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Flexural Analysis of Isotropic Rectangular Plate Resting On Variable Bi-Parametric Elastic Foundation at Uniform Speed

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Abstract

In this paper, the influence of bi-parametric elastic subgrade on the dynamic response of a rectangular plate is investigated analytically and numerically. The fourth order partial differential equation governing the system is solved using Shadnam et al method, Struble's asymptotic technique and method of integral transformation. The effects of the parameters namely; rotatory inertia, correction factor, shear modulus and foundation stiffness are investigated for all the boundary conditions considered. Numerical results in plotted curves show that all the above-mentioned parameters actually have significant effects on the dynamic response of rectangular plate resting on variable bi-parametric elastic subgrade under moving distributed masses. Finally, we deduced that the critical speed for the moving distributed mass problem is reached prior to that of moving distributed force problem for the system under consideration.

Keywords: Struble's technique, Critical speed, Bi-parametric, Stiffness, Dynamic response

1.0 Introduction

Beams and plates of various sizes are widely found in various structural engineering applications such as satellites, tanks, bridges etc. The dynamic behaviour of such structures resting on elastic foundation has attracted numerous authors in the area of transport and civil engineering practices. This present study is sequel to an earlier one [1] that considered the dynamic response of simply supported rectangular plate under moving distributed mass. In particular, this paper is a generalization of the theory advanced in paper [1].

The problem of the dynamic response of elastic plates to moving loads neglecting the moving mass effect of the load has been addressed by several authors. However, in comparison plates subjected to moving loads have been the attraction of few authors. Holl [2] was among the earliest researchers into this area. He solved the problem of a rectangular plate carrying uniformly loads and concluded that the critical velocity exerted for each mode of vibration. Willis et al [3] examined the effects of eccentricity, span length, acceleration and initial velocity of the moving load by using the finite element method to study the dynamic response under moving loads.

Most of the previous works concerned plate flexures not resting on an elastic foundation. Meanwhile, for practical purposes, it is useful to consider plate supported by an elastic foundation. Therefore, the simplest mechanical foundation model was proposed by Winkler [4]. This model provides a simplified model to approximate the reaction of the foundation without much mathematical complexity. It expresses the relation between the pressure and the deflection of the surface. It also provides a simplified model to approximate the reaction of the foundation without much mathematical complexity, for this and other reasons, majority of the studies have been denoted to Winkler type foundation. Several authors have worked in this area, [5,6,7] to mention few ones.

Furthermore, the dynamical problems of elastic plate under moving load resting on an elastic foundation are cumbersome to handle especially if the foundation stiffness varies along the structures. Oni and Awodola [8] investigated the dynamic response to moving masses of rectangular plate resting on an elastic foundation with stiffness variations. The results show that an increase in the rotatory inertia correction factor and foundation modulus decreases the displacements response of the plate.

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However, most of these investigations have been on moving concentrated load which act at a point on the structure along a single line in space. Loads in real sense are disturbed over a small segment. And for more practical purpose, it is better to model moving load problem as moving distributed load as against concentrated moving load. The works of Oni and Ogunyebi [9], Esmailzadeh and Gorashi [10], Gbadeyan and Dada [11] treated the moving distributed load extensively. But in all these, the foundations are of constant type. Therefore, this paper presents the dynamic behavior of isotropic rectangular plate subjected to moving distributed masses and resting on variable bi-parametric elastic foundation at constant speed.

2.0 Theoretical Analysis

(a) Model Equation

The dynamic behaviour of a rectangular plate incorporating the effects of rotatory inertia correction factor to moving distributed load on variable bi-parametric elastic foundation is governed by the fourth order partial differential equation

$$D\nabla^2\nabla^2 V(x,y,t)+\mu\frac{\partial^2}{\partial t^2}V(x,y,t)+F_k V(x,y,t)=\mu R^o\frac{\partial^2}{\partial t^2}\nabla^2 V(x,y,t)+P(x,y,t) \qquad (2.1)$$

where E is the Young modulus, D is the bending rigidity of the plate, h is thePlate's thickness, μ is themass per unit area of the plate, v is the poisson's ratio, x is the position co-ordinate in $x-$direction, t is time, y is theposition co-ordinate in $y-$direction, R^o is therotatory inertia correction factor, $P(x,y,t)$ is themoving distributed load and

$$D=\frac{Eh^2}{12(1-V)} \qquad (2.2)$$

The relation between the foundation reaction and the lateral deflection $V(x,y,t)$ is

$$F_k(x,y,t)=S(x)V(x,y,t)-\frac{\partial}{\partial x}[K(x)\frac{\partial}{\partial x}V(x,y,t)]-\frac{\partial}{\partial y}[K(x)\frac{\partial}{\partial y}V(x,y,t)] \qquad (2.3)$$

where $S(x)$ and $K(x)$ are the two variable parameters of the elastic foundation. Specifically, $S(x)$ is the variable foundation stiffness and $K(x)$ is the variable shear modulus.

For the constant foundation stiffness and constant shear modulus, we have

$$F_k(x,y,t)=SV(x,y,t)-K\nabla^2 V(x,y,t) \qquad (2.4)$$

Since we are concerned with the dynamical system when the foundation parameter vary along x, equation (2.3) is rewritten to take the form

$$F_k(x,y,t)=S(x)V(x,y,t)-K'(x)\frac{\partial}{\partial x}V(x,y,t)-K(x)\nabla^2 V(x,y,t) \qquad (2.5)$$

where

$K'(x)$ implies $\frac{d}{dx}K(x)$ and ∇^2 is the two-dimensional Laplace operator.

where the effect of the moving load on the response of the plate is taken into consideration, the external moving surface load takes on the form

$$P(x,y,t)=P_f(x,y,t)[1-\frac{\Delta^*}{g}V(x,y,t)] \qquad (2.6)$$

where $P_f(x,y,t)$ is the continuous moving force, Δ^* is the substance acceleration operator and g is the acceleration due to gravity.

The end-support for the plate are arbitrary and the structure under consideration is assumed to be carrying an arbitrary number say (N) of distributed masses M_i moving with constant velocities $c_i, i=1,2,3...N$ along a straight line parallel to $x-$axis (no difficulty arises by assuming that masses travel in an arbitrary path) issuing from point $y=y_1$ on the y-axis. Therefore, the moving force acting on the plate is defined as

$$P_f(x,y,t)=\sum_{i=1}^{N}m_i g H(x-c_i t)H(y-y_1) \qquad (2.7)$$

where $H(\cdot)$ is the Heaviside function. The operator Δ^* used in above for masses travelling in an arbitrary path in the $x-y$ plane is defined as

$$\Delta^* = \frac{\partial^2}{\partial t^2} + 2c_i \frac{\partial^2}{\partial x \partial t} + c_i^2 \frac{\partial^2}{\partial x^2} \tag{2.8}$$

As an example in this problem, a variable elastic foundation stiffness of the form [1]

$$S(x) = S_0(4x - 3x^2 + x^3) \tag{2.9}$$

where S_0 is the foundation stiffness, and a variable shear modulus of the form [1]

$$K(x) = K_0(12 - 13x + 6x^2 - x^3) \tag{2.10}$$

where K_0 are constants are considered.

Substituting (2.2), (2.3), (2.4), (2.6), (2.7), (2.8), (2.9), and (2.10) into (2.1), one obtains

$$D\nabla^2\nabla^2 V(x,y,t) + \mu \frac{\partial^2}{\partial t^2} V(x,y,t) = \mu R^o$$

$$\left[\frac{\partial^4}{\partial t^2 \partial x^2} + \frac{\partial^4}{\partial t^2 \partial y^2}\right] V(x,y,t) - S_0(4x - 3x^2 + x^3)V(x,y,t)$$

$$+K_0(-13 + 12x - 3x^2)\frac{\partial}{\partial x}V(x,y,t) + K_0(12 - 13x + 6x^2 - x^3)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) V(x,y,t) + \sum_{i=1}^{N}\Big[M_i g H(x - c_i t)H(y - y_1)$$

$$-M_i\left(\frac{\partial^2}{\partial t^2} + 2c_i \frac{\partial^2}{\partial x \partial t} + c_i^2 \frac{\partial^2}{\partial x^2}\right)V(x,y,t)H(x - c_i t)H(y - y_1)\Big] \tag{2.11}$$

The boundary conditions are arbitrary and the initial conditions, without loss of generality is taken as

$$V(x,y,t) = 0 = \frac{\partial}{\partial t}V(x,y,t) \tag{2.12}$$

(b) Solution Procedures

We first express Heaviside function as a Fourier series and due to the variable foundation term, the elegant method of the generalized integral transform breaks down while the generalized Galerkin's method used in one-dimensional structural problems (Beam problem) fails to handle the two-dimensional structural problem (plate problems).

