Transactions of the Nigerian Association of Mathematical Physics
Volume 3, (January, 2017), **pp 271 – 278**

Modeling the impact of Reserve Zones on MSY and Coexistence Equilibrium Points for a Bioeconomic Model of Tilapia and Nile Perch

A. Umar and S. Musa

Department of Mathematics, Modibbo Adama University of Technology,
P.M.B 2076, Yola – Nigeria.

Abstract

Fish populations are becoming increasingly limited, catches are declining due to overexploitation and extinction of fish biomass as a result of difficult sustainability of fish stock. The aim of the research is to determine an optimal harvesting strategy which fulfills the economic objective of the harvester while maintaining the population density of the fish species over a prespecified threshold values throughout the harvest. To achieve that, the research developed a modified version of a bioeconomic model for Prey-Predator interaction in Polluted Environment with Constant Harvesting by incorporating reserve zones. Maximum Sustainable Yield (MSY) and coexistence equilibrium points of both Tilapia and Nile perch are determined. The study revealed that, migration rate beyond $\phi = 0.7$ ***has no impact whatsoever on the MSY and coexistence equilibrium point of the Tilapia and Nile perch. Similarly, the study revealed that the optimal economic rent of Tilapia and Nile perch species in unreserve zone increases with varied migration rate. Thus, the study recommends creation of reserve zones for efficient control of overexploitation and extinction of fish biomass in both reserve and unreserve zones and Migration rate at*** $\phi = 0.7$ ***should be kept.***

Keywords: Bioeconomic, Reserve Zone, MSY, MEY,Coexistence equilibrium point.

1.0 Introduction

Fish is one of the major and healthiest sources of protein to human beings; not only that, it does provide man with employment, business opportunities, and recreational activities among other things. According to [1] tilapia fish farming has been an important source of protein of the world and it is well suited for farming, since they are fast growing and hardy. Many developing countries such as Nigeria evolved seriously engages its citizens in the fishing venture. In 2015, the Fishing industry in Nigeria contributed 3.5% to the national GDP contended by National Bureau of Statistics [2].Though, fish biomass has been considered globally to decline due to intensive harvesting, water pollution, and Predatory effects among the species of fish[3,4]. These lead to the phenomenon of overexploitation and extinction of fish stock.

In an attempt to provide better control strategies to curb the menace of overexploitation and extinction of fish stock, successful researchesresults are in the following studies:[4-21].

In this paper we modified the model due to [4] by incorporating reserve zones, natural death rate and Maximum Economic Yield (MEY). Our aim is to study the impact of reserve zones on Maximum Sustainable Yield (MSY), Maximum Economic Yield (MEY), and optimal economic rent of the model

2.0 Materials and Methods

The total populations of the fish species in our model is subdivided into four compartments (see Figure 1). We presented the model variables in Table 1 and parameters in Table 2.

Corresponding author: A. Umar, E-mail: ahmedumar42@gmail.com, Tel.: +2347033181737

Table 1: Model Variables and their Description

Symbol	Descriptions
$x(t)$	Number of Tilapia perch in unreserve zone at time t
$y(t)$	Number of Nile perch in unreserve zone at time t
$x_R(t)$	Number of Tilapia perch in reserve zone at time t
$y_R(t)$	Number of Nile perch in reserve zone at time t
$E_T(t)$	Economic rent for Tilapia perch populations at time t
$E_N(t)$	Economic rent for Nile perch populations at time t

Table 2: Model Parameters and their Description

Symbol	Descriptions
γ_1	Intrinsic growth rate of Tilapia perch in the unreserve zone
γ_2	Intrinsic growth rate of Nile perch in the unreserve zone
γ_3	Intrinsic growth rate of Tilapia perch in the reserve zone
γ_4	Intrinsic growth rate of Nile perch in the reserve zone
k_1	Environmental carrying capacity for Tilapia perch in the unreserve zone
k_2	Environmental carrying capacity for Nile perch in the unreserve zone
k_3	Environmental carrying capacity for Tilapia perch in the reserve zone
k_4	Environmental carrying capacity for Nile perch in the reserve zone
β_1	The maximal relative increase of predation
β_2	Conversion factor from Prey to Predator
A	Saturation constant
d_1	Death rate of Tilapia perch due to water pollution
d_2	Death rate of Nile perch due to water pollution
q_1	Catchability coefficient for Tilapia perch
q_2	Catchability coefficient for Nile perch
μ_1	Stiffness parameter for Tilapia perch
μ_2	Stiffness parameter for Nile perch
p_1	Constant price per unit biomass for Tilapia perch
p_2	Constant price per unit biomass for Nile perch
c_1	Constant cost per unit biomass for Tilapia perch
c_2	Constant cost per unit biomass for Nile perch
ϕ_1	Migration rate of Tilapia perch from reserve to unreserve zone
ϕ_2	Migration rate of Nile perch from reserve to unreserve zone
σ_1	Natural death rate of Tilapia perch in unreserve zone
σ_2	Natural death rate of Nile perch in unreserve zone
σ_3	Natural death rate of Tilapia perch in reserve zone
σ_4	Natural death rate of Nile perch in reserve zone
E_1	Harvesting effort for Tilapia perch
E_2	Harvesting effort for Nile perch

