

Equal Duration Shift Schedule Model of One Lunch Break And One Relief Break With Overlap

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Abstract

Though shift schedule models are currently being used in many organizations to provide services to customers that arrive randomly there is need to incorporate lunch and relief breaks for better quality employees output. In this paper we develop a shift schedule model that is relevant to many practical situations whereby a worker is entitled to a lunch and relief break. The relief break in particular is at that last period of a shift which is also the first period of an incoming shift. The solution to the shift schedule model in LP form shows that in each shift it is possible to know the number of workers to be on lunch/relief break at each period. By this information management can make adequate planning that will minimize total labour thereby enhancing profit.

Keywords: Shift, break, implicit scheduling, queueing model.

1.0 Introduction

In many organizations or units demand for services are random in a given time horizon (such as a day) that is much longer than the normal work duration. It becomes imperative to prepare duty rosters for workers that will indicate the shift duty each worker belongs to in a given day, week or month as the case may be. Generally, for better productivity workers should be allowed to go on lunch break. In some peculiar instances there is need for additional break(s) referred to in this paper as relief break.

In hospitals for example, patients on admission are periodically monitored and attended to throughout a daily time horizon. Hence the nurses in particular have to be scheduled on shifts. The model of this paper is particularly applicable to practical hospital situation whereby the relief break is taken at the last period of a shift and is used for the purpose of handing over each patient to the succeeding shift. During handover, the detail situation of each patient such as type of ailments, prescribed treatments at specific times are explained to nurses of succeeding shift.

Most current practice to optimize employee scheduling follows the general approach originally presented in [1, 2, 3, 4]. These papers highlighted a typical sequence of four steps in scheduling employees as demand forecast, converting the forecast to staffing requirements, optimal scheduling of shifts and assignment of employees to shifts.

Shift scheduling problem can be solved either with the use of heuristics or optimization models including linear programming and integer programming models [5]. As contained in [6] ten classifications of the solution techniques of shift scheduling models are considered as optimal solution approaches and heuristic solution approaches. Most studies use classical integer programming approach which is based on the set covering model which goes back to [7]. This formulation require a large number of decision variables, for some cases that incorporate a high degree of break placement flexibility the computation becomes prohibitive. Rather than focusing on the heuristics solution of large scale problems, other researchers have tried to model the employee scheduling problem by using a second approach in which flexibility does not appear explicitly in the variables as in explicit model but is implicitly hidden in the constraints and or the objective function. These kinds of models are known as implicit shift schedule models. This approach considerably reduces the size of the model when compared to the Dantzig explicit set covering approach [8, 9, 10, 11].

Models for employee scheduling have a long history in operations research literature. The first two distinct streams of research which are on how to set staffing requirements to optimally schedule employees are [7, 12]. It is known that [13] was the first to propose implicit shift schedule model. His approach implicitly represents shifts without considering meal breaks. However, [9] presented a compact integer model to handle meal break placement flexibility for the shift scheduling problem. Similarly, [14] presented a doubly implicit integer shift scheduling model that combined the work of [13] and [9]. Furthermore [10] considered a more general implicit shift scheduling problem with multiple breaks and disjoint break windows.

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2.0 Model Formulation

We first state the assumptions and notations as follows:

Assumptions

1. Each employee receives one lunch break
2. A work day is 24 hours or less divided into equal planning periods.
3. Each lunch break covers the shift planning period.

The following **notations** which are used in this model are defined as follows:

K = Set of all shift allowed

$t \in T$ = interval of time period in the planning horizon.

x_k = number of employees to be assigned to shift $k \in K$

C_k = Unit cost associated with shift $k \in K$

$\delta_{kt} = \begin{cases} 1 & \text{if time period } t \text{ is a working time period for shift } k \in K \\ 0, & \text{otherwise} \end{cases}$

b_t = minimum number of employee required for each time period $t \in T$

W_{kt} = number of employees assigned to shift k and starting their lunch break in period t

U_{kt} = number of employees assigned to shift k and starting their first relief break in period t .

p_0 = probability of no customer in the system.

n = number of customers.

λ = mean arrival rate.

μ = mean departure rate.