Hence, the technique of Shadnam et al [12] is used to reduce the fourth order partial differential equation governing the motion of the plate to a set of second order ordinary differential equations. The resulting equation is then simplified by the modified asymptotic method of Struble. Finally, the method of convolution theory is then employed to obtain the closed form solution of the two-dimensional dynamical problems under moving distributed loads.

To solve equation (2.11), let the deflection be written in the form [12]

$$V(x,y,t) = \sum_{m=1}^{\infty} \beta_m(x,y) Z_m(t) \tag{2.13}$$

where β_m are the known eigenfunctions of the plate with the same boundary conditions. Also β_m have the form of

$$\nabla^4 \beta_m - \omega_m^4 \beta_m = 0 \tag{2.14}$$

where

$$\omega_m^4 = \frac{\Omega_m^2 \mu}{D} \tag{2.15}$$

and $\Omega_m, m = 1, 2, 3, \ldots,$ are the natural frequencies of the dynamical system and $Z_m(t)$ are amplitude functions which have to be calculated.

Therefore eqn. (2.11) is written in the form

$$\frac{D\nabla^4}{\mu}V(x,y,t) + \frac{\partial^2}{\partial t^2}V(x,y,t) = R^o\left[\frac{\partial^4}{\partial t^2 \partial x^2} + \frac{\partial^4}{\partial t^2 \partial y^2}\right]V(x,y,t) - \frac{S_0}{\mu}(4x - 3x^2 + x^3)V(x,y,t)$$

$$+\frac{k_0}{\mu}(-13 + 12x - 3x^2)\frac{\partial}{\partial x}V(x,y,t)$$

$$+\frac{k_0}{\mu}(12-13x+6x^2-x^3)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)V(x,y,t)+\sum_{i=1}^{N}\left[\frac{M_i g}{\mu}H(x-c_it)H(y-y_1)-\frac{M_i}{\mu}\left(\frac{\partial^2}{\partial t^2}+2c_i\frac{\partial^2}{\partial x\partial t}+c_i^2\frac{\partial^2}{\partial t^2}\right)V(x,y,t)\right.$$

$$\left.H(x-c_it)H(y-y_1)\right] \qquad (2.16)$$

Re-written the right hand side of the eqn. (2.16) in the form of a series, one obtains

$$R^o\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]V(x,y,t)-\frac{S_0}{\mu}(4x-3x^2+x^3)V(x,y,t)+\frac{k_0}{\mu}(-13+12x-3x^2)\frac{\delta}{\delta x}V(x,y,t)$$

$$+\frac{k_0}{\mu}(12-13x+6x^2-x^3)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)V(x,y,t)+\sum_{i=1}^{N}\left[\frac{M_i g}{\mu}H(x-c_it)(y-y_1)-\frac{M_i}{\mu}\left(\frac{\partial^2}{\partial t^2}+2c_i\frac{\partial^2}{\partial x\partial t}+c_i^2\frac{\partial^2}{\partial x^2}\right)\right.$$

$$\left.V(x,y,t)H(x-c_it)H(y-y_1)\right]=\sum_{m=1}^{\infty}\beta_m(x,y)Z_m(t) \qquad (2.17)$$

When equation (2.13) is substituted into equation (2.17) one obtains,

$$\sum_{m=1}^{\infty}\left\{R^0\left[\beta_{m,xx}(x,y)Z_{m,tt}(t)+\beta_{m,yy}(x,y)Z_{m,t}(t)\right]-\frac{S_0}{\mu}\left[4x-3x^2+x^3\right]\beta_m(x,y)Z_m(t)+\frac{k_0}{\mu}\right.$$

$$\left[(-13+12x-3x^2)\beta_{m,x}(x,y)Z_m(t)+(12-13x+6x^2-x^3)\left(\beta_{m,xx}(x,y)Z_m(t)+\beta_{m,yy}(x,y)Z_m(t)\right)\right]$$

$$\left.\sum_{i=1}^{n}\left[\frac{M_i g}{\mu}H(x-c_it)H(y-y_1)-\frac{M_i}{\mu}(\beta_m(x,y),Z_{m,tt}(t)+2c_i\beta_{m,x}(x,y)Z_{m,t}(t)+c_i^2\beta_{m,xx}(x,y)Z_{(m,t)}H(x-c_it)(y-y_1)\right]\right\}$$

$$=\sum_{m=1}^{\infty}\beta_m(x,y)G_m(t) \qquad (2.18)$$

Multiplying both sides of the equation (2.18) by $\beta_p(x,y)$, then integrate on an area A of the plate and subsequently impose the orthogonality of $\beta_m(x,y)$, one obtains

$$G_m(t)=\frac{1}{\Omega_m^*}\sum_{m=1}^{\infty}\int_A\left\{R^0\left[\beta_{m,xx}(x,y)\beta_p(x,y)Z_{tt}(m,t)+\beta_{m,yy}(x,y)\beta_p(x,y)Z_{tt}(m,t)\right]-\frac{1}{\mu}\left(S_0\left[4x-3x^2+x^3\right]\right.\right.$$

$$+k_0\left[(-13+12x-3x^2)\beta_{m,x}(x,y)\beta_p(x,y)Z(m,t)+(12-13x+6x^2-x^3)\right.$$

$$\left.\left.\left(\beta_{m,xx}(x,y)\beta_p(x,y)Z(m,t)+\beta_{m,yy}(x,y)\beta_p(x,y)Z(m,t)\right)\right]\right)+\frac{1}{\mu}\sum_{i=1}^{N}M_i g\beta_p(x,y)H(x-c_it)H(y-y_1)$$

$$\left.-\frac{1}{\mu}M_i(\beta_m(x,y)\beta_p(x,y)Z_{tt}(m,t)+2c_i\beta_{m,x}(x,y)\beta_p(x,y)Z_t(m,t)+c_i^2\beta_{m,xx}(x,y)Z(m,t)H(x-c_it)H(y-y_1)\right]\right\}dA \qquad (2.19)$$

Where

$$\Omega_m^*=\int_A\beta_p^2 dA \qquad (2.20)$$

Further simplification of equation (2.19) gives

$$Z_{tt}(m,t)+\frac{D\omega_n^4}{\mu}Z(m,t)=\frac{1}{\Omega_m^*}\sum_{\alpha=1}^{\infty}\int_A\left\{R^0\left[\beta_{\alpha,xx}(x,y)\beta_p(x,y)Z_{\alpha,tt}(m,t)+\beta_{\alpha,yy}(x,y)\beta_p(x,y)Z_{\alpha,tt}(m,t)\right]\right.$$

$$-\frac{1}{\mu}\left(S_0\left[4x-3x^2+x^3\right]\beta_\alpha(x,y)\beta_p(x,y)Z_{\alpha,tt}(m,t)+k_0\left[(-13+12x-3x^2)\beta_{\alpha,x}(x,y)\beta_p(x,y)Z_\alpha(m,t)\right.\right.$$

$$\left.+(12-13x+6x^2-x^3)\left(\beta_{\alpha,xx}(x,y)\beta_p(x,y)Z_\alpha(m,t)+\beta_{\alpha,yy}(x,y)\beta_p(x,y)Z_\alpha(m,t)\right)\right]$$

$$+\frac{1}{\mu}\sum_{i=1}^{N}\Big[M_i g\beta_p(x,y)H(x-c_it)H(y-y_1)-\frac{1}{\mu}M_i\big(\beta_\alpha(x,y)\beta_p(x,y)Z_{\alpha,tt}(m,t)+2c_i\beta_{\alpha,x}(x,y)\beta_p(x,y)Z_{\alpha,t}(m,t)$$
$$+c_i^2\beta_{\alpha,xx}(x,y)\beta_p(x,y)Z_\alpha(m,t)\big)H(x-c_it)H(y-y_1)\Big]\Big\}dA \qquad (2.21)$$

Equation (2.21) is a set of coupled ordinary differential equation of our dynamical system.