2.1 Model Assumption

Assumptions of the model is given as follows:

i. populations of Tilapia perch and Nile perch subdivided into reserve and unreserve zones;
ii. migration is consider from reserve zone to unreserve zone ;
iii. we assumed that water in the reserve zones to be free form pollution;
iv. it is assumed that in each zone the population is homogeneous
v. natural death rate in unreserve zone is relatively greater than that of reserve zone
vi. For simplicity, it is assumed that the growth rate of Tilapia perch is relatively greater than that of Nile perch in unreserved zone, while both Tilapia and Nile perch to have equal growth rate in reserve zone.

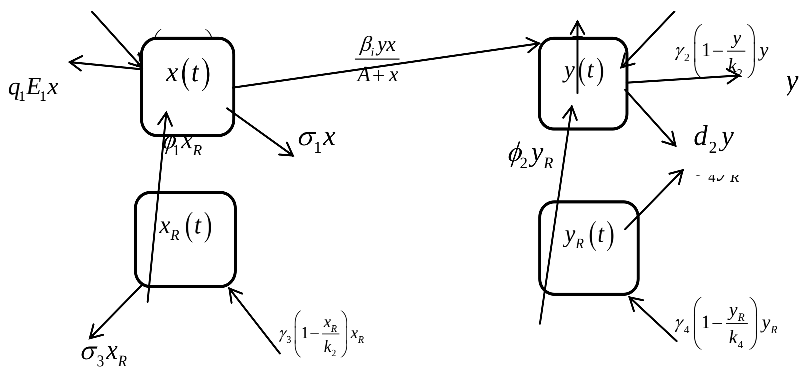

Figure 1: Flow Diagram for the Model

2.2 Model Equations

$$\dot{x}(t)=\gamma_1\left(1-\frac{x}{k_1}\right)x-q_1E_1x-\frac{\beta_1 yx}{A+x}-d_1x-\sigma_1x+\phi_1x_R \tag{1}$$

$$\dot{y}(t)=\gamma_2\left(1-\frac{y}{k_2}\right)y-q_2E_2y+\frac{\beta_2 yx}{A+x}-d_2y-\sigma_2y+\phi_2y_R \tag{2}$$

$$\dot{x}_R(t)=\gamma_3\left(1-\frac{x_R}{k_3}\right)x_R-\phi_1x_R-\sigma_3x_R \tag{3}$$

$$\dot{y}_R(t)=\gamma_4\left(1-\frac{y_R}{k_4}\right)y_R-\phi_2y_R-\sigma_4y_R \tag{4}$$

$$\dot{E}_T(t)=\mu_1(p_1q_1x-c_1)E_1 \tag{5}$$

$$\dot{E}_N(t)=\mu_2(p_2q_2y-c_2)E_2 \tag{6}$$

$$x(0)=x^0,\ y(0)=y^0,\ E_T(0)=E_T^0,\ E_N(0)=E_N^0,\ x_R(0)=x_R^0,\ y_R(0)=y_R^0,\ t=0 \tag{7}$$

3.0 Results

In this section we present the analyticaland Numerical results obtained in this work

3.1 Coexistence Equilibrium Point of the Model

In this section, we were able to establish the coexistence equilibrium point of the model equations in (1) - (6) as given below

$$P^*=\left(\omega_1,\omega_2,\frac{b_1k_3}{\gamma_3},\frac{b_2k_4}{\gamma_4},\frac{(A+\omega_1)\left[\gamma_3\left(a_1k_1+\gamma_1\omega_1\right)+k_1k_3b_1\phi_1\right]-k_1\gamma_3\beta_1\omega_2}{k_1q_1\gamma_3\left(A+\omega_1\right)},\frac{(A+\omega_1)\left[\gamma_4\left(a_2k_2+\gamma_2\omega_2\right)+k_2k_4b_2\phi_2\right]+k_2\gamma_4\beta_2\omega_1}{k_2q_2\gamma_4\left(A+\omega_1\right)}\right) \tag{8}$$

