{s1y} + A2e^{-s2y} + B1e^{\lambda \Pr y} + B2$

Where B1, B2, A1, and A2 are respectively

$B1 = \dfrac{-GrK1}{\lambda \Pr(\lambda^2 \Pr^2 - \lambda^2 \Pr - m^2)}$ (32)

$B2 = \dfrac{GrK2}{m^2}$ (33)

$A1 = -A2 - B1 - B2$ (34)

$A2 = \dfrac{B1(e^{\lambda \Pr} - e^{s1}) + B2(1 - e^{s1})}{(e^{s1} - e^{-s2})}$

Solving θ_1 following the equation below

$$\frac{d^2\Theta_1}{dy^2} - \lambda \mathrm{Pr}\frac{d\Theta_1}{dy} = -\left[1+\frac{4R}{3}(C_T+\theta_0)^3\right]\frac{d^2\theta_0}{dy^2} - 4\mathrm{R}(C_T+\theta_0)^2\left(\frac{d\theta_0}{dy}\right)^2 \qquad (35)$$

With the new boundary condition

$$-\frac{d\theta_1}{dy} = -Bi1\theta_1 \quad \text{At } y=0$$

$$-\frac{d\theta_1}{dy} = Bi2\theta_1 \quad \text{At } y=1$$

$$\theta_1 = A3 + A4e^{-\lambda \mathrm{Pr}\, y} + L1ye^{\lambda \mathrm{Pr}\, y} + L2e^{2\lambda \mathrm{Pr}\, y} + L3e^{3\lambda \mathrm{Pr}\, y} + L4e^{4\lambda \mathrm{Pr}\, y} \qquad (36)$$

Where the constant A3, A4, L1, L2, L3, and L4 are respectively (37)

$$A3 = \frac{Z1 - V1A4}{Bi1} \qquad (38)$$

$$A4 = \frac{-Bi2 - Bi1Z2}{Z3} \qquad (39)$$

$$L1 = \frac{P1}{\lambda \mathrm{Pr}} \qquad (40)$$

$$L2 = \frac{P2}{2\lambda^2 \mathrm{Pr}^2} \qquad (41)$$

$$L3 = \frac{P3}{6\lambda^2 \mathrm{Pr}^2}$$

$$L4 = \frac{P4}{12\lambda^2 \mathrm{Pr}^2}$$

Solving U1 using the equation below

$$\frac{d^2u_1}{dy^2} - \lambda\,\frac{du_1}{dy} - m^2u_1 = -G_r\Theta_1 \qquad (42)$$

Using the new boundary condition

$$U=0, \quad U_1 = 0 \quad \text{At} \quad y=0 \qquad (43)$$

$$U=0, \quad U_1 = 0 \quad \text{At} \quad y=1 \qquad (44)$$

$$U1 = a5e^{q1y} + a6e^{-q2y} + b3 + b4e^{-\lambda \mathrm{Pr}\, y} + b5ye^{\lambda \mathrm{Pr}\, y} + b6e^{2\lambda \mathrm{Pr}\, y} + b7e^{3\lambda \mathrm{Pr}\, y} + b8e^{4\lambda \mathrm{Pr}\, y} \qquad (45)$$

Where the constants a5, a6, b3, b4, b5, b6, b7, and b8 are:

$$a5 = -a6 - b3 - b4 - b6 - b7 - b8$$

$$a6 = \frac{b5e^{\lambda \mathrm{Pr}} - b3(e^{q1}+1) - b4(e^{q1} - e^{-\lambda \mathrm{Pr}}) - b6(e^{q1} - e^{2\lambda \mathrm{Pr}}) - b7(e^{q1} - e^{3\lambda \mathrm{Pr}}) - b8(e^{q1} - e^{4\lambda \mathrm{Pr}})}{(e^{q1} - e^{-q2})}$$