Expressing Heaviside function in equation (2.21) as a Fourier series i.e.

$$H(x-c_it)=\frac{1}{4}+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{Sin\big((2n+1)\pi(x-c_it)\big)}{2n+1} \qquad (2.22)$$

$$H(y-y_1)=\frac{1}{4}+\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{Sin\big((2n+1)\pi(y-y_1)\big)}{2n+1} \qquad (2.23)$$

Considering equations (2.23) and (2.23), equation (2.21) now becomes

$$\ddot{Z}(m,t)+\Phi_m^2 Z(m,t)-\frac{1}{\Omega_m^*}\sum_{\alpha=1}^{\infty}\Bigg\{R^0\phi_A\ddot{Z}_\alpha(m,t)-\frac{1}{\mu}\big(S_0\phi_{BA}+K_0\phi_{BB}\big)Z_\alpha(m,t)-\frac{1}{\mu}\sum_{i=1}^{N}\Bigg[\frac{1}{16}\phi_{CA}$$
$$+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{CB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{CC}+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CD}$$
$$-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CE}+\frac{1}{\pi^2}\Bigg(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{CE}-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CD}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{CB}$$
$$-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{CC}\Bigg)\ddot{Z}_\alpha(m,t)+2c\Bigg[\frac{1}{16}\phi_{DA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{DB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{DC}$$
$$+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{DD}-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{DE}+\frac{1}{\pi^2}\Bigg(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{DD}$$
$$-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{DE}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{DB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{DC}\Bigg)\Bigg]\dot{Z}_\alpha(m,t)$$
$$+c^2\Bigg[\frac{1}{16}\phi_{EA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{EB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{EC}+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{ED}$$
$$-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{EE}+\frac{1}{\pi^2}\Bigg(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{ED}-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{EE}$$
$$\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{EB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{EC}\Bigg)Z_\alpha(m,t)\Bigg]\Bigg\}=\sum_{i=1}^{N}\frac{M_ig}{\mu\Omega_m^*}\beta_p(c_it,y_1) \qquad (2.24)$$

Where

$$\Phi_m^2=\frac{D\omega_n^4}{\mu} \qquad (2.25)$$

and

$$\phi_A=\int_0^{Lx}\int_0^{Ly}\big[\beta_{m,xx}(x,y)+\beta_{m,yy}(x,y)\big]\beta_p(x,y)dydx\,,\quad \phi_{BA}=4S_a-S_b+S_c$$
$$\phi_{BB}=-13S_d+12S_e-3S_f+12(S_g+S_h)-13(S_i+S_j)+6(S_k+S_l)-(S_m+S_n)$$
$$\phi_{CA}=\int_0^{Lx}\int_0^{Ly}\beta_m(x,y)\beta_p(x,y)dydx \qquad \phi_{CB}=\int_0^{Lx}\int_0^{Ly}\sin(2m+1)\pi y_1\beta_m(x,y)\beta_p(x,y)dydx$$

$$\phi_{CC}=\int_0^{Lx}\int_0^{Ly}\cos(2m+1)\pi y_1\beta_m(x,y)\beta_p(x,y)dydx$$
$$\phi_{CD}=\int_0^{Lx}\int_0^{Ly}\cos(2n+1)\pi x\beta_m(x,y)\beta_p(x,y)dydx$$
$$\phi_{CE}=\int_0^{Lx}\int_0^{Ly}\sin(2n+1)\pi x\beta_m(x,y)\beta_p(x,y)dydx$$
$$\phi_{DA}=\int_0^{Lx}\int_0^{Ly}\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$\phi_{DB}=\int_0^{Lx}\int_0^{Ly}\sin(2m+1)\pi y_1\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$\phi_{DC}=\int_0^{Lx}\int_0^{Ly}\cos(2m+1)\pi y_1\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$\phi_{DD}=\int_0^{Lx}\int_0^{Ly}\sin(2n+1)\pi x\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$\phi_{DE}=\int_0^{Lx}\int_0^{Ly}\cos(2n+1)\pi x\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$\phi_{EA}=\int_0^{Lx}\int_0^{Ly}\beta_{m,xx}(x,y)\beta_p(x,y)dydx$$
$$\phi_{EB}=\int_0^{Lx}\int_0^{Ly}\sin(2m+1)\pi y_1\beta_{m,xx}(x,y)\beta_p(x,y)dydx$$
$$\phi_{EC}=\int_0^{Lx}\int_0^{Ly}\cos(2m+1)\pi y_1\beta_{m,xx}(x,y)\beta_p(x,y)dydx$$
$$\phi_{ED}=\int_0^{Lx}\int_0^{Ly}\sin(2n+1)\pi x\beta_{m,xx}(x,y)\beta_p(x,y)dydx$$
$$\phi_{EE}=\int_0^{Lx}\int_0^{Ly}\cos(2n+1)\pi x\beta_{m,xx}(x,y)\beta_p(x,y)dydx$$

and

$$S_a=\int_0^{Lx}\int_0^{Ly}x\beta_m(x,y)\beta_p(x,y)dydx,\qquad S_b=\int_0^{Lx}\int_0^{Ly}x^2\beta_m(x,y)\beta_p(x,y)dydx$$
$$S_c=\int_0^{Lx}\int_0^{Ly}x^3\beta_m(x,y)\beta_p(x,y)dydx,\qquad S_d=\int_0^{Lx}\int_0^{Ly}\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$S_e=\int_0^{Lx}\int_0^{Ly}x\beta_{m,x}(x,y)\beta_p(x,y)dydx,\qquad S_f=\int_0^{Lx}\int_0^{Ly}x^2\beta_{m,x}(x,y)\beta_p(x,y)dydx$$
$$S_g=\int_0^{Lx}\int_0^{Ly}\beta_{m,xx}(x,y)\beta_p(x,y)dydx,\qquad S_h=\int_0^{Lx}\int_0^{Ly}\beta_{m,yy}(x,y)\beta_p(x,y)dydx$$
$$S_i=\int_0^{Lx}\int_0^{Ly}x\beta_{m,xx}(x,y)\beta_p(x,y)dydx,\qquad S_j=\int_0^{Lx}\int_0^{Ly}x\beta_{m,yy}(x,y)\beta_p(x,y)dydx$$
$$S_k=\int_0^{Lx}\int_0^{Ly}x^2\beta_{m,xx}(x,y)\beta_p(x,y)dydx,\qquad S_l=\int_0^{Lx}\int_0^{Ly}x^2\beta_{m,yy}(x,y)\beta_p(x,y)dydx$$
$$S_m=\int_0^{Lx}\int_0^{Ly}x^3\beta_{m,xx}(x,y)\beta_p(x,y)dydx,\qquad S_n=\int_0^{Lx}\int_0^{Ly}x^3\beta_{m,yy}(x,y)\beta_p(x,y)dydx \tag{2.26}$$

The second order coupled differential equation (2.24) is the transformed equation governing the problem of a rectangular plate on a variable bi-parametric elastic foundation. This differential equation holds for all variants of the classical boundary conditions.

In order to solve equation (2.24) we shall consider only one mass with uniform velocity c along the line $y=y_1$. Thus for single mass M, equation (2.24) reduces to

$$\ddot{Z}(m,t)+\Phi_m^2 Z(m,t)-\frac{1}{\Omega_m^*}\sum_{\alpha=1}^{\infty}\Bigg\{R^0\phi_A\ddot{Z}_\alpha(m,t)-\frac{1}{\mu}\Big(S_0\phi_{BA}+K_0\phi_{BB}\Big)Z_\alpha(m,t)-\lambda^o\sum_{i=1}^{N}\Bigg[\frac{1}{16}\phi_{CA}$$
$$+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{CB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{CC}+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CD}$$

$$-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CE}+\frac{1}{\pi^2}\left(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{CE}-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CD}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{CB}\right.$$
$$\left.-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{CC}\right)\ddot{Z}_\alpha(m,t)+2c\left[\frac{1}{16}\phi_{DA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{DB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{DC}\right.$$
$$+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{DD}-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{DE}+\frac{1}{\pi^2}\left(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{DD}\right.$$
$$\left.\left.-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{DE}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{DB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{DC}\right)\right]\dot{Z}_\alpha(m,t)$$
$$+c^2\left[\frac{1}{16}\phi_{EA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{EB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{EC}+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{ED}\right.$$
$$-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{EE}+\frac{1}{\pi^2}\left(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{ED}-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{EE}\right.$$
$$\left.\left.\left.\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{EB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{EC}\right)Z_\alpha(m,t)\right]\right\}=\frac{m_{\dot{g}}}{\mu\Omega^*_m}\left\{\frac{L_x}{mi}\left(-Cosb_{mi}+A_{mi}Sinb_{mi}\right.\right.$$
$$\left.+B_{mi}Coshb_{mi}+C_{mi}Sinhb_{mi}+Cosb_{mi}\frac{c_it}{L_x}-A_{mi}Sinb_{mi}\frac{c_it}{L_x}-B_{mi}Coshb_{mi}\frac{c_it}{L_x}-C_{mi}Sinhb_{mi}\frac{c_it}{L_x}\right)\times\frac{L_y}{mj}\left(-Cosb_{mj}\right.$$
$$\left.\left.+A_{mj}Sinb_{mj}+B_{mj}Coshb_{mj}+C_{mj}Sinb_{mj}+Coshb_{mj}\frac{y_1}{L_y}-A_{mj}Sinb_{mj}\frac{y_1}{L_y}-B_{mj}Coshb_{mj}\frac{y_1}{L_y}-C_{mj}Sinhb_{mj}\frac{y_1}{L_y}\right)\right]$$

(2.27)

Where

$$\lambda^o=\frac{M}{\mu L_x L_y} \tag{2.28}$$

Equation (2.27) is now the fundamental equation of our dynamical problem when the rectangular plate on variable bi-parametric elastic foundation has arbitrary end conditions. In what follows, we shall discuss two special cases of the equation (2.27) namely; the moving distributed force and the moving distributed mass problems respectively.

