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Solution of Certain Boundary-Value Problems In A Semi-Finite Domain By Adomian Sumudu Transform Decomposition Method

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Abstract

This paper presents a simple modification of Sumudu Transform method for the solution of the generalized extended Blasius equation with the two forms of boundary conditions. Pade approximation is used to deal with the first form of boundary conditions while Wang Transformation and Pade approximation are used for the second form of boundary conditions. Adomian Polynomials are employed to decompose the nonlinear terms involved. Comparison of the results obtained with the existing results show the reliability and effectiveness of the method.

Keywords: Adomian Polynomials, Blasius equation, Pade approximation, Sumudu Transform

1. Introduction:

The Blasius differential equation arises in the theory of fluid boundary layer, and in general, must be solved numerically and because of this it has been the major concern of many researchers and therefore much progress has been made to solve this equation. Blasius [1] was the first to show that this problem provided a special solution to the Prandtl boundary layer equations.

The main applications of boundary-layer theory is devoted to the calculation of the skin-friction drag acting on a body moving through a fluid, for example the drag of: an airplane wing, a turbine blade, or a complete ship. Schlichting and Gersten [2] and Blasius main interest then was to compute, without worrying about existence or uniqueness of its BVP solution, the value of $\lambda = \frac{d^2 f(0)}{d\eta^2}$ i. e., the skin-friction coefficient. Due to the increasing number of applications of microelectronics devices, boundary-layer theory has found a renewal of interest within the study of gas and liquid flows at the micro-scale regime [3, 4].

The generalized (extended) Blasius equation considered is given as:

$$f'''(\eta) + \beta f(\eta) f''(\eta) = 0 \tag{1}$$

subject to either the first form of boundary condition

$$f(0) = 0, \qquad f'(0) = 1, \qquad f'(\infty) = 0 \tag{2}$$

or the second form of boundary conditions

$$f(0) = 0, \qquad f'(0) = 0, \qquad f'(\infty) = 1 \tag{3}$$

In recent time, many analytical, semi-analytical and numerical methods had been devised to provide the needed solution to the problem of Blasius equation among which are variational iterative and Hybrid variation iteration methods[5-9], Pade approximation[10-11], Shooting Method via Taylor Series[12], Differential Transform method [13], Homotopy Analysis method [14], Adomian Sumudu Transform method[15] and many other methods. Therefore, this paper aims at providing solution to the generalized extended Blasius equation with the two forms of boundary conditions via simple modification of Adomian Sumudu Transform method and compare the results with the existing ones.

2.0 Mathematical Formulation of Adomian Sumudu Transform Decomposition Method(ASTDM)

Given a general nonlinear non-homogeneous differential equation

$$LU(x) + RU(x) + NU(x) = g(x), \tag{4}$$

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where L is the highest order linear differential operator, R is the linear differential operator of order less than L, N is the nonlinear differential operator, U is the dependent variable, x is an independent variable and g(x) is the source term.
Application of the Sumudu Transform on equation (4) resulted into

$$S[LU(x)]+S[RU(x)]+S[NU(x)]=S[g(x)], \tag{5}$$

where S denotes the Sumudu Ttransform.
Using the differentiation property of the Sumudu transform in (5) we get

$$\frac{S[U(x)]}{u^m}-\sum_{k=o}^{m-1}\frac{U(x)^{(k)}(0)}{u^{(m-k)}}+S[RU(x)]+S[NU(x)]=S[g(x)], \tag{6}$$

where

$$\sum_{k=o}^{m-1}\frac{U(x)^{(k)}(0)}{u^{(m-k)}}=\sum_{k=o}^{m-1}\frac{U(0)^{(k)}}{u^{(m-k)}}$$

Further simplification of (6) gives

$$S[U(x)]-u^m\sum_{k=o}^{m-1}\frac{U(x)^{(k)}(0)}{u^{(m-k)}}+u^m[S[RU(x)]+S[NU(x)]-S[g(x)]]=0 \tag{7}$$

Application of Sumudu inverse Transform on (7) yields

$$U(x)=G(x)-S^{-1}\left[u^m\left[[S[RU(x)]+S[NU(x)]]\right]\right], \tag{8}$$

where

$$G(x)=S^{-1}\left[u^m\left[\sum_{k=o}^{m-1}\frac{U(x)^{(k)}(0)}{u^{(m-k)}}+S[g(x)]\right]\right] \tag{9}$$

Thus, G(x) represents the term arising from the source term and the prescribed initial conditions.
The representation of the solution (8) as an infinite series is given below.

