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Numerical Solution of Hydromagnetic Flow Past An Infinite Vertical Porous Plate

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Abstract

Numerical solution of hydromagnetic fluid flow of a viscous incompressible electrically conducting fluid flow past an infinite vertical porous plate in presence of constant suction and heat sink has been investigated. The partial differential equations governing the flow field has been derived and transformed to non-dimensional form. The equations and their respective initial and boundary conditions are then non-dimensionalized and solved numerically using finite difference method specifically, the Crank-Nicolson method. The effects of varying various flow parameters on the velocity, temperature, and concentration profiles were studied. This research is an extension of the work of Maina et al. [7]. In this research chemical reaction term, heat sink and suction parameters are added to the governing equations which are not included her work.

Keywords: MHD, Heat and mass transfer, vertical plate, suction, porous medium

10. Introduction

The science of hydromagnetic fluid flow with heat and mass transfer has been a subject of interest of many researchers because of its possible applications in the field of astrophysical sciences, engineering sciences and also in industry.

In view of its applications, Sudhakar et al [1] studied Chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo. They observed that the numerical results for some special cases were compared with Chaudhary et al. [2] and were found to be in good agreement. Sailam [3] analyzed the finite element method solution of an unsteady mhd free convection flow past an infinite vertical plate with constant suction and heat absorption. He observed that increase in magnetic field strength decreases the velocity of the fluid. Also the skin friction and rate of heat transfer of the conducting fluid decrease with increase in magnetic field strength. Prakashi et al. [4] investigated thermo-diffusion and chemical reaction effects on mhd three dimensional free convective couette flow. They found that the velocity profiles increased due to increase in thermo-diffusion parameter, chemical reaction parameter, the Schmidt number, thermal Grashof number, mass Grash of number, Reynolds number and Prandtl number. Murali [5] studied unsteady mhd free convection viscous dissipative flow past an infinite vertical plate with constant suction and heat source/sink.He observed that among other things that the rate of heat transfer for both mercury and electrolytic solution is decreasing with increasing of Hartmann number and increases with increasing of heat source parameter. Sushila and Swagatika [6] investigated unsteady magneto hydro dynamic flow, heat and mass transmit through an accelerate vertical porous plate in the pressure of viscous debauchery heat supply and variable suction. Maina et al. [7] analysed unsteady hydromagnetic free convective flow past an infinite vertical porous plate in porous medium mathematical theory and modeling.They found that It has been observed that, increasing injection parameter accelerates the velocity of the flow field, increases the concentration and reduces the temperature of the flow field. It also increases the skin friction at the wall and decreases the rate of heat transfer.

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A growing Hartmann number (M) retards the velocity of the fluid, increases the skin friction at the wall and reduces the rate of heat transfer. Increase in heat source parameter increases the temperature of the flow field, skin friction at the wall and reduces the rate of heat transfer. A growing permeability parameter accelerates the velocity and decreases the temperature of the flow field when Grθ>0 and reverses when Grθ<0. It also reduces the skin friction and increases the rate of heat transfer at the wall. Increase in Schmidt number has an effect of reducing the concentration of the flow field. It also increases the skin friction at the wall and the rate of heat transfer decreases. Mwangi et al. [8] studied effects of temperature dependent viscosity on magnetohydrodynamic natural convection flow past an isothermal sphere. They observed that among things that increase in reynolds number (re) leads to an increase in the secondary velocity (v) and heat transfer in the fluid but leads to a decrease in primary velocity (u), temperature (t) and skin friction of the fluid. Asogwa and Joachim [9] carrird out analytical study of mhd viscoelastic fluid past vertical porous plate with radiation, variable suction and heat sink. They observed that the rate of mass transfer and heat transfer is not significantly affected by the viscoelastic parameter in the fluid flow region and increasing the values of magnetic parameter, prandtl number, grashof number for mass transfer and heat source parameter accelerate the fluid flow past a heated plate but a reverse behavior is experienced during the flow past a cooled plate. Asogwa et al. [10]estimated heat and mass transfer over a vertical plate with periodic suction and heat sink. They found graphically that the velocity increases with increase in heat sink and suction parameters.

This research is an extension of the work of Maina et al. [7]. In this research chemical reaction term, heat sink and suction parameters are added to the governing equations which are not included her work.Approximate solutions are obtained for velocity field, temperature and concentration fields, skin friction,sherwood and rate of heat transfer using numerical method. The effects of the flow parameters on the flow field are analyzed with the help of figures and tables. The problem has some relevance in the geophysical and astrophysical studies.