The implicit shift model is of the form:

Minimize $\sum_{k \in K} C_k x_k$

subject to

$$\sum_{k \in K} \delta_{kt} x_k - \sum_{k \in TL_t} W_{kt} - \sum_{k \in TR_t} U_{kt} \geq b_t, \forall t \in T \tag{1}$$

$$x_k - \sum_{t \in BL_k} W_{kt} = 0, \forall k \in K$$

$$x_k - \sum_{t \in BR_k} U_{kt} = 0, \forall k \in K$$

$$x_k, W_{kt}, U_{kt} \geq 0 \text{ and integers, } \forall k \text{ and } t \tag{2}$$

3.0 Numerical Illustration

Suppose an organization provides services for 24 hours and employees are on duty for eight- hour shifts and are given one hour off during the shift for lunch. An integer programming problem could be developed to minimize the total labour cost while satisfying various requirements.

Consider three scheduling variables; the time a shift may begin, the time for lunch break and the time for relief break for each employee. If it is possible to begin shift only at three times i.e. 8am, 4pm and midnight, if lunch times can only be taken at the third, fourth, fifth or sixth hour of a shift and if relief break could be taken at the end of each shift i.e. 8am, 4pm and midnight.

If the unit cost is the same for all shifts the Model is of the form:

Minimize $\sum_{k=1}^3 x_k$ (3)

subject to

$$x_1 + x_3 - U_{3,0} \geq b_0$$

$$x_1 \geq b_1$$

$$x_1 - W_{1,2} \geq b_2$$

$$x_1 - W_{1,3} \geq b_3$$

$$x_1 - W_{1,4} \geq b_4$$

$$x_1 - W_{1,5} \geq b_5$$

$$x_1 \geq b_6$$

$$\begin{aligned}
 &x_1 \geq b_7 \\
 &x_1 + x_2 - U_{1,8} \geq b_8 \\
 &x_2 \geq b_9 \\
 &x_2 - W_{2,10} \geq b_{10} \\
 &x_2 - W_{2,11} \geq b_{11} \\
 &x_2 - W_{2,12} \geq b_{12} \\
 &x_2 - W_{2,13} \geq b_{13} \\
 &x_2 \geq b_{14} \\
 &x_2 \geq b_{15} \\
 &x_2 + x_3 - U_{2,16} \geq b_{16} \\
 &x_3 \geq b_{17} \\
 &x_3 - W_{3,18} \geq b_{18} \\
 &x_3 - W_{3,19} \geq b_{19} \\
 &x_3 - W_{3,20} \geq b_{20} \\
 &x_3 - W_{3,21} \geq b_{21} \\
 &x_3 \geq b_{22} \\
 &x_3 \geq b_{23} \\
 &x_1 - W_{1,2} - W_{1,3} - W_{1,4} - W_{1,5} = 0 \\
 &x_2 - W_{2,10} - W_{2,11} - W_{2,12} - W_{2,13} = 0 \\
 &x_3 - W_{3,18} - W_{3,19} - W_{3,20} - W_{3,21} = 0 \\
 &x_1 - U_{1,8} = 0 \\
 &x_2 - U_{2,16} = 0 \\
 &x_3 - U_{3,0} = 0 \\
 &x_k, W_{kt}, U_{kt} \geq 0 \text{ and integer, } \forall k \text{ and } t
 \end{aligned}
 \tag{4}$$

All shifts are of equal duration.

4.0 Minimum Service Requirements (b_t)

First, we assume a low probability value, say $\varphi \leq 0.1$, specified in [15, 16] as threshold probability of customers delay. Secondly, we assume that a steady state is attained at each period by approximating the varying arrival rate by its average for each one hour interval for the entire time horizon to obtain λ_t for each period t. For this work we obtained the following arrival rates (λ_t) of customers (demand for services) from a filling station for 24 mutually exclusive periods: 9.8, 9.6, 8.7, 8.1, 6.7, 5.3, 4.1, 3.3, 2.5, 2.5, 2.9, 3.8, 4.3, 5.0, 5.9, 6.6, 7.8, 8.6, 9.4, 9.8, 10.2, 10.4, 10.2, and 10.0.