$$b3 = \frac{Gra3}{m^2}$$

$$b4 = \frac{-Gra4}{\mathrm{Pr}^2\lambda^2 + \lambda^2 \mathrm{Pr} - m^2}$$

$$b5 = \frac{-GrL1}{\lambda^2 \mathrm{Pr}^2 - \lambda^2 \mathrm{Pr} - m^2}$$

$$b6 = \frac{-GrL2}{4\lambda^2 \mathrm{Pr}^2 - 2\lambda^2 \mathrm{Pr} - m^2}$$

$$b7 = \frac{-GrL3}{9\lambda^2 \mathrm{Pr}^2 - 3\lambda^2 \mathrm{Pr} - m^2}$$

$$b8 = \frac{-GrL4}{9\lambda^2 \mathrm{Pr}^2 - 3\lambda^2 \mathrm{Pr} - m^2}$$

$$L1 = \frac{P1}{\lambda \ \Pr}$$

$$L2 = \frac{P2}{2\lambda^2 \Pr^2}$$

$$L3 = \frac{P3}{6\lambda^2 \Pr^2}$$

$$L4 = \frac{P4}{12\lambda^2 \Pr^2}$$

$$P1 = \frac{-4J^3 \lambda \Pr K1}{3}$$

$$P2 = -8J^2 K1^2$$

$$P3 = -4JK1^3 - \frac{8JK1^3}{\lambda \Pr}$$

$$P4 = \frac{-4K1^4}{3} - \frac{4K1^4}{\lambda \Pr}$$

$$J = C_T + K2 \tag{46}$$

4.Result and Discussion

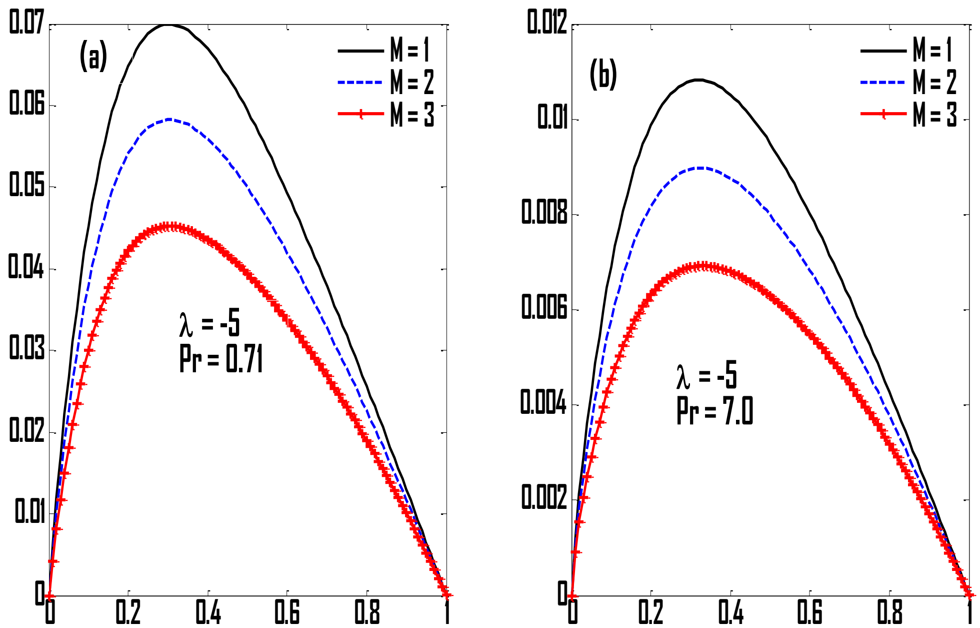

Figure1: Effect of Magnetic Parameter (M) on Velocity when: Bi1 = 0.5, Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, Gr = 5

Figure1a,b It is observed in Figure 1 a,b that velocity decreases with the increase of the Magnetic Parameter M due to suction (λ<0). demostrate the fact that at lower Prandtl number in the case of air (0.71) the Magnetic Parameter (M) has more Effect on velocity in comparison with Pr = 7.0 as shown in Figure 1b.

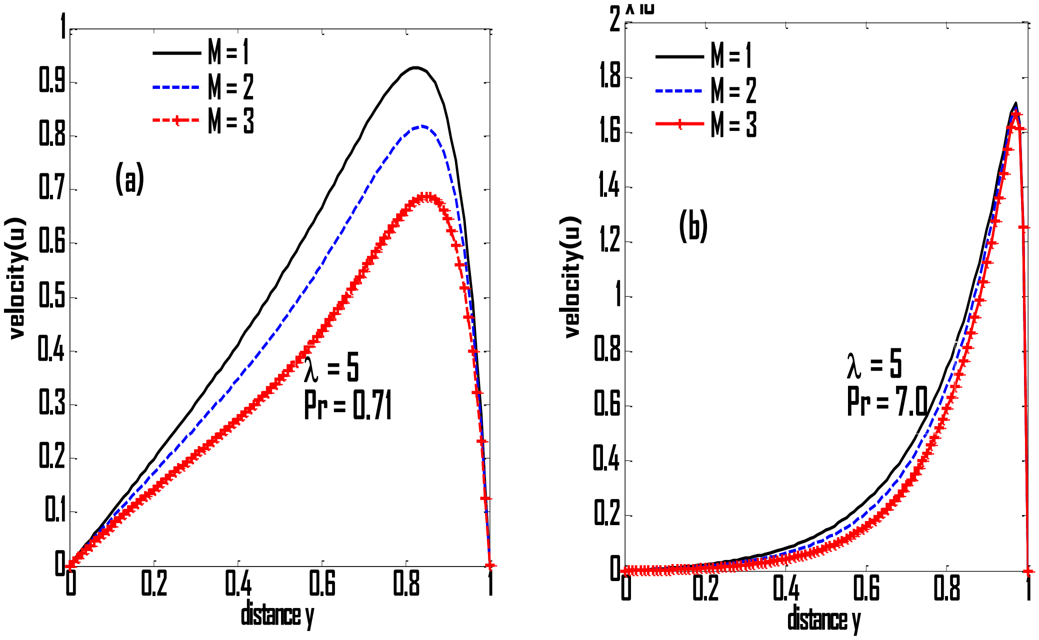

figure.2 Effect of magnetic Parameter (M) on velocity when: Bi1 = 0.5, Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, Gr =5