2.0 Mathematical Formulations Of The Problem

Consider the unsteady flow of an incompressible viscous electrically conducting fluid which is initially at rest past an infinite vertical plate with variable temperature through a porous medium. The flow is assumed to be in x-direction, which is taken along the vertical plate in the upward direction .The y-axis is taken be normal to the plate. Initially the plate and the fluid are at same Temperature T∞′ with same concentration C∞′ level at all points. At time t′ > 0 the plate starts oscillating in its own plane with a velocity u = 0. The plate Temperature is raised to Tw ′ and the levels of concentration near the plate are raised to Cw′ linearly with time t. The viscosity is taken into account with constant permeability of porous medium. Then by usual Boussinesq's approximation the unsteady flow is governed by the following equations.

$$\frac{\partial u}{\partial t}-v_0\frac{\partial u}{\partial y}=g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)+v\frac{\partial^2 u}{\partial y^2}-\frac{v}{k}u-\frac{\sigma\beta_0^2 u}{\rho} \tag{1}$$

$$\frac{\partial T}{\partial t}-v_0\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2-Q_0\left(T-T_\infty\right) \tag{2}$$

$$\frac{\partial C}{\partial t}-v_0\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y}-K'C \tag{3}$$

where u is the velocity of the fluid, v_0 is the suction term parameter, and t is time, K is chemical reaction term, v is the kinematics viscosity, g is the gravitational constant, β is the thermal conductivity, β^* is modified thermal conductivity, T is the temperature of the fluid, k is thermal conductivity, ρ is density, C_p is heat capacity at constant pressure, D is diffusion term, C is the mass concentration, Q_0 is the heat sink term, y is distance

With the following initial and boundary conditions

$$\left.\begin{array}{l} t\le 0,\ u=0,\ \ T=T_\infty,\ \ C=C_\infty\ \ \forall\ y \\ t>0\quad \begin{cases} u=0,\ \ T=T_w,\ \ C=C_w\ \ at\ \ y=0 \\ u\to 0,\ T\to T_\infty,\ \ C\to C_\infty\ \ as\ \ y\to\infty \end{cases} \end{array}\right\} \tag{4}$$

This research is an extension of the work of Maina *et al.* [7]

$$\left.\begin{array}{l} u' = \dfrac{u}{u_0}, \quad t' = \dfrac{tu_0^2}{\nu}, \quad y' = \dfrac{yu_0}{\nu}, \quad \theta = \dfrac{T - T_\infty}{T_\varpi - T_\infty}, \quad Gr = \dfrac{g\beta\nu\left(T_w - T_\infty\right)}{u_0^3}, \quad C = \dfrac{C - C_\infty}{C_w - C_\infty}, \\ Gc = \dfrac{g\beta^*\nu\left(C_w - C_\infty\right)}{u_0^3}, \quad \Pr = \dfrac{\mu c_\rho}{\kappa}, \quad Sc = \dfrac{\nu}{D}, \quad s = \dfrac{\nu_0}{u_0}, \quad Q = \dfrac{Q_0\nu}{u_0^2}, \quad K = \dfrac{K'\nu}{\left(C_w - C_\infty\right)u_0^2} \end{array}\right\} \quad (5)$$

Substituting the dimensionless variables of equation (5) into (1) to (4), gives (6) to (8),

$$\frac{\partial u'}{\partial t'} - s\frac{\partial u'}{\partial y'} = Gr\theta + GcC + \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{1}{k_p} + M\right)u' \quad (6)$$

$$\frac{\partial \theta}{\partial t'} - s\frac{\partial \theta}{\partial y'} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y'^2} + Ec\left(\frac{\partial u'}{\partial y'}\right)^2 - Q\theta \quad (7)$$

$$\frac{\partial C}{\partial t'} - s\frac{\partial C}{\partial y'} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y'^2} - KC \quad (8)$$

Where s is the suction parameter, Q is heat sink, K is the chemical reaction parameter Gr ,is the thermal Grashof number, *Gc* is the mass grashof number, *Sc* is the Schmidt number, Pr is the Prandtl number.

