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Unsteady Oscillatory Couette Flow Embeded in Porous Medium

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Abstract

In this paper, flow due to oscillatory movement of one of the plate is considered in horizontal parallel plates embedded in porous medium. Brinkman- extended- Darcy equation was utilized to model the flow. Laplace transform technique was utilized to obtain solutions describing the flow at small and large values of time for steady and transient flows. A special case when the value of $Da \to \infty$ is presented.

Keywords: Oscillatory Couette flow, Laplace, steady, transient, channel.

1.0 Introduction

The study on oscillating fluid flow exists in many practical applications. Example of the applications is acoustic streaming around an oscillating body [1].In [2] the problem is termed viscous fluid flow caused by oscillation of flat plate as Stoke's second problem. Penton [3] presented solution of transient flow due to the oscillation of plate, and the large times steady state flow is set up with the same frequency of plane boundary. The starting solution is obtained from the addition of transient solution to steady solution, since the problem is linear.

Recently, Singh [4] studied generalized Couette flow of two viscous, incompressible, immiscible fluids with heat transfer in presence of heat source through two straight parallel horizontal walls. The lower wall is bounded below, by a naturally permeable material of high porosity and the flow inside the porous medium is assumed to be of moderate permeability, modeled by Brinkman equation.

In addition, Pantokratoras[5] studied the steady laminar flow in afluid-saturated porous medium channel bounded by two parallel plates withconstant but unequal temperatures. One plate is moving with constantvelocity while the other is stationary using Brinkman–Darcy–Forchheimer model. Jaballah et al [6] studied the numerical simulation of the heat transfer and the mixed convection of an incompressible fluid filling a horizontal channel where some porous blocks are intermittently inserted in transverse to the channel axis. Eldabe et al [7] analyzed the steady magneto hydrodynamicflow of an incompressible electrically conducting visco-elasticfluidthrough a porousmedium between two porous parallel plates under the influence of a transverse magnetic field using Brinkman-Forchheimer extension of Darcy's momentum equation for flow. Jain et al [8] studied Couette flow through a highly porous medium between two horizontal parallel porous flat plates with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion has been analyzed.

Similarly, Fang [9] presented unsteady and steady velocity profiles for an incompressible Couette flow with mass transfer in simple horizontal channel. They discussed and solved the steady state temperature. Also they discussed and solved the steady state temperature. Similarly, Fang [10] presented pressure-driven unsteady and steady velocity profiles for an incompressible Couette flow with mass transfer in a simple horizontal channel. Das [11] studied and presented effect of suction and injection on MHD three dimensional Couette flow and heat transfer through a porous medium. Sharma [12] studied unsteady free convection oscillatory Couette flow through a porous medium with periodic wall temperature.

Erdogan [13] recently, studied fluid flow due to moving boundary in its own plane. Sinusoidal variation of the velocity was considered. The time required to attain steady flows for the cosine oscillation of the boundary is one-half cycle and it was observed that for sine oscillation it is a full cycle.Other studies carried out recently among many others in porous media are [14] and [15]. So far no study is presented for oscillatory Couette flowin horizontal channel embedded in porous medium. Hence the present study.

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2.0 Governing Equation

A horizontal channel filled with uniform porous material between y=0 and y=h is considered. At y=0 the wall is initially oscillatory moving. The governing equation in non dimensional form is

$$\frac{\partial u}{\partial t} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{u}{Da} \quad (1)$$

where $u(y,t)$ is the velocity, γ viscosity of the fluid, y is the coordinate and t is the time. The initial and boundary conditions are

$$\begin{aligned} &t \le 0: \quad u = 0 \text{ for all } y \\ &t > 0: u = exp(i\omega t) \text{ at } y = 0 \\ &u = 0 \quad \text{at } y = H \end{aligned} \quad (2)$$

The non dimensional parameters used in the governing equation and conditions are defined as:

$$u = \frac{u'}{U}, T = \omega t, y = y'\sqrt{\left(\frac{\omega}{\vartheta}\right)}, H = h\sqrt{\left(\frac{\omega}{\vartheta}\right)}, Da = \frac{k'}{h^2}$$