Figure 2a.b shows the effect of magnetic parameter due to injection (λ>0), the values of velocity reflect to be higher in figure 2bwhen Pr = 7.0 in comparison with figure 2a (Pr = 0.71)

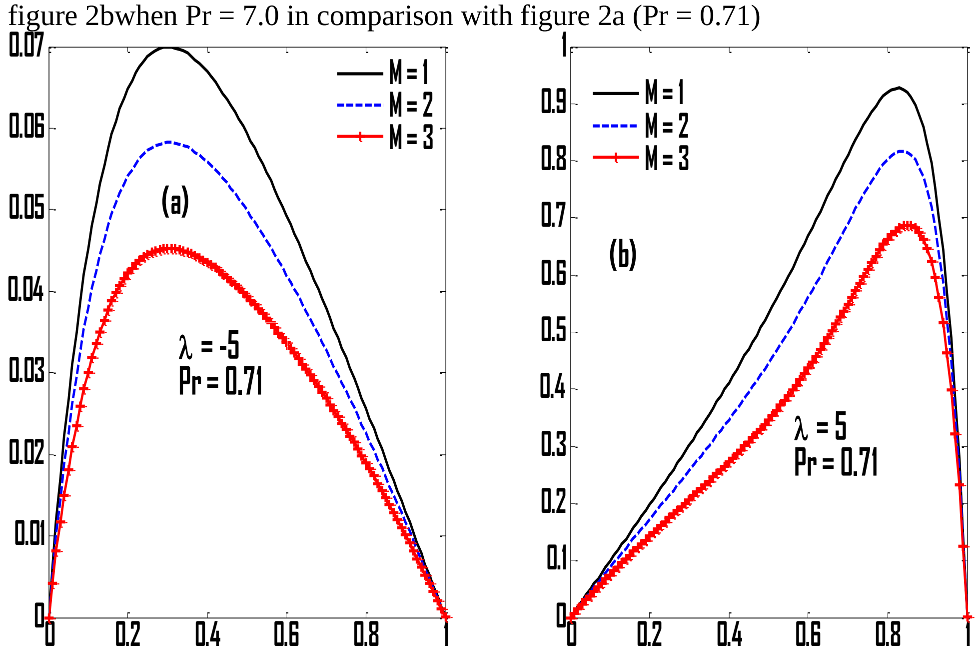

Figure3:Effect of magnetic Parameter (M) on velocity for Bi1 = 0.5, Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, Gr = 5

Figure 3a,b depict the effect of magnetic parameter (M) due to suction (λ<0) / injection (λ>0) when Pr = 0.7. from the figures It is observed that velocity decreases with the increase of the Magnetic Parameter M however, the value of velocity appears to be higher in case of (λ>0) in comparison with (λ<0) see figure 3a and b above.

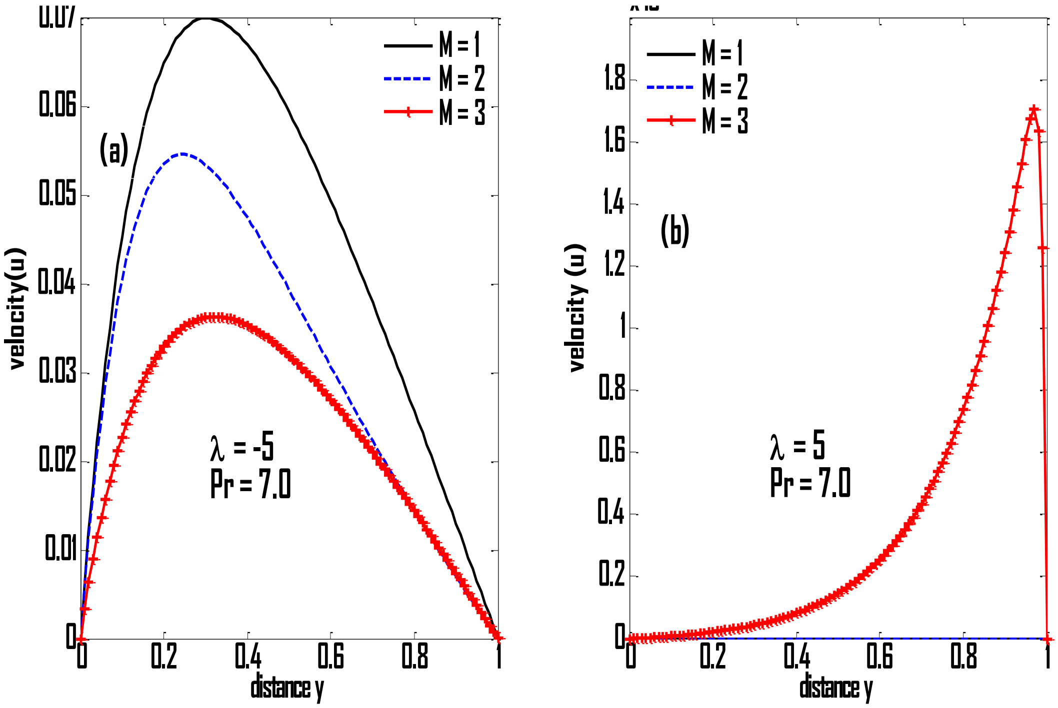

Figure4:Effect of magnetic parameter (M) on velocity when: Bi1 = 0.5, Bi2 = 0.5**,** C_T = 0.01, rt = 0.01, R = 0.0001, Gr = 5