The initial and boundary conditions (4) of the non-dimensional form are:

$$\left.\begin{array}{ll} t \le 0, & u' = 0, \quad \theta = 0, \quad C = 0 \ \forall \ y \\ t > 0 & \begin{cases} u' = 0, \quad \theta = 1, \quad C = 1 \ \ at \ \ y = 0 \\ u' \to 0, \ \theta \to 0, \ C \to 0 \ \ as \ \ y \to \infty \end{cases} \end{array}\right\} \quad (9)$$

3.0 Method Of Solution

Numerical Solutions By Finite Difference Method Using Crank-Nicolson Method

The equations (6) to (9) are solved numerically using the Finite difference method that applies Crank-Nicolson algorithm.

$$u_j'^{i+1} = \frac{\left[\dfrac{Gr\Delta t}{2}\left(\theta_j^{i+1} + \theta_j^i\right) + \dfrac{Gc\Delta t}{2}\left(C_j^{i+1} + C_j^i\right) + u_j'^{i+1} + \dfrac{s_j^i\Delta t}{2\Delta y}\left(-u_{j-1}'^{i+1} + u_j'^i - u_{j-1}'^i\right) + \dfrac{\Delta t}{2\left(\Delta y\right)^2}\left(u_{j-1}'^{i+1} + u_{j+1}'^{i+1} + u_{j-1}'^i - 2u_j'^i + u_{j+1}'^i\right) - \dfrac{u_j'^i\Delta t}{2}\left(\dfrac{1}{k_p} + M\right)\right]}{\left(1 - \dfrac{s_j^i\Delta t}{2\Delta y} + \dfrac{\Delta t}{\left(\Delta y\right)^2} + \dfrac{\Delta t}{2}\left(\dfrac{1}{k_p} + M\right)\right)} \quad (10)$$

$$\theta_j^{i+1} = \left[\theta_j^i + \frac{s_j^i\Delta t}{2\Delta y}\left(-\theta_{j-1}^{i+1} + \theta_j^i - \theta_{j-1}^i\right) + \frac{\Delta t}{2\Pr\left(\Delta y\right)^2}\left(\theta_{j-1}^{i+1} + \theta_{j+1}^{i+1} + \theta_{j-1}^i - 2\theta_j^i + \theta_{j+1}^i\right) + Ec\Delta t\left(\frac{\theta_j^{i+1} - \theta_{j-1}^{i+1} + \theta_j^i - \theta_{j-1}^i}{2\Delta y}\right)^2 - \frac{\theta_j^i Q\Delta t}{2}\right] \div \left(1 - \frac{s_j^i\Delta t}{2\Delta y} + \frac{\Delta t}{\Pr\left(\Delta y\right)^2} + \frac{Q\Delta t}{2}\right) \quad (11)$$

$$C_j^{i+1} = \left[C_j^i + \frac{s_j^i \Delta t}{2\Delta y}\left(-C_{j-1}^{i+1} + C_j^i - C_{j-1}^i\right) + \frac{\Delta t}{Sc\left(\Delta y\right)^2}\left(C_{j-1}^{i+1} + C_{j+1}^{i+1} + C_{j-1}^i - 2C_j^i + C_{j+1}^i\right) - \frac{C_j^i K \Delta t}{2} \right]$$

$$\div \left(1 - \frac{s_j^i \Delta t}{2\Delta y} + \frac{\Delta t}{Sc\left(\Delta y\right)^2} + \frac{K\Delta t}{2}\right) \qquad (12)$$

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.
The skin-friction coefficient at the wall along x - axis is given by

$$\tau = \left(\frac{\partial u'}{dy'}\right)_{y=0} \qquad (13)$$

The rate of heat transfer coefficient (Nusselt number) due to temperature profiles is given

$$Nu = \left(\frac{\partial \theta}{dy}\right)_{y=0} \qquad (14)$$

And the rate of mass transfer coefficient (Sherwood number) due to concentration profiles is given

$$Sh = \left(\frac{\partial C}{dy}\right)_{y=0} \qquad (15)$$

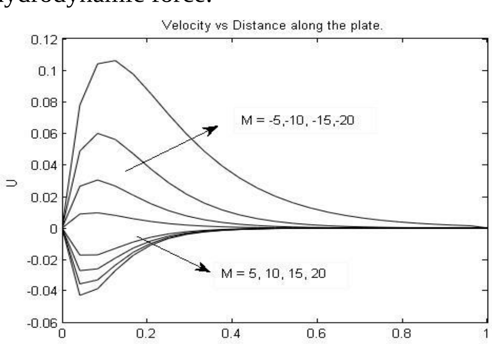
Equations (10) - (15) are then solved using Matrix Laboratory (MATLAB R2010a), a computer program.