Where $a_i=\left(\gamma_i-d_i-\sigma_i\right),\ i=1,2.\ b_i=\left(\gamma_j-\phi_i-\sigma_j\right),\ i=1,2.\ \ j=3,4.\quad \omega_i=\frac{c_i}{p_iq_i},\ i=1,2.$

3.2 Maximum Sustainable Yield (MSY) Points and Maximum Economic Yield (MEY) Point of the Model and Conditions for Existence.

The right hand side (RHS) of the model equation (1) could be viewed as sum of function $f(x)$, $F(x,y)$, and $f(x_R)$. We have

$$\dot{x}(t) = f(x) + F(x,y) + f(x_R) \tag{9}$$

Such that $$f'(x)\begin{cases} >0, & x < x_{msy} \\ =0, & x = x_{msy} \end{cases} \tag{10}$$

Note that: Maximum Sustainable Yield (for single stock)

$$f(x) = a_1 x - \frac{\gamma_1}{k_1}x^2 - q_1 E_1 x \tag{11}$$

$$f'(x) = a_1 - 2\frac{\gamma_1}{k_1}x - q_1 E_1 \tag{12}$$

Thus, at $x = x_{msy}$ we get $f'(x) = 0$, implying that $x_{msy} = \frac{k_1}{2\gamma_1}\left(a_1 - q_1 E_1\right)$ (13)

In order to get the positive value of x_{msy}, the following condition must be satisfied

$$\gamma_1 > d_1 + \sigma_1 + q_1 E_1 \tag{14}$$

Condition (14) suggests that the intrinsic growth rate of Tilapia perch in unreserve zone must exceed the sum of death rate of Tilapia perch due to water pollution, Natural death rate and harvesting rate of Tilapia perch; otherwise Tilapia specie in unreserve zone will go extinction.

Similarly, The right hand side (RHS) of the model equation (2) could be viewed as a functional sum of $g(x)$, $G(x,y)$, and $g(x_R)$. We have

$$\dot{y}(t) = g(y) + G(x,y) + g(y_R) \quad \text{Such that } g'(y)\begin{cases} >0, & y < y_{msy} \\ =0, & y = y_{msy} \end{cases}$$

$$g(y) = a_2 y - \frac{\gamma_2}{k_2}y^2 - q_2 E_2 y$$

$$g'(y) = a_2 - 2\frac{\gamma_2}{k_2}y - q_2 E_2$$

Thus, at $y = y_{msy}$ we get $g'(y) = 0$, implying that $y_{msy} = \frac{k_2}{2\gamma_2}\left(a_2 - q_2 E_2\right)$ (15)

In order to get the positive value of y_{msy}, the following condition must be satisfied

$$\gamma_2 > d_2 + \sigma_2 + q_2 E_2 \tag{16}$$

Condition (16) suggests that the intrinsic growth rate of Nile perch in unreserve zone must exceed the sum of death rate of Nile perch due to water pollution, Natural death rate and harvesting rate of Nile perch; otherwise Nile perch specie in unreserve zone will go extinction.

By the same token, the right hand side (RHS) of the model equation (3) could be viewed as a function of $h(x_R)$. We have

$$\dot{x}_R(t) = h(x_R)$$

Such that $$h'(x_R)\begin{cases} >0, & x_R < x_{R(msy)} \\ =0, & x_R = x_{R(msy)} \end{cases}$$

$$h(x_R) = b_1 x_R - \frac{\gamma_3}{k_3} x^2{}_R$$

$$h'(x_R) = b_1 - 2\frac{\gamma_3}{k_3} x_R$$

Thus, at $x_R = x_{R(msy)}$ we get $h'(x_R) = 0$, implying that $x_{R(msy)} = \dfrac{b_1 k_3}{2\gamma_3}$ (17)

In order to get the positive value of $x_{R(msy)}$, the following condition must be satisfied

$$\gamma_3 > \phi_1 + \sigma_3 \qquad (18)$$

Condition (19) suggests that the intrinsic growth rate of Tilapia perch in reserve zone must exceed the sum of migration rate of Tilapia perch from reserve to unreserve zone and Natural death rate of Tilapia perch in reserve zone; otherwise Tilapia perch specie in the reserve zone will go extinction.