$$U(x)=\sum_{n=o}^{\infty}U_n(x) \tag{10}$$

The nonlinear term is been decomposed as:

$$NU(x)=\sum_{r=0}^{\infty}A_n(u_0,u_1,...u_n), \tag{11}$$

where A_n are the Adomian polynomials of functions U_0, U_1, $U_2 \ldots U_n$ and can be calculated by formula given in [16] as:

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{r=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0} \quad n=0,1,2,... \tag{12}$$

Substituting (10) and (11) into (8) yields

$$\sum_{n=o}^{\infty}U_{n+1}(x)=U_0(x)-S^{-1}\left[Su^m\left[[[R\sum_{n=o}^{\infty}U_n(x)]+[\sum_{n=o}^{\infty}A_n]]\right]\right], \tag{13}$$

where

$$U_0(x)=G(x)=S^{-1}\left[u^m\left[\sum_{k=o}^{m-1}\frac{U(x)^{(k)}(0)}{u^{(m-k)}}+S[g(x)]\right]\right]. \tag{14}$$

We obtain from equation (13) the recursive relation given by

$$U_{n+1}(x)=-S^{-1}\left[Su^m\left[[[RU_n(x)]+[A_n]]\right]\right], \qquad n\geq 0 \tag{15}$$

Simplifying equation (15), we obtain in turn U_0, U_1, $U_2 \ldots U_n$. With these, the general solution is obtained as

$$U(x)=U_0(x)+U_1(x)+U_2(x)+U_3(x)+... \tag{16}$$

3.0 Numerical Application of STDM

In this section we apply STDM illustrated in previous section on the generalized Blasius equation with the two forms of boundary conditions for the particular cases $\beta=\frac{1}{2}$ and $\beta=1$ to demonstrate the efficiency of the method. The results obtained are compared with existing results in the literature.

3.1 Blasius Equation with the First Form of Boundary Conditions

Consider

$$f'''(\eta)+\beta f(\eta)f''(\eta)=0, \tag{17}$$

subject to the initial-boundary conditions

$$f(0)=0,\ f'(0)=1,\ f'(\infty)=0 \tag{18}$$

Let $f''(0)=\alpha$

we obtain the solution of $f(\eta)$ in terms of α for $\beta=\frac{1}{2}$ and $\beta=1$ respectively as follows:

$$f(\eta)=\eta+\frac{1}{2}\alpha\eta^2-\frac{1}{48}\alpha\eta^4-\frac{1}{240}\alpha^2\eta^5+\frac{1}{960}\alpha\eta^6+\frac{11}{20160}\alpha^2\eta^7+$$
$$(-\frac{1}{21504}\alpha+\frac{11}{161280}\alpha^3)\eta^8-\frac{43}{967680}\alpha^2\eta^9+... \tag{19}$$

$$f(\eta)=\eta+\frac{1}{2}\alpha\eta^2-\frac{1}{24}\alpha\eta^4-\frac{1}{120}\alpha^2\eta^5+\frac{1}{240}\alpha\eta^6+\frac{11}{5040}\alpha^2\eta^7+$$
$$(-\frac{1}{2688}\alpha+\frac{11}{40320}\alpha^3)\eta^8-\frac{43}{120960}\alpha^2\eta^9+..., \tag{20}$$

where $\alpha=f''(0)$.

In order to determine the unknown value αin (19) and (20) we have to apply the third boundary condition, that is $f'(\infty)=0$. For that, Pade approximants of (19) and (20) which enlarge convergence radius of the solution are used. The values of $f''(0)$ obtained for $\beta=\frac{1}{2}$ and $\beta=1$ are tabulated in tables 1 and 2 respectively.

Table 1: Comparison of numerical value of $\alpha=f''(0)$ for $\beta=\frac{1}{2}$

Pade Approximants	**Our Method**	**Khan & Hussain [17]**	**Wazwaz [18]**	**Wazwaz [19]**	**Chun *et al.*[20]**
[2/2]	0.5773502692	0.5773315430	0.5773502692	0.577350693	0.577350693
[3/3]	0.5163977795	0.5163574219	0.5163977795	0.5163777793	0.5163777795
[4/4]	0.5227030798	0.5227050781	0.5227030798	0.52277030796	0.5227030798

Table 2: Numerical value of $\alpha=f''(0)$ for $\beta=1$

Pade Approximants	**Our Method**
[2/2]	0.816496580
[3/3]	0.7302967333
[4/4]	0.7392137845