4.0 Results And Discussion

Numerical solution of hydromagnetic fluid flow of a viscous incompressible electrically conducting fluid flow past an infinite vertical porous plate in presence of constant suction and heat sink has been formulated, analysed and solved numerically. In order to point out the effects of physical parameters namely; suction parameter s, Heat sink Q, thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, Hartmann number M, and chemical reaction parameter K. on the flow patterns, the computation of the flow fields are carried out. The value of the Prandtl number Pr is chosen to represent air (Pr = 0.71). The value of Schmidt number is chosen to represent water vapour (Sc = 0.6). The values of velocity, temperature and concentration are obtained for the physical parameters as presented below**.**

The velocity profiles is studied for different values of Hartmann number (M=-5,-10,-15,-20, 5,10,15,20) and is presented in **Figure 1**. It is observed that an increase in the values of Hartmann number M, causes a decrease in the magnitude of the velocity profile. Which implies that a decrease in the values of Hartmann number M, causes an increase in the magnitude of the velocity profile.When magnetic field is applied to an electrically conducting fluid, it gives rise to a force called the Lorentz force. This resistive force has a tendency to slow down the motion of the fluid. This observation agrees closely with that of [7].

The velocity profiles is studied for different values of mass Grashof number (Gc=10, 20, 40,100) as Gr>0 and is presented in **Figure 2**. It is observed that velocity increases with decreasing Gc. Hence its increase implies a reduction in viscous hydrodynamic force.



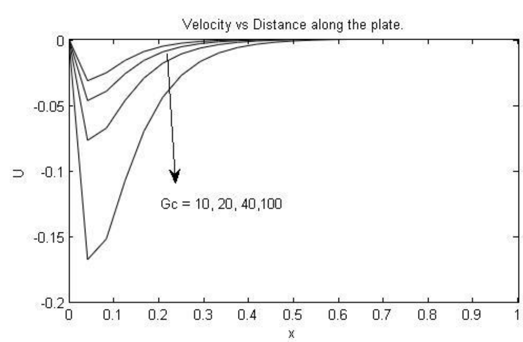


Figure 1. Velocity profile for Gr>0 with variation of M   Figure 2 Velocity profile for Gr>0 with variation of Gc

The velocity profiles is studied for various values of permeability parameter (k_p = 1, 3, 5,10) and is presented in **Figure 3**. It is observed that velocity increases with increasing permeability parameter. Which implies an increase in the rate of heat transfer.
The velocity profiles is studied for different values of suction parameter (s = 10, 15, 25, 35) and is presented in **Figure 4**. It is observed that velocity increases with decreasing suction parameter. Which culminate into a decrease in the rate of mass concentration.