In the same way, the right hand side (RHS) of the model equation (4) could be viewed as a function of $l(y_R)$. We have

$$\dot{y}_R(t) = l(y_R)$$

Such that $l'(y_R)\begin{cases} > 0, & y_R < y_{R(msy)} \\ = 0, & y_R = y_{R(msy)} \end{cases}$

$$l(y_R) = b_2 y_R - \frac{\gamma_4}{k_4} y^2{}_R$$

$$l'(y_R) = b_2 - 2\frac{\gamma_4}{k_4} y_R$$

Thus, at $y_R = y_{R(msy)}$ we get $l'(y_R) = 0$, implying that $y_{R(msy)} = \dfrac{b_2 k_4}{2\gamma_4}$ (19)

In order to get the positive value of $y_{R(msy)}$, the following condition must be satisfied

$$\gamma_4 > \phi_2 + \sigma_4 \qquad (20)$$

Condition (20) suggests that the intrinsic growth rate of Nile perch in reserve zone must exceed the sum of migration rate of Nile perch from reserve to unreserve zone and Natural death rate of Nile perch in reserve zone; otherwise Nile perch specie in reserve zone will go extinction.

Proposition 3.2.1

The existence of MSY points for Tilapia and Nile perch in both reserve and unreserve zone must satisfied conditions in (14), (16), (18) and (20) respectively.

3.3 The Maximum Economic Yield Point and its Conditions for Existence

Gordon's model established that the net revenue (Sustainable Economic Rent) derived from fishing as a function of Total Sustainable Rent (TSR) and Total Costs (TC) is given by

Sustainable Economic Rent= $TSR - TC$

$$SER = pqkE\left(1 - \frac{qE}{\gamma}\right) - cE \qquad (21)$$

From equation (21), the maximum Sustainable Economic Rent occurs at fishing effort is

$$\frac{d(SER)}{dE} = E_{MEY} = \frac{\gamma}{2q}\left(1 - \frac{c}{pqk}\right) \qquad (22)$$

In order to get the positive value of E_{MEY}, the following condition must be satisfied

$$\frac{c}{p} < qk \qquad (23)$$

Proposition 3.3.1: The existence of MEY points for both Tilapia and Nile perch must satisfied the condition in (23). It suggests that the fishing cost price ratio is less than the product of effort exerted and the carrying capacity, such that fish resources could be exploited.

3.4 Numerical Results

In this section, we present the numerical results of the model by establishing the equilibrium points, Maximum Sustainable Yield, Maximum Economic Yield and Optimal economic rent of the model. We used the baseline values for the variables and parameters as in Table 3 for computed results. We also computed the impact of reserve zones on MSY, coexistence equilibrium point and optimal economic rent with varied migration rate from reserve to unreserve zones (see table 4-5). In addition, the computed results for MEY at equilibrium effort is presented in Table 6.

Table 3: The baseline value for Variables and Parameters for the Model for Prey-Predator Interaction in Polluted Environment with Constant Harvesting Strategy and Reserve Zones

Parameter	Value	Source
γ_1	0.80	[4]
γ_2	0.65	[4]
γ_3	0.90	Assumed
γ_4	0.90	Assumed
k_1	600000	[4]
k_2	500000	[4]
k_3	600000	[4]
k_4	500000	[4]
β_1	0.000005	[4]
β_2	0.000003	[4]
A	60000	[4]
d_1	0.2	[4]
d_2	**0.2**	**[4]**

Continuation of table 3

q_1	**0.000005**	**[4]**
q_2	0.000012	[4]
μ_1	0.1	[4]
μ_2	0.12	[4]
p_1	750	[4]
p_2	700	[4]
c_1	500	Assumed
c_2	500	Assumed
ϕ_1	0.5	Assumed
ϕ_2	0.5	Assumed
σ_1	0.2	Assumed
σ_2	0.2	Assumed
σ_3	0.1	Assumed
σ_4	0.1	Assumed
E_1	1.20	[4]
E_2	1.50	[4]

Table 4: Computed Results of the Impact of Reserve Zone on Maximum Sustainable Yield and Coexistence equilibrium point of the Fish Species with varied Migration Rate from Reserve to Unreserve Zone