Solution

In order to obtain analytical solution at small and large times, Laplace transform technique is utilized. The Laplace transform of u is defined by

$$\bar{u} = \int_0^\infty u e^{-st} dt$$

Therefore the equation and the conditions take the form:

$$\frac{d^2\bar{u}}{dy^2} = \left(S - \frac{1}{Da}\right)\bar{u} \quad (4)$$

$$\begin{aligned} &S \le 0: \bar{u} = 0 \ for\ all\ y \\ &s > 0: \quad \bar{u} = \frac{1}{S-i}\ aty = 0 \\ &\bar{u} = 0at\ y = H \end{aligned} \quad (5)$$

The solution of the differential equation (4) subject to the boundary condition (5) is

$$\bar{u} = \left[-\frac{1}{(S-i)}\frac{exp\left(-2H\sqrt{S+\frac{1}{Da}}\right)}{\left[1-\exp\left(-2H\sqrt{S+\frac{1}{Da}}\right)\right]}\right]\exp\left(y\sqrt{S+\frac{1}{Da}}\right) + \left[\frac{1}{(S-i)}\frac{1}{\left[1-\exp\left(-2H\sqrt{S+\frac{1}{Da}}\right)\right]}\right]\exp\left(-y\sqrt{S+\frac{1}{Da}}\right) \quad (6)$$

Where $i = \sqrt{-1}$

Taking the Laplace inverse of the equation (6) utilizing shifting and convolution theorems of the Laplace transform [16]

$$u(y,T) = \frac{1}{2}exp\left(\left(i+\frac{1}{Da}\right)T\right)\sum_{n=0}^{\infty}\left[exp\left(ay\sqrt{i+\frac{1}{Da}}\right)erfc\left(\frac{a}{2\sqrt{T}}+\sqrt{(i+\frac{1}{Da})T}\right) + exp\left(-a\sqrt{i+\frac{1}{Da}}\right)erfc\left(\frac{a}{2\sqrt{T}}-\sqrt{(i+\frac{1}{Da})T}\right) - exp\left(by\sqrt{i+\frac{1}{Da}}\right)erfc\left(\frac{b}{2\sqrt{T}}+\sqrt{(i+\frac{1}{Da})T}\right) - exp\left(-b\sqrt{i+\frac{1}{Da}}\right)erfc\left(\frac{b}{2\sqrt{T}}-\sqrt{(i+\frac{1}{Da})T}\right)\right] \quad (7)$$

Where $erfc(x+iy)$the complementary error is function of the complex argument given by $erfc(x+iy) = 1 - erf\ (x+iy)$ and $erf\ (x+iy)$is error function[13].

At large $Ti.e\ T \to \infty$

$$erf\left[\left(\frac{y}{2\sqrt{T}}+\sqrt{\frac{T}{2}}\right)+\left(i+\frac{1}{Da}\right)\sqrt{\frac{T}{2}}\right] \to 0$$

$$erf\left[\left(\frac{y}{2\sqrt{T}}-\sqrt{\frac{T}{2}}\right)-\left(i+\frac{1}{Da}\right)\sqrt{\frac{T}{2}}\right] \to 2$$

$$u_S(y,T) = exp(iT)\sum_{n=0}^{\infty}\left[exp\left(ay\sqrt{i+\frac{1}{Da}}\right) - exp\left(-b\sqrt{i+\frac{1}{Da}}\right)\right] \quad (8)$$