The above periodic arrival rates and service rate of two customers per minute (i.e. $\mu = 2$) were obtained from [17] and were used to compute the minimum service RHS values (b_t). Consequently, the periodic arrival rates and their corresponding minimum service RHS values which are calculated in [17] are tabulated in Table 1.

Table 1: Periodic arrival rates and their corresponding RHS values

Period t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Arrival rate of Customers λ_t	9.8	9.6	8.7	8.1	6.7	5.3	4.1	3.3	2.5	2.5	2.9	3.8	4.3	5.0	5.9	6.6	7.8	8.6	9.4	9.8	10.2	10.4	10.2	10.0
Minimum number of Employees (b_t)	9	9	8	8	7	6	5	4	4	4	4	5	5	6	6	7	8	8	9	9	9	9	9	9

The model with the RHS imputed from Table 1 is stated as follows:

Minimize $z = x_1 + x_2 + x_3$ (6)

subject to

$$\left. \begin{aligned} x_1 + x_3 - U_{3,0} &\geq 9 \\ x_1 &\geq 9 \\ x_1 - W_{1,2} &\geq 8 \\ x_1 - W_{1,3} &\geq 8 \\ x_1 - W_{1,4} &\geq 7 \\ x_1 - W_{1,5} &\geq 6 \\ x_1 &\geq 5 \\ x_1 &\geq 4 \end{aligned} \right\}$$

$$\left. \begin{aligned} x_1 + x_2 - U_{1,8} &\geq 4 \\ x_2 &\geq 4 \\ x_2 - W_{2,10} &\geq 4 \\ x_2 - W_{2,11} &\geq 5 \\ x_2 - W_{2,12} &\geq 5 \\ x_2 - W_{2,13} &\geq 6 \\ x_2 &\geq 6 \\ x_2 &\geq 7 \\ x_2 + x_3 - U_{2,16} &\geq 8 \\ x_3 &\geq 8 \\ x_3 - W_{3,18} &\geq 9 \\ x_3 - W_{3,19} &\geq 9 \\ x_3 - W_{3,20} &\geq 9 \\ x_3 - W_{3,21} &\geq 9 \\ x_3 &\geq 9 \\ x_3 &\geq 9 \\ x_1 - W_{1,2} - W_{1,3} - W_{1,4} - W_{1,5} &= 0 \\ x_1 - U_{1,8} &= 0 \\ x_2 - W_{2,10} - W_{2,11} - W_{2,12} - W_{1,13} &= 0 \\ x_2 - U_{2,16} &= 0 \\ x_3 - W_{3,18} - W_{3,19} - W_{3,20} - W_{3,21} &= 0 \\ x_3 - U_{3,0} &= 0 \\ x_k, U_{kt}, W_{kt} &\geq 0 \text{ and integers.} \end{aligned} \right\} \tag{7}$$

(8)

Columns 33 through 44

0	0	0	0	0	0	0	0	0	0	-0.5000	2.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	1.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	1.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	3.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	4.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	5.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	6.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	10.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	7.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	3.0000
1.0000	-1.0000	0	0	0	0	0	0	0	0	0	1.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	3.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	2.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	2.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	1.0000
0	-3.0000	0	0	0	0	0	0	0	0	0	1.0000
0	0	0	0	-0.3333	-0.3333	-0.3333	-0.3333	0	0	0	12.0000
0	0	1.0000	0	-0.3333	-0.3333	-0.3333	-0.3333	0	0	0	4.0000
0	0	0	0	-0.3333	-0.3333	-0.3333	-0.3333	0	1.0000	0	3.0000
0	0	0	1.0000	-0.3333	-0.3333	-0.3333	-0.3333	0	0	0	4.0000
0	0	0	0	-0.3333	0.6667	-0.3333	-0.3333	0	0	0	3.0000
0	0	0	0	-0.3333	-0.3333	0.6667	-0.3333	0	0	0	3.0000
0	0	0	0	-0.3333	-0.3333	-0.3333	0.6667	0	0	0	3.0000
0	0	0	0	-0.3333	-0.3333	-0.3333	-0.3333	1.0000	0	0	3.0000
0	0	0	0	0	0	0	0	0	0	-0.5000	10.0000
0	0	0	0	0	0	0	0	0	0	1.0000	1.0000
0	-1.0000	0	0	0	0	0	0	0	0	0	7.0000
0	2.0000	0	0	0	0	0	0	0	0	0	2.0000
0	0	0	0	-0.3333	-0.3333	-0.3333	-0.3333	0	0	0	12.0000
0	0	0	0	0.6667	-0.3333	-0.3333	-0.3333	0	0	0	3.0000
0	0	0	0	0	0	0	0	0	0	-1.5000	1.0000
0	1.0000	0	0	0.3333	0.3333	0.3333	0.3333	0	0	0.5000	-29.0000