2.1 Rectangular Plate on Variable Foundation Traversed by a Moving Distributed Force

In this section, an approximate model which assumes the inertia effect of the moving distributed mass M as negligible is obtained when the mass ratio $\lambda^o=0$ is set to zero in equation (2.27). Thus, setting $\lambda^o=0$, equation (2.27) reduces to

$$\frac{d^2Z(m,t)}{dt^2}+\phi_m^2 Z(m,t)-R^0\sum_{\alpha=1}^{\infty}\frac{\phi_A}{\Omega_m^*}\frac{d^2}{dt^2}Z(m,t)-\frac{1}{\mu}\left(\frac{S_0}{\Omega_m^*}\sum_{\alpha=1}^{\infty}\left[\phi_A-\frac{K_0}{S_0}\right]Z(m,t)\right)=\frac{M_g}{\mu\Omega_m^*}a_{\alpha i}(ct)a_{\alpha j}(y_1) \tag{2.29}$$

This is an approximate model which assumes the inertia effect of the moving mass as negligible.

Evidently, an exact analytical solution to this equation is not possible. Consequently, the approximate analytical solution technique, which is a modification of the asymptotic method of Struble shall be used.

To solve equation (2.29), first, we neglect the rotatory inertia term and rearrange the equation to take the form

$$\frac{d^2Z(m,t)}{dt^2}+\left[\phi_m^2+\eta^o\left(\phi_{BA}-\frac{K_0\phi_{BB}}{S_0}\right)\right]Z_\alpha(m,t)-\eta^o\sum_{\substack{\alpha=1\\ \alpha\neq 1}}^{\infty}\left[\phi_{BA}-\frac{K_0\phi_{BB}}{S_0}\right]Z_\alpha(m,t)=\frac{M_g}{\mu\Omega_m^*}a_{\alpha i}(ct)a_{\alpha j}(y_1) \tag{2.30}$$

Where

$$\eta^o = \frac{S_0}{\mu\Omega_m^*} \qquad (2.31)$$

By means of the Strubles' technique, one seeks the modified frequency corresponding to the frequency of the free system due to the presence of the shear modulus S_0 . An equivalent free system operator defined by the modified frequency then replaces equation (2.30). Thus, consider a parameter η^o for any arbitrary ratio defined by

$$\Gamma_m = \frac{\eta^o}{1+\eta^o} \qquad (2.32)$$

$$\eta^o = \Gamma_m + 0(\Gamma_m^2) \qquad (2.33)$$

Substituting equation (2.33) into the homogenous part of equation (2.30) yields

$$\frac{d^2Z(m,t)}{dt^2} + \left(\Phi_m^2 - \Gamma_m\left(\phi_{BA} - \frac{K_0\phi_{BB}}{S_0}\right)\right)Z_\alpha(m,t) - \Gamma_m\sum_{\substack{\alpha=1\\ \alpha\neq 1}}^{\infty}\left(\phi_{BA} - \frac{K_0\phi_{BB}}{S_0}\right)Z_\alpha(m,t) = 0 \qquad (2.34)$$

When $\Gamma_m = 0$ in (2.34), a situation corresponding to the case in which the effect of the foundation reaction is regarded as negligible is obtained. In such a case the solution is of the form

$$Z(m,t) = D_m\cos\left[\phi_m t - \alpha_o\right] \qquad (2.35)$$

where D_m and α_o are constants.

since $\Gamma_m = 0$ for any arbitrary constants mass ratio, Struble's technique requires that the asymptotic solution of the homogeneous part of equation (2.34) be of the form

$$Z(m,t) = \Delta_m(t)\cos\left[\phi_m t - \psi_m\right] + \Gamma_m Z(1,t) + 0(\Gamma_m^2) \qquad (2.36)$$

where Δ_m and ψ_m are slowly varying functions of time or equivalently

$$\frac{d\Delta_m(t)}{dt} \to 0(\Gamma_m) \quad ; \quad \frac{d^2\Delta_m(t)}{dt^2} \to 0(\Gamma_m^2) \qquad (2.37)$$

$$\frac{d\psi_m t}{dt} \to 0(\Gamma_m) \quad ; \quad \frac{d^2\psi_m t}{dt^2} \to 0(\Gamma_m^2) \qquad (2.38)$$

Where $\to$ implies "is of ".

To obtain the modified frequency equation (2.36) and its derivatives are substituted into the homogeneous part of equation (2.34) where terms higher than Γ_m are neglected. The variational equations are obtained by equating the coefficients of $\sin\left[\Delta_m t - \psi_m\right]$ and $\cos\left[\Delta_m t - \psi_m\right]$ terms on both sides of the resulting equation to zero. The resulting variational equations describing the behaviour of $\Delta_m(t)$ and $\psi_m(t)$ during the motion of the system is determine by the modified frequency. In particular, the variational equations are

$$-2\phi_m t\dot{\Delta}_m(t) = 0 \qquad (2.39)$$

and

$$2\phi_m t\Delta_m(t)\dot{\psi}_m - \Gamma_m\phi_{BA}\Delta_m(t) + \Gamma_m\frac{K_0}{S_0}\phi_{BB}\Delta_m(t) = 0 \qquad (2.40)$$

Solving equation (2.39) and (2.40) respectively, one obtains

$$\Delta_m(t) = K_m \qquad (2.41)$$

and

$$\psi_m = \frac{\Gamma_m\left(\phi_{BA} - \frac{K_0\phi_{BA}}{S_0}\right)t}{2\phi_m} + \eta_m \qquad (2.42)$$

Where $\Delta_m(t)$ and η_m are constant.
Then equation (2.36) becomes

$$Z(m,t) = D_o \cos\left[\tau_m t - \varphi_o\right] + Z(1,t) + 0(\Gamma_m^2) \tag{2.43}$$

where

$$\tau_m = \phi_m - \frac{\Gamma_m\left(\phi_{BA} - \frac{K_0\phi_{BB}}{S_0}\right)}{2\phi_m} \tag{2.44}$$

Is called the modified frequency due to the effect of the shear modulus of the foundation. Thus, equation (2.44) now becomes

$$\frac{d^2Z(m,t)}{dt^2} + \tau_m^2 Z(m,t) = 0 \tag{2.45}$$

Using (2.45), equation (2.29) can be written as

$$\frac{d^2Z(m,t)}{dt^2} + \tau_m^2 Z(m,t) - \frac{R^o}{\Omega_m^*}\phi_A \sum_{\alpha=1}^{\infty} \frac{d^2Z(m,t)}{dt^2} = \frac{M_g}{\mu 0\Omega_m^*} a_{\alpha i}(ct) a_{\alpha j}(y_1) \tag{2.46}$$

In what follows, we seek the modified frequency corresponding to the frequency of the free system due to the presence of the effect of rotatory inertia correction factor R^0. An equivalent free system operator defined by the modified frequency then replaces equation (2.46). To this end, the homogeneous part of equation (2.46) is rearranged to take the form

$$\frac{d^2Z(m,t)}{dt^2} + \frac{\tau_m^2 Z(m,t)}{(1 - A_m\phi_A)} - \frac{A_m\phi_A}{(1 - A_m\phi_A)} \sum_{\substack{\alpha=1 \\ \alpha\neq 1}}^{\infty} \frac{d^2Z(m,t)}{dt^2} = 0 \tag{2.47}$$

Where

$$A_m = \frac{R^0}{\Omega_m^*} \tag{2.48}$$

Now consider the parameter R^0 for any arbitrary ratio defined as

$$\varepsilon_o = \frac{A_m}{1 + A_m} \tag{2.49}$$

It can be shown that

$$A_m = \varepsilon_o + 0(\varepsilon_o^2) \tag{2.50}$$

and

$$\frac{1}{1 - \varepsilon_o} = 1 + \varepsilon_o\phi_A + 0(\varepsilon_o^2) \tag{2.51}$$

where

$$\left|\varepsilon_o\phi_A\right| < 1 \tag{2.52}$$

Substituting equations (2.49) and (2.50) into equation (2.47), one obtains

$$\frac{d^2Z(m,t)}{dt^2} + \tau_m^2 Z(m,t) + \varepsilon_o\phi_A\tau_m^2 Z(m,t) - \varepsilon_o\phi_A \sum_{\substack{\alpha=1 \\ \alpha\neq 1}}^{\infty} \frac{d^2Z(m,t)}{dt^2} = 0 \tag{2.53}$$

to ε_o only.

Since $\varepsilon_0 < 1$, an asymptotic solution of the homogeneous part of equation (2.45)
can be written in the form

$$Z(m,t) = B_0(t)\cos\left[\tau_m t - \phi_m\right] + \varepsilon_0 Z(1,t) + 0(\varepsilon_0^2) \tag{2.54}$$

where $B_0(t)$ and ϕ_m are slowly varying function of time.