1.1 Blasius equation with the second form of boundary conditions.

Consider

$$f'''(\eta)+\beta f(\eta)f''(\eta)=0 \tag{21}$$

subject to the initial-boundary conditions

$$f(0)=0,\ f'(0)=0,\ f'(\infty)=1 \tag{22}$$

Let $f''(0)=\alpha$

Following the same procedure above we obtain the solution $f(\eta)$ for (21)-(22) in terms α for $\beta=\frac{1}{2}$ and $\beta=1$ respectively as:

$$f(\eta)=\frac{1}{2}\alpha\eta^2-\frac{1}{240}\alpha^2\eta^5+\frac{11}{161280}\alpha^3\eta^8-\frac{5}{4257792}\alpha^4\eta^{11}+\frac{9299}{464950886400}\alpha^5\eta^{14}-$$
$$\frac{1272379}{3793999233024000}\alpha^6\eta^{17}+\frac{19241647}{3460127300517888000}\alpha^7\eta^{20}+\ldots \qquad (23)$$

$$f(\eta)=\frac{1}{2}\alpha\eta^2-\frac{1}{120}\alpha^2\eta^5+\frac{11}{40320}\alpha^3\eta^8-\frac{5}{532224}\alpha^4\eta^{11}+\frac{9299}{29059430400}\alpha^5\eta^{14}-$$
$$\frac{1272379}{118562476032000}\alpha^6\eta^{17}+\frac{19241647}{54064489070592000}\alpha^7\eta^{20}+\ldots \qquad (24)$$

To determine the unknown value αin (23) and (24), Wang [18] has shown that the transformation to (18) is given as

$$y''(x)+\beta\frac{x}{y(x)}=0 \qquad (25)$$

subject to the boundary conditions

$$y(0)=\alpha, y(1)=0, y'(0)=0, \quad 0\le x\le 1 \qquad (26)$$

Solving (25) by STDM alongside the condition (26) we have the truncated $y(x)$ for $\beta=\frac{1}{2}$ and $\beta=1$ respectively

$$y(x)=\alpha-\frac{x^3}{12\alpha}-\frac{x^6}{720\alpha^3}-\frac{x^9}{17280\alpha^5}-\frac{x^{12}}{304128\alpha^7}-\frac{2099x^{15}}{9580032000\alpha^9}-$$
$$\frac{31453x^{18}}{1954326528000\alpha^{11}}-\frac{46061x^{21}}{36480761856000\alpha^{13}}-\frac{62749793x^{24}}{604121416335360000\alpha^{15}}+\ldots \qquad (27)$$

and

$$y(x)=\alpha-\frac{x^3}{6\alpha}-\frac{x^6}{180\alpha^3}-\frac{x^9}{2160\alpha^5}-\frac{x^{12}}{19008\alpha^7}-\frac{2099x^{15}}{299376000\alpha^9}-$$
$$\frac{31453x^{18}}{30536352000\alpha^{11}}-\frac{46061x^{21}}{285005952000\alpha^{13}}-\frac{62749793x^{24}}{2359849282560000\alpha^{15}}+\ldots \qquad (28)$$

The unknown parameter $\alpha=f''(0)$ can be found by solving (27) and (28) with $y(1)=0$ to obtain the results obtained are tabulated in tables 3 and 4 respectively.

Table 3: Comparison of numerical value of $\alpha=f''(0)$ for $\beta=\frac{1}{2}$

Our Method	**Sajid [9]**	**Oderinu & Aregbesola [12]**	**Howarth [21]**	**He [22]**	**Agbakhani *et al.* [23]**
0.3315634093	0.3320573373	0.3320573372	0.33206	0.54360	0.33205

Table 4: Comparison of numerical value of $\alpha=f''(0)$ for $\beta=1$

Our Method	**Oderinu & Aregbesola [12]**	**Zhang & Chen [24]**	**Salama[25]**	**Asaithambi[26]**	**Asaithambi [27]**
0.4689014702	0.4695999884	0.469600	0.469600	0.46900	0.469601

4.0 Conclusion

The simple modification of Sumudu Transform Decomposition Method for the solution of the generalized extended Blasius equation with the two forms of boundary conditions is proposed in this paper. The developed algorithm combines the features of Sumudu Transform, Adomian Decomposition, Pade Approximation and Wang Transformation. The comparison of the skin-friction drag acting on a body moving through a fluid with the existing methods showthat thereis good agreement between the present method and the existing one.

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