Change in Migration Rate	$(x_{msy}, y_{msy}, x_{R(msy)}, y_{R(msy)})$	(x^*, y^*, x_R^*, y_R^*)
0.5	(150000, 96153, 100000, 100000)	(133333, 59524, 200000, 166667)
0.6	(150000, 96153, 666667, 666667)	(133333, 59524, 133333, 111111)
0.7	(150000, 96153, 33333, 33333)	(133333, 59524, 66667, 55556)
0.8	(150000, 96153, 0, 0)	(133333, 59524, 0, 0)
0.9	(150000, 96153, 0, 0)	(133333, 59524, 0, 0)
1	(150000, 96153, 0, 0)	(133333, 59524, 0, 0)

Table 5: Computed Results of the Impact of Reserve Zone on Optimal Economic Rent of the Fish Species with varied Migration Rate from Reserve to Unreserve Zone

Change in migration rate	E_T^*	E_N^*
0.5	20,000,115,591	66,754,772,177
0.6	24,000,115,591	80,105,669,726
0.7	28,000,116,697	93,456,567,276
0.8	32,000,115,591	106,807,464,825
0.9	36,000,114,485	120,158,362,374
1	40,000,116,144	133,509,254,607

Table 6: Computed Results for Gordon-Schaefer Economic model Using of Parameter Values as Provided in Table 3

Specie	E_{MEY}	E_{MSY}	E_{BE}
Tilapia perch	62,222	80,000	124,444
Nile perch	23,359	27,083	47,718

4.0 Discussion

In the computed results of the model in the presence of reserve zone, with varied migration rate at Maximum Sustainable Yield point and coexistence equilibrium points of the fish species in the unreserve zone changes respectively. Harvesting is restricted in the reserve zone we no longer need threshold values to control harvesting. As such, add third coordinate to the first coordinate and fourth to the second coordinate in both MSY and coexistence equilibrium points. Migration rate beyond 0.7 in both MSY and coexistence equilibrium points is similar to migration rate at 0.5 (see Table 4). Equivalently in the computed results, increasing migration rate contemporaneously increases optimal economic rentof Tilapia and Nile perch species in the unreserve zone (see Table 5).The study revealed that, migration rate beyond 0.7 has no impact whatsoever on the MSY and coexistence equilibrium point of the fish species. Thus, optimal economic rentof Tilapia and Nile perch species in unreserve zone is attain at migration rate 0.7.

In this work, we computed the equilibrium effort at MEY, MSY, and Bionomic equilibrium as provided in Table 6. The study revealed that, $E < E_{BE}$ meaning that the fishery is more profitable and hence in an open access fishery, it would attract more and more fishermen. As such, it has an increasing effect on harvesting effort. Therefore, in as much as the harvesting effort increases as the results of invariably influx of fishermen; then, Both MSY and MEY declines. [20]contended that, the situation of $E < E_{BE}$ cannot be maintained indefinitely. That contention quite agrees with our results which revealed that as the unreserve zone receives spillover from the reserve zone with varied migration rate; then, there would be an increase in optimal economic effort portrayed the influx of fishermen. Nonetheless, by reason of high harvesting pressure from the fishermen; it adversely affect the change from $E < E_{BE}$ to $E > E_{BE}$ (i.e Sustainable economic rent is negative) meaning that some fisheries are losing money and therefore drops out of market, thus decreasing total harvesting effort.

5.0 Conclusion

A model due to [4] have been improved, proposed and studied. We establishing the coexistence equilibrium point of the improved model. Computed results of the impact of reserve zone on MSY and coexistence equilibrium points are presented in Table 4. We also computed results of the impact of reserve zone on optimal economic rents presented in Table 5. Similarly, we computed the equilibrium effort at MEY, MSY, and Bionomic equilibrium as provided in Table 6.

The computed results of impact of reserve zone on optimal economic rents, show that with varied migration rateincreases the optimaleconomic rent of both fish species. However, this alters MSY, and coexistence equilibrium points of both fish species in the unreserve zones. In fact, migration rate beyond 0.7 provide similar value toMSY, and coexistence equilibrium points of the Tilapia and Nile perch species in reserve zone. The study however suggested that, migration rate at 0.7 should be sustained in the reserve zone in order to prevent resurfacing of overexploitation and extinction of fish stock.

6.0 Recommendation

We recommend the creation of reserve zones to both private and public fishing industries. Nevertheless, in order to prevent intensive harvesting efforts which obviously causes overexploitation and extinction of fish biomass; therefore $\phi = 0.7$ as migration rate should be retain.