Similarly, therefore, when the effects of the rotatory inertia is considered, the first approximation to the homogenous system is

$$Z(m,t) = f_o \cos\left[\tau_{mm} t - \phi_o\right] \tag{2.55}$$

where

$$\tau_{mm} = \tau_m \left(1 + \frac{\varepsilon_o \phi_A}{2}\right) \tag{2.56}$$

Equation (2.56) is the modified frequency corresponding to the frequency of the free system due to the presence of the rotatory inertia.

In order to solve the non-homogeneous equation (2.53), the differential operator which acts on $Z(m,t)$ is replaced by the equivalent free system operator defined by the modified frequency τ_{mm}. Thus,

$$\frac{d^2}{dt^2} Z(m,t) + \tau_{mm}^2 Z(m,t) = H_m \frac{L_x L_y}{mimj}\left[-\cos b_{mi} + A_m \sin b_{mi} + B_m \sinh b_{mi} + C_m \cosh b_{mi} + \cos b_{mi} \frac{c_i t}{L} - A_m \sin b_{mi} \frac{c_i t}{L} - B_m \cosh b_{mi} \frac{c_i t}{L} - C_m \sin b_{mi} \frac{c_i t}{L}\right] \times E_m(y_1) \tag{2.57}$$

where

$$E_m(y_1) = -\cos b_{mj} + A_m \sin b_{mj} + B_m \sinh b_{mj} + C_m \cosh b_{mj} + \cos b_{mj} \frac{c_i t}{L} - A_m \sin b_{mj} \frac{c_i t}{L} - B_m \cosh b_{mj} \frac{c_i t}{L} - C_m \sin b_{mj} \frac{c_i t}{L} \tag{2.58}$$

and

$$b_{mi} = \frac{i\pi c}{L_x} \tag{2.59}$$

The ordinary differential equation, when solved using Laplace transformation and convolution theory, it can be shown that

$$Z(m,t) = \frac{H_m L_x L_y}{m_i m_j} \cdot E_m(y_1)\left[-\frac{1}{\tau_{mm}} E_m^o \sin \tau_{mm} t - \frac{\sin b_{mi} t + \sin \tau_{mm} t}{\tau_{mm}^2 - b_{mi}^2} - \frac{\cos b_{mi} t + \cos \tau_{mm} t}{\tau_{mm}^2 - b_{mi}^2} - \frac{B_{mi} 2\tau_{mm}^2 b_{mi} \sin 2\tau_{mm} t \cosh b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} - \frac{B_{mi} b_{mi}^2 \cos 2\tau_{mm} t \sin b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{B_{mi} \tau_{mm}^2 \sin b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{B_{mi} b_{mi} \sin \tau_{mm} t (b_{mi}^2 - \tau_{mm}^2)}{\tau_{mm}^4 - b_{mi}^4} - \frac{C_{mi} 2\tau_{mm}^2 b_{mi} \sin 2\tau_{mm} t \sinh b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{C_{mi} b_{mi}^2 \cos 2\tau_{mm} t \cos h b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{C_{mi} \tau_{mm}^2 \cosh b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{C_{mi}(b_{mi}^2 - \tau_{mm}^2)}{\tau_{mm}^4 - b_{mi}^4}\right] \tag{2.60}$$

where

$$E_m^o = -\cos b_{mi} - A_{mi} \sin b_{mi} + B_{mi} \cosh b_{mi} + C_{mi} \sinh b_{mi} \tag{2.61}$$

and in view of equation (2.13),one obtains

$$V(x,y,t) = \sum_{mi=1}^{\infty}\sum_{mj=1}^{\infty} \frac{H_m L_x L_y}{mimj} \cdot E_m(y_1)\left[-\frac{1}{\tau_{mm}} E_m^o \sin \tau_{mm} t - \frac{\sin b_{mi} t + \sin \tau_{mm} t}{\tau_{mm}^2 - b_{mi}^2} - \frac{\cos b_{mi} t + \cos \tau_{mm} t}{\tau_{mm}^2 - b_{mi}^2} - \frac{B_{mi} 2\tau_{mm}^2 b_{mi} \sin 2\tau_{mm} t \cosh b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} - \frac{B_{mi} b_{mi}^2 \cos 2\tau_{mm} t \sin b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{B_{mi} \tau_{mm}^2 \sin b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{B_{mi} b_{mi} \sin \tau_{mm} t (b_{mi}^2 - \tau_{mm}^2)}{\tau_{mm}^4 - b_{mi}^4} - \frac{C_{mi} 2\tau_{mm}^2 b_{mi} \sin 2\tau_{mm} t \sinh b_{mi} t}{\tau_{mm}^4 - b_{mi}^4} + \frac{C_{mi} b_{mi}^2 \cos 2\tau_{mm} t \cos h b_{mi} t}{\tau_{mm}^4 - b_{mi}^4}$$

$$+\frac{C_{mi}\tau_{mm}^2\cosh b_{mi}t}{\tau_{mm}^4-b_{mi}^4}+\frac{C_{mi}(b_{mi}^2-\tau_{mm}^2)}{\tau_{mm}^4-b_{mi}^4}\Bigg]\times\Bigg[\Bigg(\sin\frac{b_{mi}x}{L_x}+A_{mi}\cos\frac{b_{mi}x}{L_x}+B_{mi}\sinh\frac{b_{mi}x}{L_x}$$

$$+C_{mi}\cosh\frac{b_{mi}x}{L_x}\Bigg)\cdot\Bigg(\sin\frac{b_{mj}y_1}{L_y}-A_{mj}\cos\frac{b_{mj}y_1}{L_y}-B_{mj}\sinh\frac{b_{mj}y_1}{L_y}+C_{mj}\cosh\frac{b_{mj}y_1}{L_y}\Bigg)\Bigg]\tag{2.62}$$

Equation (2.62) represents the transverse displacement response to a moving distributed force of a rectangular plate resting on a variable bi-parametric elastic foundation with constant velocity.

2.2 Rectangular Plate on Variable Foundation Traversed by Moving Distributed Mass

In this section, one is required to solve the entire equation (2.24) when no term of the coupled differential equation is neglected. This is termed the moving distributed mass problem.

To this end, the approximate analytical solution method of Struble that has been used to solve this form of coupled differential equation in the previous section shall be employed to obtain its closed form solution.

Thus equation (2.24) after arrangements can be rewritten in the form

$$\ddot{Z}(m,t)+\frac{\varepsilon_{mf}H_{a2}(n,m,t)}{\left[1+\varepsilon_{mf}H_{a1}(n,m,t)\right]}\dot{Z}(m,t)+\frac{\tau_{mm}^2+\varepsilon_{mf}H_{a3}(n,m,t)}{\left[1+\varepsilon_{mf}H_{a1}(n,m,t)\right]}Z(m,t)$$

$$+\frac{\varepsilon_{mf}}{\left[1+\varepsilon_{mf}H_{a1}(n,m,t)\right]}\sum_{\substack{\alpha=1\\ \alpha\neq 1}}^{\infty}\Big[H_{a1}(n,m,t)\ddot{Z}_\alpha(m,t)+H_{a2}(n,m,t)\dot{Z}_\alpha(m,t)$$

$$+H_{a3}(n,m,t)Z_\alpha(m,t)\Big]=\frac{\varepsilon_{mf}gL_xL_y}{\left[1+\varepsilon_{mf}H_{a1}(n,m,t)\right]\Omega_m^*}a_{\alpha i}(ct)a_{\alpha j}(y_1)\tag{2.63}$$

Where

$$\varepsilon_{mf}=\frac{M}{\mu L_xL_y}\tag{2.64}$$

$$H_{a1}(n,m,t)=\frac{\varepsilon_{mf}}{\Omega_m^*}\Bigg[\Bigg(\frac{1}{16}\phi_{CA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{CB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{CC}$$

$$+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CD}-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{CE}+\frac{1}{\pi^2}\Bigg(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{CE}$$

$$-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{CD}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{CB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{CC}\Bigg)\Bigg)\Bigg]\tag{2.65}$$

$$H_{a2}(n,m,t)=\frac{2c\varepsilon_{mf}}{\Omega_m^*}\Bigg[\frac{1}{16}\phi_{DA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{DB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{DC}$$

$$+\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{DD}-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{DE}+\frac{1}{\pi^2}\Bigg(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{DD}$$

$$-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{DE}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{DB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{DC}\Bigg)\Bigg]\tag{2.66}$$

and

$$H_{a3}(n,m,t)=\frac{c^2\varepsilon_{mf}}{\Omega_m^*}\Bigg[\frac{1}{16}\phi_{EA}+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{EB}-\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{EC}$$

$$+\frac{1}{4\pi}\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{ED}-\frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{EE}+\frac{1}{\pi^2}\left(\sum_{n=0}^{\infty}\frac{\cos(2n+1)}{2n+1}\pi y_1\phi_{ED}\right.$$
$$\left.-\sum_{n=0}^{\infty}\frac{\sin(2n+1)}{2n+1}\pi y_1\phi_{EE}\times\sum_{m=0}^{\infty}\frac{\cos(2m+1)}{2m+1}\pi y_1\phi_{EB}-\sum_{m=0}^{\infty}\frac{\sin(2m+1)}{2m+1}\pi y_1\phi_{EC}\right)\Bigg] \quad (2.67)$$

In what follows, we shall first consider the homogenous part of equation (2.63) and obtain a modified frequency corresponding to the frequency of the free system due to the presence of the moving mass M. An equivalent free system operator defined by the modified frequency then replaces equation (2.63). To this end, we set as before

$$\varepsilon_{mf} = \lambda_1 + 0(\lambda_1^2) \quad (2.68)$$

setting $\lambda_1 = 0$, we obtain a case corresponding to the case when the inertia effect of the mass of the moving system is neglected. The solution of the homogeneous part of equation (2.24) can be written as

$$Z(m,t) = C_{mm}\cos\left[\tau_{mm}t-\lambda_1\right] \quad (2.69)$$

where C_{mm} and λ_1 are constants.

setting $\lambda_1 < 0$, an asymptotic solution of the homogeneous part of equation (2.24) can be written in the form

$$Z(m,t) = A_m(t)c\,os\left[\tau_{mm}t-\phi_m\right]+Z(1,t)+0(\lambda_1^2) \quad (2.70)$$

where $A_m(t)$ and ϕ_m are slowly varying function of time.