Figure.4a,b demostrate the effect of the magnetic parameter (M) due to suction (λ<0)/ injection (λ>0) in the case of water (Pr = 7.0) from figure 4b it is observed that an increase in the magnetic parameter (M) has negligible influence with reference to injection (λ>0)

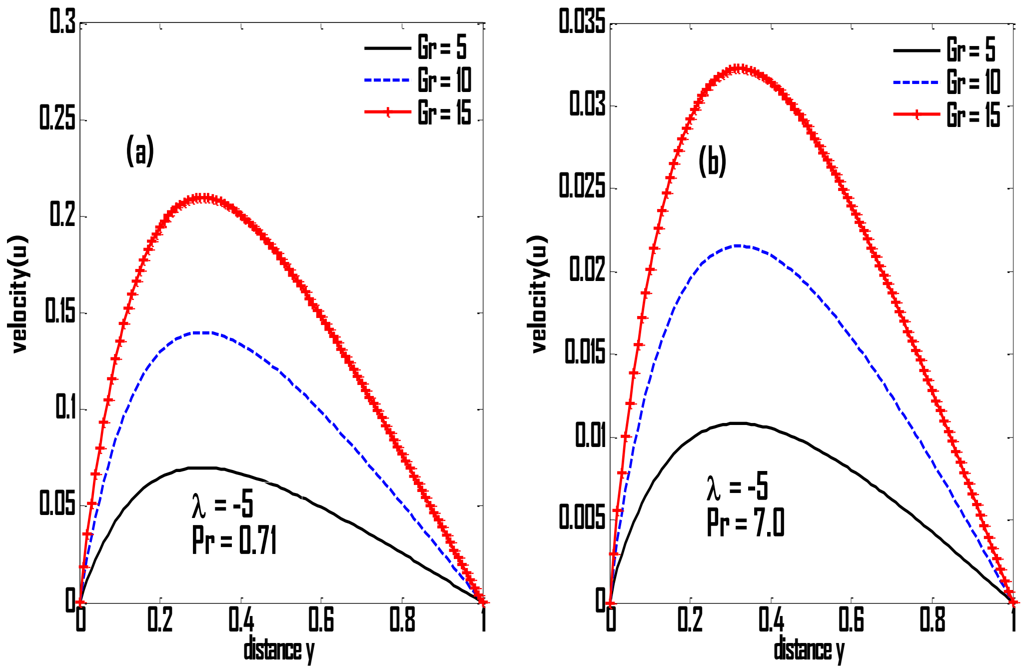

Figure5:Effect of Grashof number (Gr) on velocity when: Bi1 = 0.5, Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, M = 1

The effect of Grashof number on the non-dimensional velocity u(y) is shown in fig.5a,b it is noticed that when the Grashof number increases the non-dimensional velocity also increases due to suction (λ<0). in the case of air (Pr = 0.71) and of water also (Pr = 7.0). but in case of air is higher.