press any key to continue ...

Problem has a finite optimal solution

Values of the legitimate variables:

- x(1)= 10
- x(2)= 7
- x(3)= 12
- x(4)= 1
- x(5)= 2
- x(6)= 3
- x(7)= 4
- x(8)= 2
- x(9)= 2
- x(10)= 2
- x(11)= 1
- x(12)= 3
- x(13)= 3
- x(14)= 3
- x(15)= 3
- x(16)= 10
- x(17)= 7
- x(18)= 12

Objective value at the optimal point:

$$z = 29.000000$$

Fig. 1: Optimal Tableau.

The optimal tableau (Fig. 1) is obtained after 33 iterations

It can be seen in the optimal tableau (33rd iteration) that the optimal solution exists and is

$$x_1 = 10, x_2 = 7, x_3 = 12$$

$$W_{1,2} = 1, W_{1,3} = 2, W_{1,4} = 3, W_{1,5} = 4$$

$$U_{1,8} = 10$$

$$W_{2,10} = 2, W_{2,11} = 2, W_{2,12} = 2, W_{2,13} = 1$$

$$U_{2,16} = 7$$

$$W_{3,18} = 3, W_{3,19} = 3, W_{3,20} = 3, W_{3,21} = 3$$

$$U_{3,0} = 12$$

This problem requires a minimum of 29 employees for all the shifts with break variables (U_{kt} and W_{kt}) summarized in Table 2:

Table 2: Different Break Windows, Their Periods and No. of Employees Involved.

Shift k	Number of employees to be assigned to shift k	Lunch Break (W_{kt}) and its period	Relief Break (U_{kt}) and its period	Number of employee involved
1	10	2:00 - 3:00am(2) 3:00 - 4:00am(3) 4:00 - 5:00am(4) 5:00 - 6:00am(5)	8:00 - 9:00am(8)	1 2 3 4 10
2	7	10:00-11:00am (10) 11:00 -12:00noon(11) 12:00 - 1:00pm(12) 1:00 - 2:00pm (13)	4:00 - 5:00pm(16)	2 2 2 1 7
3	12	6:00 - 7:00pm(18) 7:00 - 8:00pm(19) 8:00 - 9:00pm(20) 9:00 - 10:00pm(21)	12:00 - 1:00am(0)	3 3 3 3 12

6.0 Discussion of Results

This Model which is a multiple shift schedule model of one lunch break and one relief break which serves as a handover period from one shift to its successor. This is a common shift schedule model applicable to real life situations. The optimal solution to this Model is summarized in Table 2. It can be seen in Table 2 that all employees of each shift observe their handover break at the last period of their shift and also observe their lunch break at the assigned period. For example, from the output summary in Table 2, all the ten employees assigned to shift 1 should observe their relief break at the last period (8.00-9.00a.m.). Also in shift 3 only three employees should have their lunch break in periods 18 to 21 inclusive. Any violation of this will result in increased total labour cost.

7.0 Conclusion

The numerical illustration of the model was done using three shifts daily each of nine-hour duration. The last period of a shift is called the relief break or handover period for the outgoing shift which is also the first period for the incoming shift, hence the overlap. It is important that the optimal solution of a shift schedule model should specify the number of employees to be assigned to each shift. Furthermore, there is need to know the number of employees in a given shift that should be on break in each period while the shift lasted such that the total labour cost is minimized. These among others have been achieved in the practical application of this model.

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