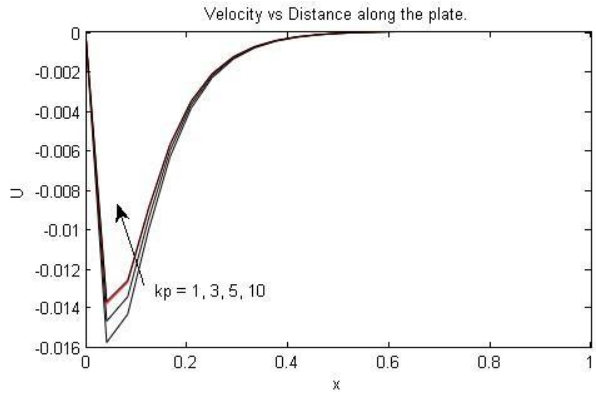


Figure 3 Velocity profiles for different values of k_p

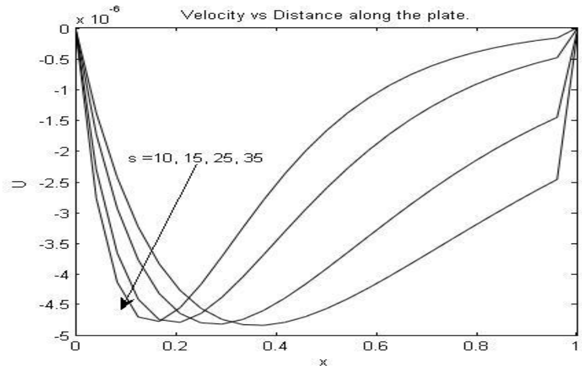


Figure 4 Velocity profiles for different values of s

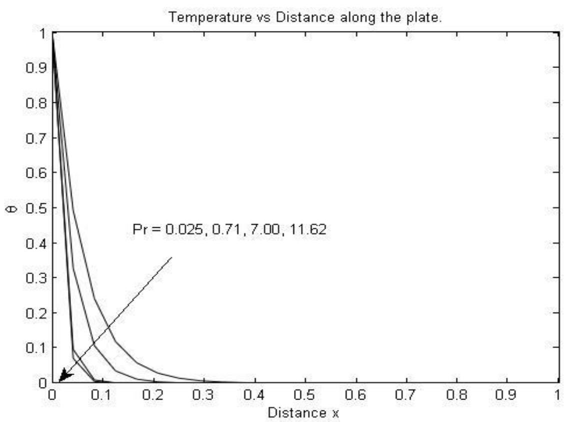


Figure 5 Temperature profiles for different values of Pr

The temperature profiles for different values of Prandtl number (Pr= 0.025, 0.71, 7.00, 11.62) is presented in **Figure 5** It is observed that the temperature increases with decreasing prandtl number. A decrease in Prandtl number (Pr) causes the skin friction to decrease and Nusselt number (Nu) to shoot up. This is because Prandtl number is the ratio of viscous force to thermal force. Therefore the viscous force will decrease holding the thermal force constant thus decreasing the shear stress.

The concentration profiles for different values of Schmidt number (Sc = 0.22, 0.30, 0.60, 0.78) is presented in **Figure 6.** It is observed that the concentration increases with decreasing Schmidt number. It is observed that concentration distribution is found to decrease faster as the diffusing foreign species becomes heavier.

Table 1. Skinfriction τ , Nusselt number Nu and Sherwood number Sh respectively

K	k_p	Pr	Sc	Ec	Gc	Gr	s	Q	τ	Nu	Sh
0.1	1	0.71	0.22	0.5	5	0.01	5	0.1	7.3011	1.505	1.465
0.1	3	0.71	0.22	0.5	5	0.01	10	0.1	7.4704	1.518	1.467
0.1	3	0.71	0.22	0.5	10	0.01	10	0.1	7.4877	1.602	1.477
0.1	5	0.71	0.22	0.5	10	0.01	10	0.1	7.4895	1.622	1.479
0.1	5	0.71	0.22	0.5	20	0.01	10	0.1	7.5011	1.650	1.484
0.1	10	0.71	0.22	1	20	0.02	25	0.3	10.1090	1.660	1.485
0.1	10	7	0.60	1	20	0.02	25	0.3	12.1490	6.234	3.005
1	10	7	0.60	1	40	0.02	25	0.3	12.3868	6.301	3.101
1	10	7	0.78	1	40	0.05	25	0.5	12.3896	6.325	4.267

5.0 Conclusion

Numerical solution of hydromagnetic fluid flow of a viscous incompressible electrically conducting fluid flow past an infinite vertical porous plate in presence of constant suction and heat sink has been formulated, analysed and solved numerically. This research is an extension of the work of Maina *et al.* [7]. In this research chemical term, heat sink and suction parameters are added to the governing equations which are not included her work. the following are my findings.

i. It is observed that velocity increases with decreasing suction parameter. Which culminate into a decrease in the rate of mass concentration which invariably affect the rate of chemical reaction K. Hence an increase in the skin friction (τ) at the wall and decreases the rate of heat transfer Nu .

ii. An increase values of heat sink parameter generate loss of heat in the boundary layer. Which causes the temperature of the fluid to decrease. This decrease in temperature produces a decrease in the flow field due to the buoyancy effect.

iii. An increase values of Hartmann number (M) retards the velocity of the fluid, increases the skin friction (τ) at the wall and reduces the rate of heat transfer Nu . Increase in heat sink parameter decreases the temperature of the flow field, skin friction (τ) at the wall and reduces the rate of heat transfer.

iv. An increase values of permeability parameter accelerates the velocity and decreases the temperature of the flow field. Increase in Schmidt number has an effect of reducing the concentration of the flow field. It also increases the skin friction (τ) at the wall and the rate of heat transfer decreases. Which invariably has effect on the Sherwood number Sh.

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