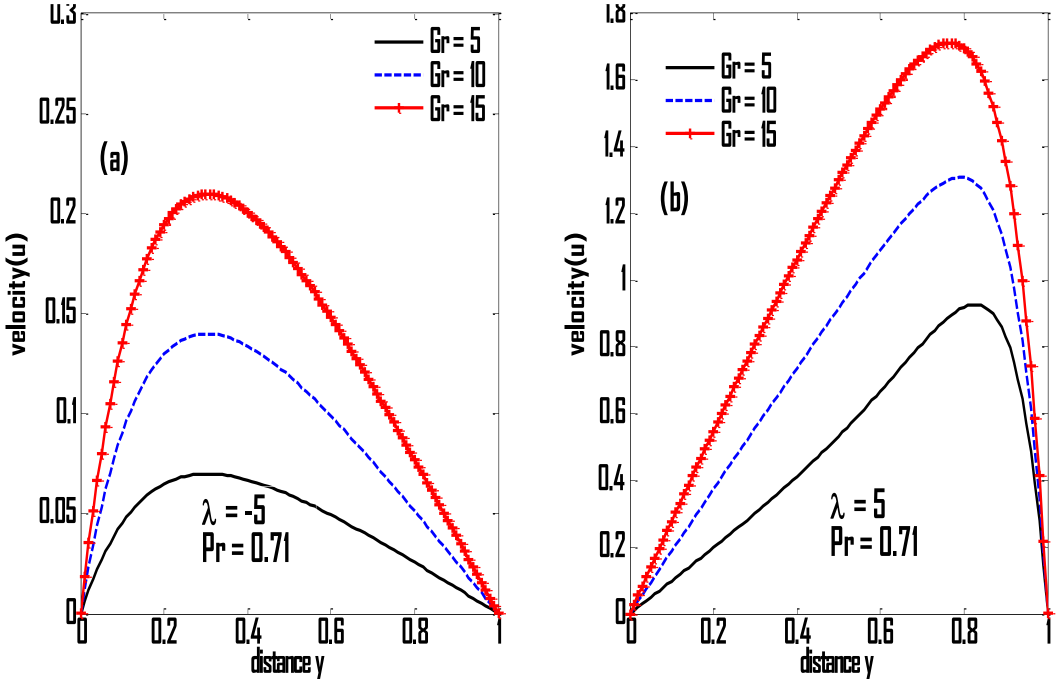

Figure6:Effect of Grashof number (Gr) on velocity when: Bi1 = 0.5, Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, M = 1

The effect of Grashof number on the non-dimensional velocity u(y) is shown in fig.6a,b it is noticed that when the Grashof number increases the non-dimensional velocity also increases due to suction (λ<0)/ injection (λ>0) in the case of air (Pr = 0.71) but when λ<0 it appears to be higher in comparison with injection (λ>0).

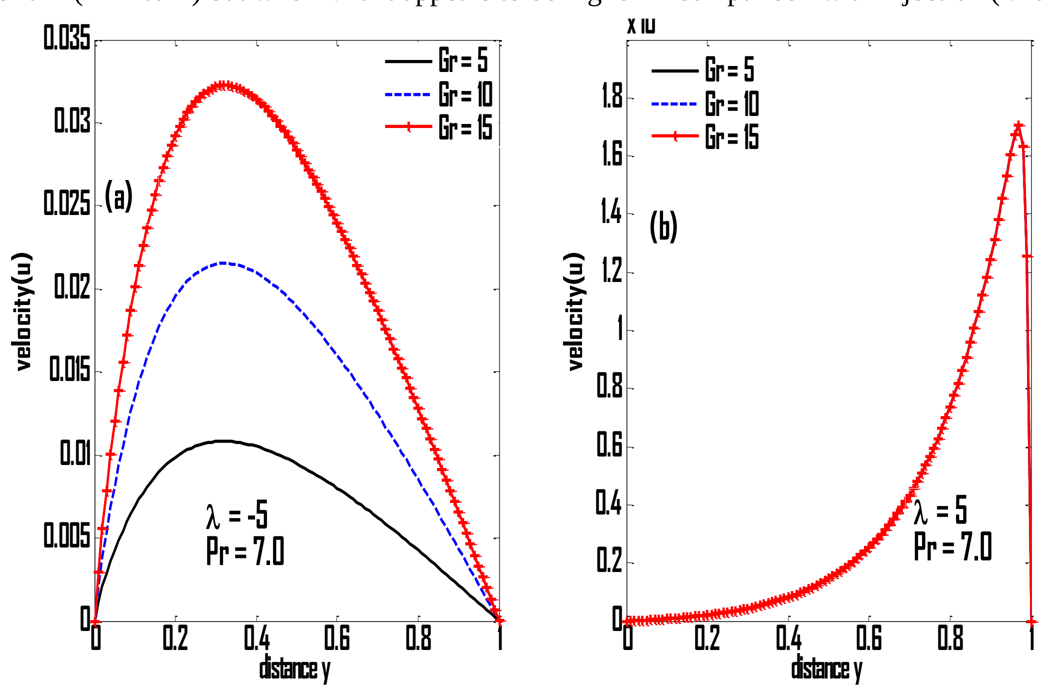

Figure7:Effect of Grashof number (Gr) on velocity when: Bi1 = 0.5, Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, M = 1

The effect of Grashof number on the non-dimensional velocity u(y) is shown in fig.7a,b it is noticed that when the Grashof number increases the non-dimensional velocity also increases to suction (λ<0) but it has negligible effect due to injection (λ>0) in the case of water (Pr = 7.0).