7.0 References

[1] Gertjan, D.G., Pieter, J.D., Bram, H. & Johan A.J.V. (2005). Simulation of Nile tilapia culture in ponds, through individual based modeling using a population dynamic approach. Aquaculture Research **36**: 455-471.

[2] National Bereau of Statistics (2015). Nigerian Gross Domestic Product Report Quarter one. Nigeria

[3] Massey, D. D., Newbold, S. C., & Gentner, B. (2006). Valuating water quantity charges using a bioeconomic model of a coastal recreational fishery.In Mayengo, M. M., Luboobi, L.S., and Kuznetsov, D. (2014). Bioeconomic model for Tilapia-Nile perch fishery in polluted Environment with constant effort in Tanzania water lake Victoria. Mathematical theory and Modelling, **4**(7), 113-128

[4] Mayengo, M. M., Luboobi, L. S., & Kuznetsov, D. (2014). Bioeconomic model for Tilapia-Nile perch fishery in polluted Environment with constant effort in Tanzania water Lake Victoria. Mathematical theory and Modelling, **4**(7), 113-128

[5] Roughgarden, J. & Smith, F. (1996). Why fisheries Collapse and What to do about it. Proc. Natl. Acad. Sci., USA, **99**(10), 5079-5083

[6] Trexler, J., & Travis, J. (2000). Can marine protected areas conserve stock attributes? Bulletin of marine Science. Press.

[7] Xu, C., Boyce, M., S., & Daley, D., J., (2005). Harvesting in seasonal environments. Journal of Mathematical Biology, **50**(6), 663–682.

[8] Clark C., W., Munro G., R., &Sumaila U., R. (2005) Subsidies, buybacks, and sustainable fisheries. J. Environ. Econ. Manag. 50, 47–58

[9] Vijavasanya, H. G. M., Rappon, S., Murugean, R., & Srivini, V. (2015). A prey-predator model with Vulnerable infected prey consisting of non-linear feedbacks. Applied mathematical Sciences, 42, 2091-2102

[10] Kar, T. K., & Chaudhuri, K. S. (2003). Regulation of predator-prey fishery by taxation: A dynamic reaction model. Journal of Biological System, **11**(2), 173-187

[11] Kar, T. K., & Matsuda, H. (2006). Controlability of a harvested predator-prey system with time delay. Journal of Biology system, **14**(2), 243-253

[12] Kar, T. K., &Matsuda, H. (2007). Global dynamics and controllability of a harvested predator-prey system with Holling type Ⅲ functional responses, Non-linear Analysis. Hybrid system, 1, 59-67.

[13] Dubey,B. (2007). A prey-predator model with a reserve area. Non-linear Analysis Modelling and Control, **12**(4), 479-494

[14] Zhang, J. R., & Yang, H. (2007). Analysisofaprey-predator fishery model with prey reserve. Applied Mathematical Sciences, **1**(49–52), 2481–2492.

[15] Kar, T. K., &Chakraborty, K. (2010). Effot dynamics in a predator-prey model with harvesting. International Journal of information and System Sciences, **6**(3), 318-332

[16] Daga, N., Singh, B., Jai, S., & Ujjainkar, G. (2012). Ecological System of a Prey-Predator model with Prey reserve. International Journal of Science and Research. **3**(7), 1-5. ISSN (Online): 2319-7064, Impact Factor (2012): 3.398

[17] Kar, T., K., & Chakraborty, K. (2012). Economic persprctive of marine reserves in fisheries: A bioeconomic model. Journal of Mathematical Bioscience,**240**(2012), 212-222. Http://dr.doi.org/10.1016/j.mbs.2012.07.008

[18] Zhang, Y., Zhang, Q., and Bai, F. (2012). Dynamic and Optimal taxation Control in a Bioeconomic model with Stage Structure and Gestation Delay. Mathematical Problem in Engineering, 2012, 1-17, Article ID 803270. Doi:10.1155/2012/803270

[19] Sharma, A., & Gupta, B. (2014). Harvesting model for fishery resource with reserve area and Bird Predator. Journal of Marine Biology, 2014,1-7, Article ID 218451. http://dx.doi.org/10.1155/2014/218451

[20] Dubey, B**.,** Chandra, P., & Sinha,P., (2003). A model for fishery resource with reserve area. Nonlinear Analysis Real World Applications,**4**(4), 625–637.

[21] Clark, C.W. (1990). Mathematical Bioeconomics: the optimal management of renewable resources. John Wiley and Sons Publishers: New York