Substituting equation (2.70) and its derivatives into the homogenous part of equation (2.24), we note the following in oder to obtain variational equations

$$\frac{cos(2m+1)\pi y_1}{2m+1}sin\left[\tau_{mm}(t)-\phi_m\right]=\frac{1}{2(2m+1)}sin\left[(2m+1)\pi y_1+\left(\tau_{mm}(t)-\phi_m\right)2m+1\right]$$
$$-\frac{1}{2(2m+1)}sin\left[(2m+1)\pi y_1-\left(\tau_{mm}(t)-\phi_m\right)2m+1\right] \quad (2.71)$$

$$\frac{\sin(2m+1)\pi y_1}{2m+1}sin\left[\tau_{mm}(t)-\phi_m\right]=\frac{1}{2(2m+1)}sin\left[(2m+1)\pi y_1-\left(\tau_{mm}(t)-\phi_m\right)2m+1\right]$$
$$-\frac{1}{2(2m+1)}\cos\left[(2m+1)\pi y_1+\left(\tau_{mm}(t)-\phi_m\right)2m+1\right] \quad (2.72)$$

$$\frac{cos(2m+1)\pi y_1}{2m+1}cos\left[\tau_{mm}(t)-\phi_m\right]=\frac{1}{2(2m+1)}sin\left[(2m+1)\pi y_1+\left(\tau_{mm}(t)-\phi_m\right)2m+1\right]$$
$$+\frac{1}{2(2m+1)}\cos\left[(2m+1)\pi y_1-\left(\tau_{mm}(t)-\phi_m\right)2m+1\right] \quad (2.73)$$

and

$$\frac{\sin(2m+1)\pi y_1}{2m+1}\cos\left[\tau_{mm}(t)-\phi_m\right]=\frac{1}{2(2m+1)}sin\left[(2m+1)\pi y_1+\left(\tau_{mm}(t)-\phi_m\right)2m+1\right]$$
$$+\frac{1}{2(2m+1)}\sin\left[(2m+1)\pi y_1-\left(\tau_{mm}(t)-\phi_m\right)2m+1\right] \quad (2.74)$$

Similarly, the same results are obtained when $m = n$ in equations (2.71) to (2.72) above.
Therefore when the effect of the mass of the load is considered, the first approximation to the homogeneous system is

$$Z(m,t) = A_1e^{-\varepsilon_m t}\cos\left[\tau_{mk}t-\phi_m\right] \quad (2.75)$$

Where

$$\tau_{mk} = \tau_{mm}\left[1-\frac{\lambda_1}{8}\left(\phi_{CA}-\frac{c^2\phi_{EA}}{\tau_{mm}}\right)\right] \quad (2.76)$$

is the modified frequency corresponding to the frequency of the free system due to the presence of moving distributed mass. us to solve the non-homogeneous equation (2.24), the differential operator which acts on $Z(m,t)$ and $Z_\alpha(m,t)$, is replaced by the equivalent free system operator defined by the modified frequency τ_{mk}. That is

$$\ddot{Z}(m,t)+\tau_{mk}^2 Z(m,t) = \frac{\lambda_1 g L_x L_y}{\Omega_m^*} a_{mi}(ct)a_{mj}(y_1) \quad (2.77)$$

It is noticed that equation (2.77) is prototype of equation (2.70) with τ_{mk} replacing τ_{mm} respectively. Therefore the solution to the entire equation (2.77) when inverted is given by

$$V(x,y,t) = \sum_{mi=1}^{\infty}\sum_{mj=1}^{\infty}\frac{H_m L_x L_y}{m_i m_j}\cdot E_m(y_1)\left[-\frac{1}{\tau_{mk}}E_m^o \sin\tau_{mk}t - \frac{\sin b_{mi}t+\sin\tau_{mk}t}{\tau_{mk}^2-b_{mi}^2} - \frac{\cos b_{mi}t+\cos\tau_{mk}t}{\tau_{mk}^2-b_{mi}^2}\right.$$
$$-\frac{B_{mi}2\tau_{mk}^2 b_{mi}\sin 2\tau_{mk}t\cosh b_{mi}t}{\tau_{mk}^4-b_{mi}^4} - \frac{B_{mi}b_{mi}^2\cos 2\tau_{mk}t\sin b_{mi}t}{\tau_{mk}^4-b_{mi}^4} + \frac{B_{mi}\tau_{mk}^2\sin b_{mi}t}{\tau_{mk}^4-b_{mi}^4}$$
$$+\frac{B_{mi}b_{mi}\sin\tau_{mk}t(b_{mi}^2-\tau_{mk}^2)}{\tau_{mk}^4-b_{mi}^4} - \frac{C_{mi}2\tau_{mk}^2 b_{mi}\sin 2\tau_{mk}t\sinh b_{mi}t}{\tau_{mk}^4-b_{mi}^4} + \frac{C_{mi}b_{mi}^2\cos 2\tau_{mk}t\cos hb_{mi}t}{\tau_{mk}^4-b_{mi}^4}$$
$$\left.+\frac{C_{mi}\tau_{mk}^2\cosh b_{mi}t}{\tau_{mk}^4-b_{mi}^4} + \frac{C_{mi}(b_{mi}^2-\tau_{mk}^2)}{\tau_{mk}^4-b_{mi}^4}\right]\times\left[\left(\sin\frac{b_{mi}x}{L_x}+A_{mi}\cos\frac{b_{mi}x}{L_x}+B_{mi}\sinh\frac{b_{mi}x}{L_x}\right.\right.$$
$$\left.\left.+C_{mi}\cosh\frac{b_{mi}x}{L_x}\right)\cdot\left(\sin\frac{b_{mj}y_1}{L_y}-A_{mj}\cos\frac{b_{mj}y_1}{L_y}-B_{mj}\sinh\frac{b_{mj}y_1}{L_y}+C_{mj}\cosh\frac{b_{mj}y_1}{L_y}\right)\right] \quad (2.78)$$

Equation (2.78) represents displacement response to a moving distributed mass of a rectangular plate resting on variable bi-parametric elastic foundation with constant velocity.

3.0 Application

We shall now illustrate the foregoing analysis by practical example. Particularly, we shall consider plates clamped at edges $x=0, x=L_x$ with simple supports at edges $y=0, y=L_y$ and plates clamped at all edges.

3.1 Rectangular Plate Clamped at Edges $x=0, x=L_x$ With Simple Supports At Egdes $y=0, y=L_y$.

In this section, a rectangular plate clamped at edges $x=0, x=L_x$ with simple support at edges $y=0, y=L_y$, the boundary conditions at such opposite edges are

$$V(0,y,t)=0, \qquad V(L_x,y,t)=0 \quad (3.1)$$

$$V(x,y,t)=0, \qquad V(x,L_y,t)=0 \quad (3.2)$$

$$\frac{\partial V(0,y,t)}{\partial x}=0, \qquad \frac{\partial V(L_x,y,t)}{\partial x}=0 \quad (3.3)$$

$$\frac{\partial^2 V(x,y,t)}{\partial y^2}=0, \qquad \frac{\partial^2 V(x,L_y,t)}{\partial y^2}=0 \quad (3.4)$$

and hence for the normal modes, one obtains

$$U_{mi}(0)=0, \qquad U_{mi}(L_x)=0 \quad (3.5)$$

$$U_{mj}\left(0\right)=0, \qquad U_{mj}\left(L_y\right)=0 \tag{3.6}$$

$$\frac{\partial U_{mi}\left(0\right)}{\partial x}=0, \qquad \frac{\partial U_{mi}\left(L_x\right)}{\partial x}=0 \tag{3.7}$$

$$\frac{\partial^2 U_{mj}\left(0\right)}{\partial y^2}=0, \qquad \frac{\partial^2 U_{mj}\left(L_y\right)}{\partial y^2}=0 \tag{3.8}$$