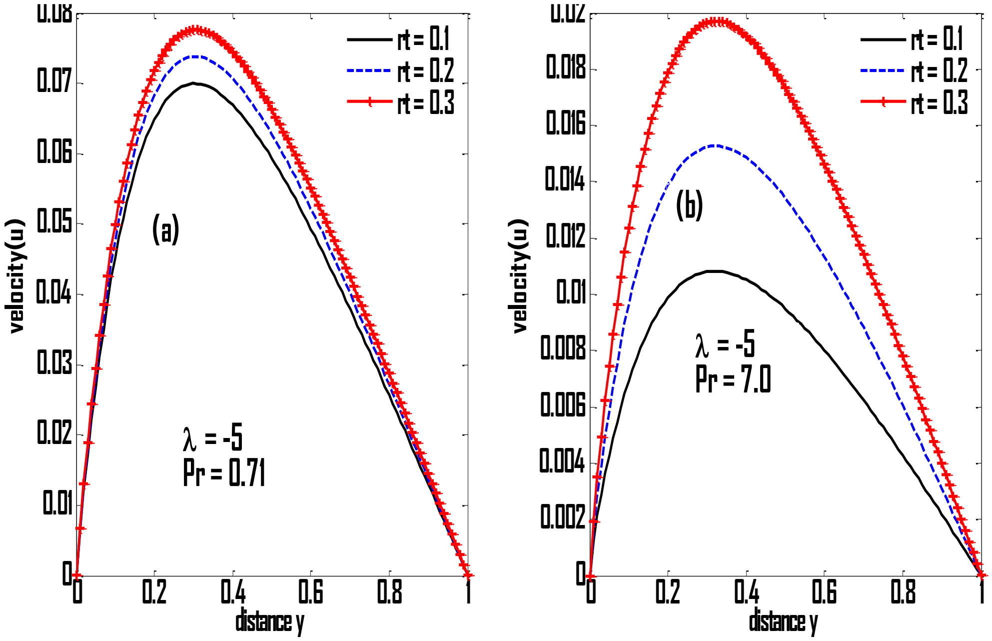

Figure8:Effect of rt on velocity when: Bi1 = 0.5, Bi2 , C_T = 0.01, Gr = 5, R = 0.0001, M = 1

Fig 8a,b demonstrate the effect of ambient temperature parameter (rt) on velocity, when rt increase the velocity will increase due to suction (λ<0), in the case of air (Pr = 0.71)/ water (Pr = 7.0). but in the case of air it appears to be higher in comparison with water (Pr = 7.0).

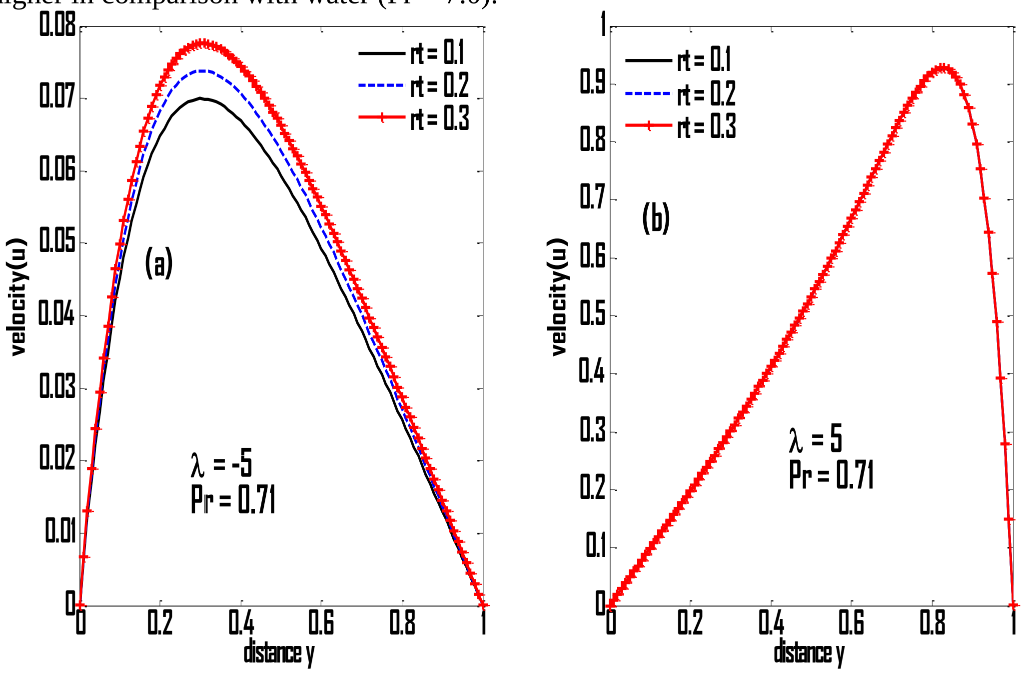

Figure 9:Effect of rt on velocity when: Bi1 = 0.5, Bi2 = 0, C_T = 0.01, Gr = 5, R = 0.0001, M = 1

Figure 9a,b demonstrate the effect of ambient temperature parameter (rt) on velocity, when rt increase the velocity will increase, but it has a negligible effect due to suction (λ<0), but it has a negligible effect due to injunction (λ>0) in the case of air (Pr = 0.71)