Therefore transient solution can be calculated using

$$u_t(y,T) = u(y,T) - u_s(y,T) \quad (9)$$

Taking the real part of eqn. (7) and (8) gives the initial and periodic Couette motion due to $Cos\ (T)$, the transient solution in initial periodic is

$$u_t(y,T) = \frac{1}{2}exp(iT)\sum_{n=0}^{\infty}\left[exp\left(ay\sqrt{(i+\frac{1}{Da})}\right)erf\left(\frac{a}{2\sqrt{T}}+\sqrt{(i+\frac{1}{Da})T}\right) - exp\left(by\sqrt{i+\frac{1}{Da}}\right)erf\left(\frac{b}{2\sqrt{T}}+\sqrt{(i+\frac{1}{Da})T}\right) + exp\left(-ay\sqrt{i+\frac{1}{Da}}\right)\left\{erf\left(\frac{a}{2\sqrt{T}}-\sqrt{(i+\frac{1}{Da})T}\right)-2\right\} - exp\left(-by\sqrt{i+\frac{1}{Da}}\right)\left\{erf\left(\frac{b}{2\sqrt{T}}-\sqrt{(i+\frac{1}{Da})T}\right)-2\right\}\right] \quad (10)$$

Taking the imaginary part of eqn. (7) and (8) gives the initial and periodic Couette motion due to $sin\,(T)$, the transient solution in initial periodic is

$$u_t(y,T) = \frac{1}{2}exp\left(i+\frac{1}{Da})T\right)\sum_{n=0}^{\infty}\left[exp\left(ay\sqrt{i+\frac{1}{Da}}\right)erf\left(\frac{a}{2\sqrt{T}}+\sqrt{\left(i+\frac{1}{Da}\right)T}\right) - exp\left(by\sqrt{i+\frac{1}{Da}}\right)erf\left(\frac{b}{2\sqrt{T}}+\sqrt{\left(i+\frac{1}{Da}\right)T}\right) - exp\left(-a\sqrt{i+\frac{1}{Da}}\right)erf\left(\sqrt{\left(i+\frac{1}{Da}\right)T}-\frac{a}{2\sqrt{T}}\right) + exp\left(-b\sqrt{i+\frac{1}{Da}}\right)erf\left(\sqrt{\left(i+\frac{1}{Da}\right)T}-\frac{b}{2\sqrt{T}}\right)\right] \quad (11)$$

Where $a = 2nH + y \quad and \quad b = 2nH + 2H - y$

MATLAB is then utilized to study the effects of parameters involved in oscillatory Couette flow. The transient solution is initially periodically oscillatory Couette flow with sine or cosine oscillation. In figure 1 variation of the velocity with porous material for sine oscillation is depicted. It is noted that the velocity decreases periodically and became steady at large time. Similarly, variation of velocity for cosine oscillation is depicted in figure 2. Here it is noted that velocity increases periodically and became steady at large time.

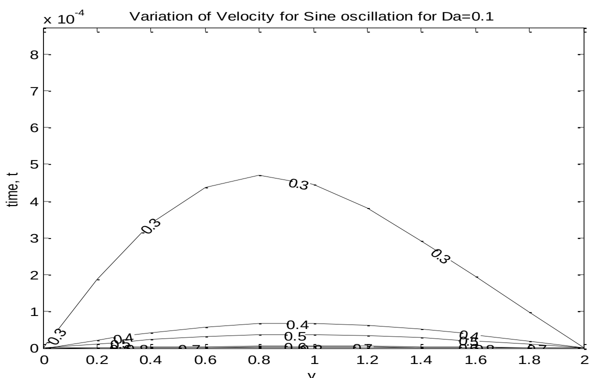

Figure 1: Variation of the velocity for sine Couette oscillation with porous material

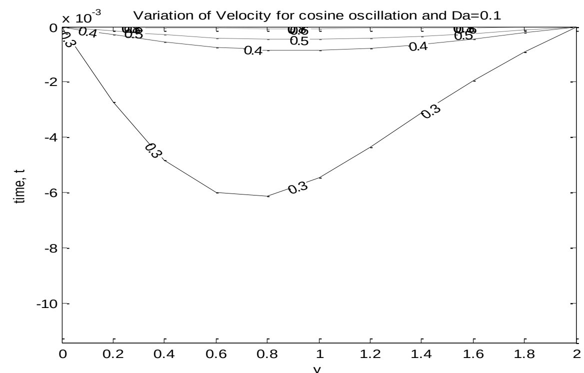

Figure 2: Variation of the velocity for cosine Couette oscillation with porous material