For simplicity, the initial conditions is of the form

$$U(x,y,0)=0=\frac{\partial U(x,y,0)}{\partial t} \tag{3.9}$$

Using the boundary conditions (3.1) to (3.4) and (3.5) and(3.6)in Beam functions. Hence, the following values of the constants are obtained for the clamped edges

$$A_{mi}=-\frac{\sinh\lambda_{mi}-\sin\lambda_{mi}}{\cosh\lambda_{mi}-\cos\lambda_{mi}} \Rightarrow A_{pi}=-\frac{\sinh\lambda_{pi}-\sin\lambda_{pi}}{\cosh\lambda_{pi}-\cos\lambda_{pi}} \tag{3.10}$$

$$B_{mj}=-1 \quad \Rightarrow \quad B_{pi}=-1 \tag{3.11}$$

$$C_{mj}=-A_{mi} \quad \Rightarrow \quad C_{pi}=-A_{pi} \tag{3.12}$$

Where

$$\lambda_{mi}=b_{mi} \tag{3.13}$$

The frequency equation of the clamped edges is given by the following determinant equation

$$\begin{vmatrix} \cos\lambda_{mi}-\cosh\lambda_{mi} & \sin\lambda_{mi}-\sinh\lambda_{mi} \\ \sin\lambda_{mi}+\sinh\lambda_{mi} & -\cos\lambda_{mi}-\cosh\lambda_{mi} \end{vmatrix}=0 \tag{3.14}$$

Which when simplified yields

$$\cosh\lambda_{mi}\cos\lambda_{mi}-1=0 \tag{3.15}$$

such that

$$\lambda_{1i}=4.73004, \lambda_{2i}=7.85320, \lambda_{3i}=10.99561 \tag{3.16}$$

and
$$U_{pi}=\frac{\mu_0 L_x}{2}\Big\{1+A_{pi}^2-B_{pi}^2+C_{pi}^2+\frac{1}{\lambda_{pi}}\Big[2C_{pi}-2A_{pi}B_{pi}-B_{pi}C_{pi}-\frac{1}{2}\left(1-A_{pi}^2\right)\sin 2\lambda_{pi}+2A_{pi}\sin^2\lambda_{pi}$$
$$+\left(B_{pi}^2+C_{pi}^2\right)\sinh\lambda_{pi}\cosh\lambda_{pi}+2\left(B_{pi}+A_{pi}C_{pi}\right)\cosh\lambda_{pi}\sin\lambda_{pi}+2\left(-B_{pi}+A_{pi}C_{pi}\right)\sinh\lambda_{pi}\cos\lambda_{pi}$$
$$+2\left(C_{pi}+A_{pi}B_{pi}\right)\sinh\lambda_{pi}\sin\lambda_{pi}+2\left(-C_{pi}+A_{pi}B_{pi}\right)\cosh\lambda_{pi}\cos\lambda_{pi}+B_{pi}C_{pi}\cosh\lambda_{pi}\Big]\Big\} \tag{3.17}$$

U_{pj} is obtained by replacing subscript i with j in equation (3.17). For the simple edges, it is

$$A_{mj}=0 \quad \Rightarrow \quad A_{pj}=0 \tag{3.18}$$

$$B_{mj}=0 \quad \Rightarrow \quad B_{pj}=0 \tag{3.19}$$

$$C_{mj}=0 \quad \Rightarrow \quad C_{pj}=0 \tag{3.20}$$

while the corresponding frequency equation is

$$\lambda_{mj}=mj\pi \quad \Rightarrow \quad \lambda_{pj}=pj\pi \tag{3.21}$$

and

$$U_{pj}=\frac{\mu_0 L_y}{2} \tag{3.22}$$

Thus, the general solutions of the associated moving distributed force and moving distributed mass problems of the simple-clamped rectangular plate are obtained by substituting the above results in (3.10) to (3.22)into equations (2.62) and (2.78).

3.2 Rectangular Plate Clamped at All Edges

For a rectangular plate clamped at all edges, both the deflection and the slope vanish at such ends. Thus the following boundary conditions pertains

$$V(0,y,t)=0, \qquad V(L_x,y,t)=0 \tag{3.23}$$

$$V(x,0,t)=0, \qquad V(x,L_x,t)=0 \tag{3.24}$$

$$\frac{\partial V(0,y,t)}{\partial x}=0, \qquad \frac{\partial V(L_x,y,t)}{\partial x}=0 \tag{3.25}$$

$$\frac{\partial U(x,0,t)}{\partial y}=0, \qquad \frac{\partial U(x,L_y,t)}{\partial y}=0 \tag{3.26}$$

And hence for the normal modes, one obtains

$$U_{mi}\left(0\right)=0, \qquad U_{mi}\left(L_x\right)=0 \tag{3.27}$$

$$U_{mj}\left(0\right)=0, \qquad U_{mj}\left(L_y\right)=0 \tag{3.28}$$

$$\frac{\partial U_{mi}\left(0\right)}{\partial x}=0, \qquad \frac{\partial U_{mi}\left(L_x\right)}{\partial x}=0 \tag{3.29}$$

$$\frac{\partial U_{mj}(0)}{\partial y}=0 \qquad \frac{\partial U_{mj}(L_y)}{\partial y}=0 \tag{3.30}$$

The initial conditions are taken to be of the form given by the equation (2.12). Using the boundary conditions (3.23) to (3.26) and (3.27) to (3.28) in Beam functions, one obtains the following values of the constants and the frequency equations for the clamped edge $x=0$ sand $x=L_x$

$$A_{mi}=-\frac{\sinh\lambda_{mi}-\sin\lambda_{mi}}{\cosh\lambda_{mi}-\cos\lambda_{mi}} \Rightarrow A_{pi}=-\frac{\sinh\lambda_{pi}-\sin\lambda_{pi}}{\cosh\lambda_{pi}-\cos\lambda_{pi}} \tag{3.31}$$

$$B_{mj}=-1 \quad \Rightarrow \quad B_{pj}=-1 \tag{3.32}$$

$$C_{mj}=-A_{mj} \quad \Rightarrow \quad \mathrm{C}_{pj}=-A_{pj} \tag{3.33}$$

The frequency equation of the clamped edges is given by the following determinant equation

$$\begin{vmatrix} \cos\lambda_{mi}-\cosh\lambda_{mi} & \sin\lambda_{mi}-\sinh\lambda_{mj} \\ \sin\lambda_{mi}+\sinh\lambda_{mj} & -\cos\lambda_{mi}-\cosh\lambda_{mi} \end{vmatrix}=0 \tag{3.34}$$

Which when simplified yields

$$\cosh\lambda_{mi}\cos\lambda_{mi}-1=0 \tag{3.35}$$

Such that

$$\lambda_{1i}=4.73004, \lambda_{2i}=7.85320, \lambda_{3i}=10.99561 \tag{3.36}$$

It follows that for the pj^th mode of vibration

$$\cosh\lambda_{pj}\cos\lambda_{pj}-1=0 \tag{3.37}$$

Similarly, for the clamped edges, $y=0$ and $y=L_y$ the same process is followed to obtain

$$A_{mj}=-\frac{\sinh\lambda_{mj}-\sin\lambda_{mj}}{\cosh\lambda_{mj}-\cos\lambda_{mj}} \Rightarrow A_{pi}=-\frac{\sinh\lambda_{pj}-\sin\lambda_{pj}}{\cosh\lambda_{pj}-\cos\lambda_{pj}} \tag{3.38}$$

$$B_{mj}=-1 \quad \Rightarrow \quad B_{pj}=-1 \tag{3.39}$$

$$C_{mj}=-A_{mj} \quad \Rightarrow \quad \mathrm{C}_{pj}=-A_{pj} \tag{3.40}$$

The frequency equation of the clamped edges is given by the following determinant equation

$$\begin{vmatrix} \cos\lambda_{pj}-\cosh\lambda_{pj} & \sin\lambda_{pj}-\sinh\lambda_{pi} \\ \sin\lambda_{pj}+\sinh\lambda_{pi} & -\cos\lambda_{pj}-\cosh\lambda_{pj} \end{vmatrix}=0 \tag{3.41}$$

which when simplified yields

$$\cosh\lambda_{pj}\cos\lambda_{pj}-1=0 \tag{3.42}$$

Similarly, for pith mode of vibration, we have

$$\cosh\lambda_{pi}\cos\lambda_{pi}-1=0 \tag{3.43}$$

Using arguments similar to the previous ones, U_{pi} is given by the equation (3.17) when the values of constants, A_{pj} , B_{pj} , C_{pj} and λ_{pj} and are approximately substituted into the equation. U_{mj} is obtained by replacing subscript pi by mj in equation (3.17).