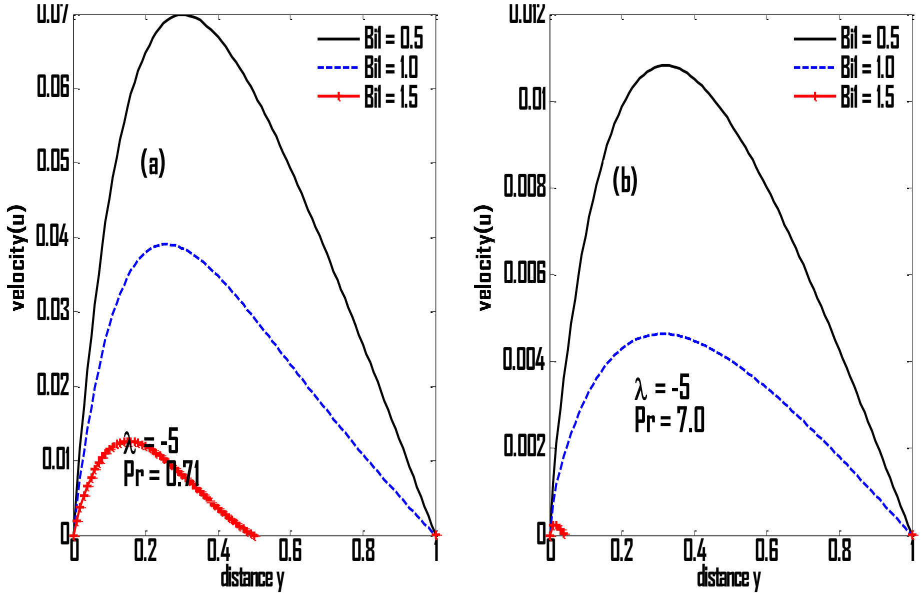

Figure10: Effect of Bi1 on velocity when Bi2 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, M = 1, Gr = 5

Figure10a,b demonstrate the effect of convective heat transfer parameter at $y = 0$ ($Bi1$) that the increases in convective heat transfer parameter at $y = 0$ ($Bi1$) will decreases the velocity due to suction (λ<0) in the case of air (Pr = 0.71) and water (Pr = 7.0), but appears to be higher in the case of air in comparison with water.

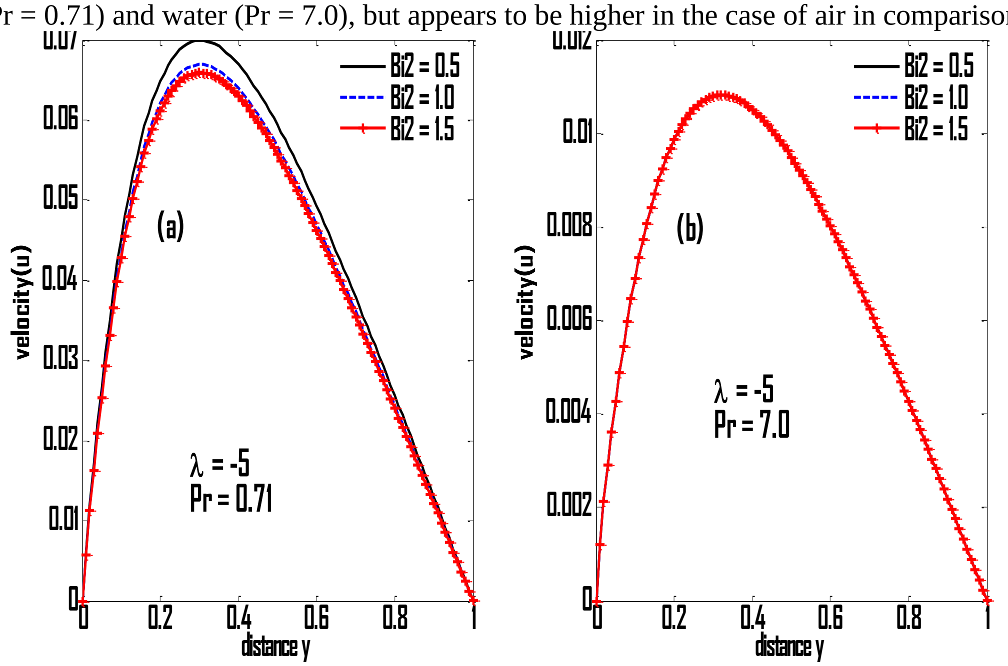

Figure 11: Effect of Bi2 on Velocity when Bi1 = 0.5, C_T = 0.01, rt = 0.01, R = 0.0001, M = 1, Gr = 5

Figure 11a,b demonstrate that the increases in the convective heat transfer parameter at $y = 1$ i.e $Bi2$ leads to very small decrease in velocity in the case of air (Pr = 0.71) due to suction (λ<0) While negligible effect in case of water.