PARTICULAR CASE $Da \to \infty$

The governing equation in non dimensional form when $Da \to \infty$ is

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} \quad (12)$$

With the initial and boundary conditions as

$t \le 0$: $u = 0$ for all y

$t > 0$: $u = exp(i\omega t)$ at $y = 0$ (13)

$u = 0$ at $y = H$

$$u(y,T) = \frac{1}{2}exp(iT)\sum_{n=0}^{\infty}\left[exp(ay\sqrt{i})\,erfc\left(\frac{a}{2\sqrt{T}}+\sqrt{iT}\right) + exp(-a\sqrt{i})\,erfc\left(\frac{a}{2\sqrt{T}}+\sqrt{iT}\right) - exp(by\sqrt{i})\,erfc\left(\frac{b}{2\sqrt{T}}+\sqrt{iT}\right) - exp(-b\sqrt{i})\,erfc\left(\frac{b}{2\sqrt{T}}-\sqrt{iT}\right)\right]$$

At large $Ti.e\ T \to \infty$

$$\mathrm{erf}\left(\frac{\mathrm{y}}{2\sqrt{\mathrm{T}}}+\sqrt{\mathrm{iT}}\right) \to 0$$

$$\mathrm{erf}\left(\frac{\mathrm{y}}{2\sqrt{\mathrm{T}}}+\sqrt{\mathrm{iT}}\right) \to 2$$

$$u_S(y,T) = exp(iT)\sum_{n=0}^{\infty}\left[exp(ay\sqrt{i}) - exp(-b\sqrt{i})\right] \quad (14)$$

Therefore transient solutions are calculated using Eqs (9). In figure 3variation of the velocity for sine oscillation is depicted. It is noted that the velocity decreases periodically and became steady at large time. Similarly, variation of velocity for cosine oscillation is depicted in figure 4. Here it is noted that velocity increases periodically and became steady at large time.

In both figures 3 and 4, it can be noticed that in comparison with figure 1 and 2, the flow is strongly dependent on the oscillatory movement of the plate and porous material.

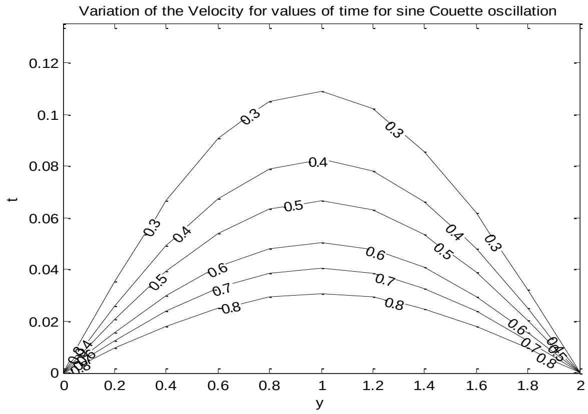

Figure 3: Variation of the velocity for sine Couette oscillation

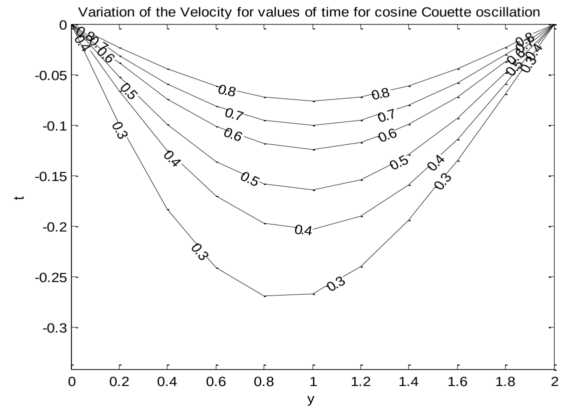

Figure 4: Variation of the velocity for cosine Couette oscillation

3.0 Conclusion

Oscillatory Couette flow of viscous fluid in horizontal channel filled with uniform porous material was presented. Laplace transform technique was utilized to present transient and steady solution at small and large times. A special case $Da \to \infty$ is considered. It is found that the porous material and oscillatory movement of the plate are strongly affecting the flow in the channel.

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