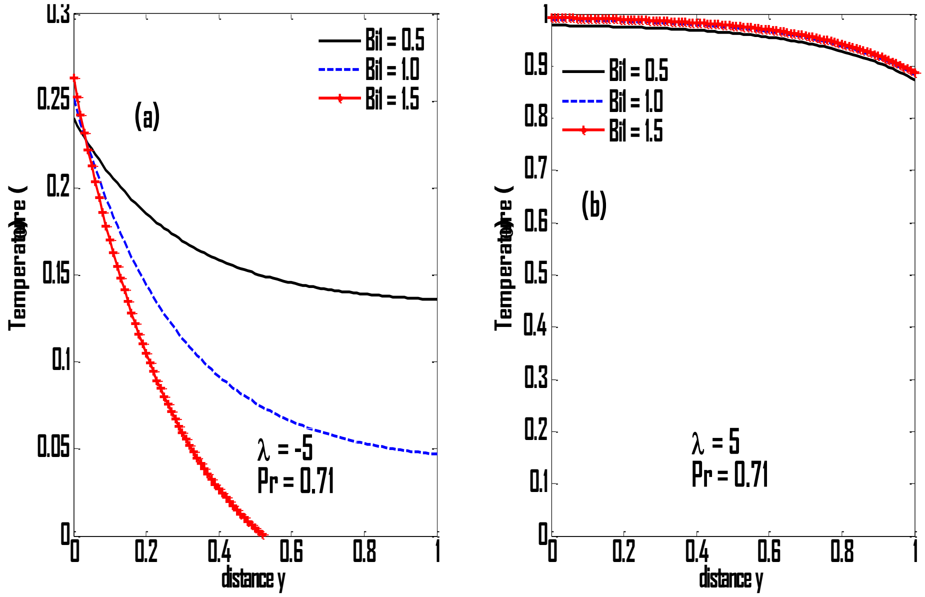

Figure12: Effect of Bi1 on temperature when Bi2 = 0.5, $\boldsymbol{\lambda = -5}$, Pr = 0.71, C_T = 0.01, rt = 0.01, R = 0.0001, M = 1, Gr = 5

Fig 12a,b demonstrate that as Biot number (heat transfer coefficient) increases, the temperature decreases, due to suction (λ<0). But due to injection (λ>0) the case reverses in the case of air (Pr = 0.71) with convective boundary condition.

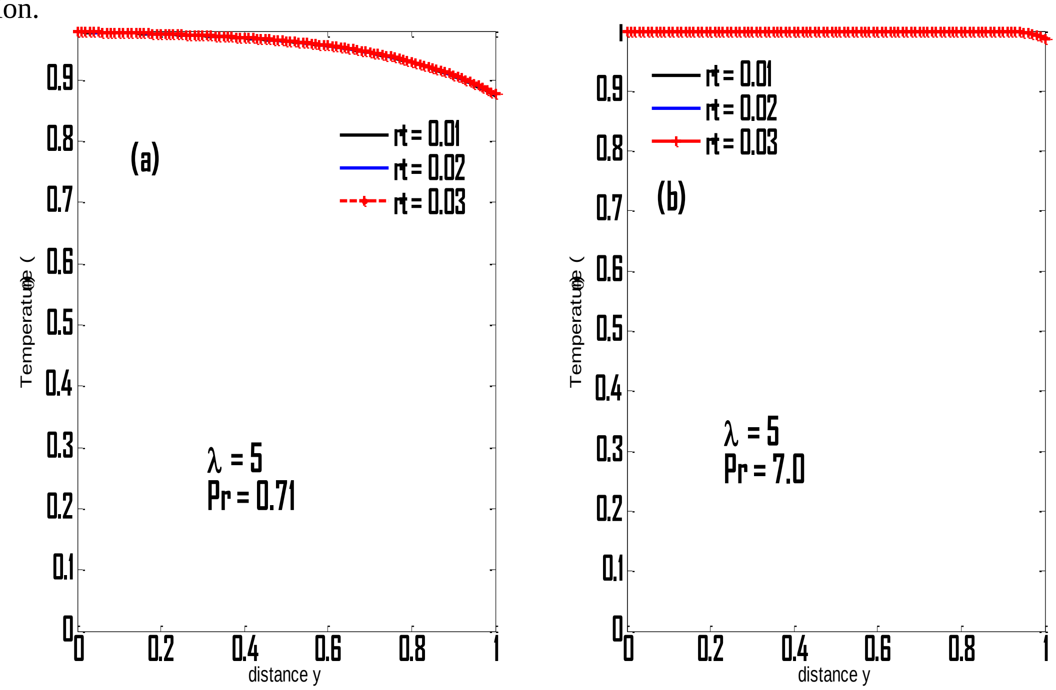

Figure13 Effect of rt on temperature when Bi1 = 0.5, Bi2 = 0.5, $\lambda = -5$, Pr = 0.71, C_T = 0.01, Gr = 5, R = 0.0001, M =1

Figure 13a,b demonstrate that the increase in an ambient temperature parameter (rt) has no effect on temperature but occurs at very high temperature in the case of air (Pr = 0.71)/ water (Pr = 7.0) due to injection (λ>0) but more higher in the case of water.

5.0 Conclusion

The problem of steady state heat and mass transfer on a mixed convection flow of an exothermic fluid in a vertical channel with convective boundary condition has been investigated. The governing equations are solved by using perturbation method. The expression of velocity, temperature, rate of heat and mass transfer have been presented. The computations show that, flow formation is strongly depend on the mixed convective parameter, symmetric wall temperature and velocity.

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