Thus, the general solutions of the associated moving distributed force and moving distributed mass problems of the clamped-clamped rectangular plate are obtained by substituting the above results in (3.31) to (3.43) into equations (2.62) and (2.78).

4.0 Comments on Closed form Solutions

In studying the undamped system such as this, it is desirable to examine the phenomenon of resonance. Equation (2.62) clearly shows that the isotropic rectangular plate traversed by a moving distributed force at constant velocity reaches a state of resonance whenever

$$\tau_{mm}=\frac{\lambda_{\alpha i}c}{L_x} \tag{4.1}$$

while equation (2.78) shows that the same plate under the action of a moving distributed mass experiences resonance effect whenever

$$\tau_{mk}=\frac{\lambda_{\alpha i}c}{L_x} \tag{4.2}$$

Where

$$\tau_{mk}=\tau_{mm}\left[1-\frac{\lambda_1}{8}\left(\phi_{CA}-\frac{c^2\phi_{EA}}{\tau_{mm}}\right)\right] \tag{4.3}$$

Equations (4.2) and (4.3) imply

$$\tau_{mk}=\tau_{mm}\left[1-\frac{\lambda_1}{8}\left(\phi_{CA}-\frac{c^2\phi_{EA}}{\tau_{mm}}\right)\right]=\frac{\lambda_{\alpha i}c}{L_x} \tag{4.4}$$

Therefore, it can be deduced from equations (4.4) that, for the same natural frequency, the critical speed (and the natural frequency) for the system traversed by a moving distributed mass is smaller than that of the system traversed by a moving distributed force. Thus, resonance is reached earlier in the moving distributed mass system than in the moving distributed force system.

5.0 Numerical Results and Discussion

In order to carry out the calculations of practical interests in the dynamics of structures and engineering design for all the illustrative examples considered in this section. A rectangular plate resting on variable bi-parametric elastic foundation of length $L_y=0.914m$,and breath $L_x=0.457m$ is considered. The mass is assumed to travel at the constant velocity $c=8.123\text{m/s}$. Furthermore, values for $\alpha,\Gamma_{mp},\mathrm{E},$ and y_1 are chosen to be $2.109\times10^9 kg/m^2$, 0.5, 0.004 be and 0.4m respectively. The results are as presented on the various classes of boundary condition considered.

The deflection profile of simple-clamped rectangular plate resting on variable bi-parametric elastic subgrade under the action of moving distributed force is given in figure 1 for various values of rotatory inertia correction factor and fixed values of foundation stiffness So=20000 and shear modulus Ko=40000. The result shows that as Ro increases, the deflection profile of the isotropic rectangular plate with constant velocity decreases. Similar results are obtained when the simple-clamped plate is subjected to a moving distributed mass as shown in figure 2.

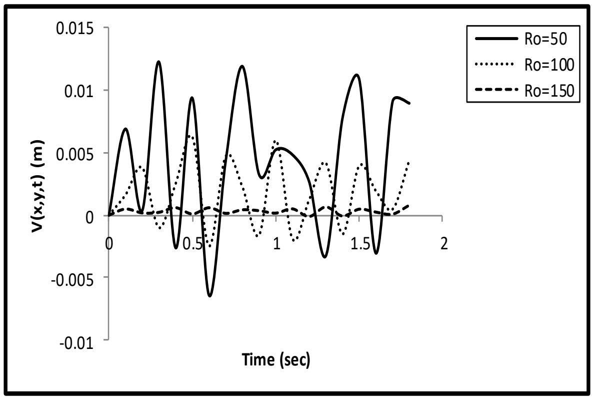

Fig. 1: Deflection profile of simple-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed force for Ko=40000, So=20000 and various values of Ro.

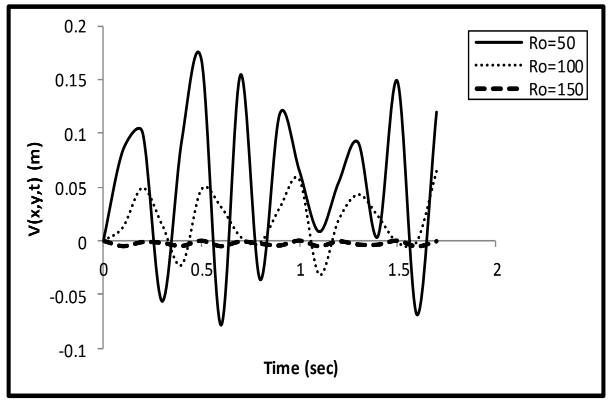

Fig. 2: Deflection profile of simple-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed mass for Ko=40000, So=20000 and various values of Ro.

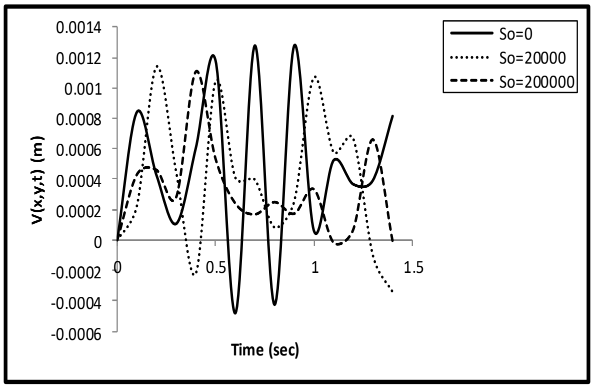

Fig. 3: Transverse displacement response of simple-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed force for Ko=40000, Ro=0.4 and various values of So.

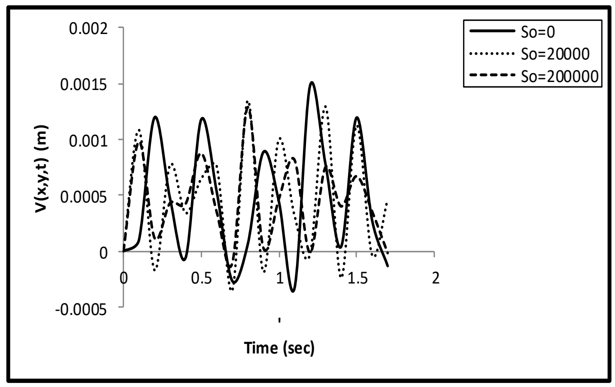

Fig. 4: Transverse displacement response of simple-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed mass for Ko=40000, Ro=0.4 and various values of So.

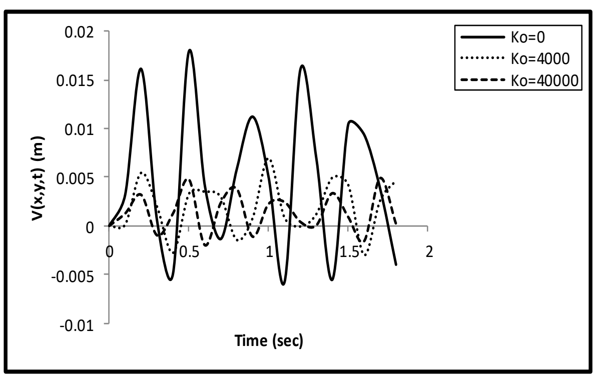

Fig. 5: Displacement response of simple-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed force for So=20000, Ro=0.4 and various values of Ko.

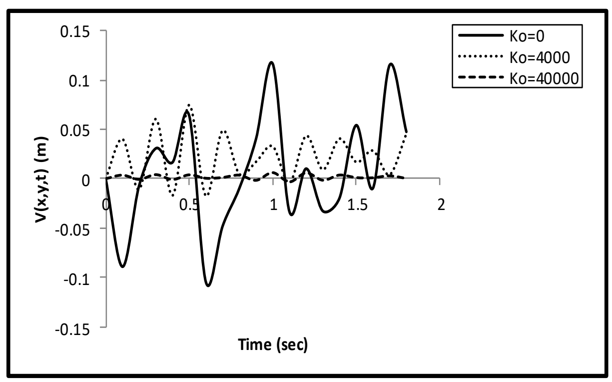

Fig. 6: Displacement response of simple-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed mass for So=20000, Ro=0.4 and various values of Ko.

In figure 3, the displacement response of a simple-clamped rectangular plate traversed by moving distributed force for various values of shear modulus Ko and for fixed values of rotatory inertia correction factor Ro=0.4 and foundation stiffness So=20000. It is observed that higher values of shear modulus Ko reduce the deflection profile of the rectangular plate. The same results are obtained for the simple-clamped rectangular plate under moving distributed mass at constant velocity for various value of shear modulus Ko as shown in figure 4.

Figures 5 and 6 show the transverse displacement response of the simple-clamped rectangular plate to moving distributed forces and masses respectively for various values of foundation stiffness So and for fixed values of rotatory inertia correction factor Ro=0.4 and shear modulus Ko=40000. From the figures, it is observed that as the values of foundation stiffness increase, the deflection profile of the simple-clamped plate under the action of moving distributed forces and moving distributed masses decreases.

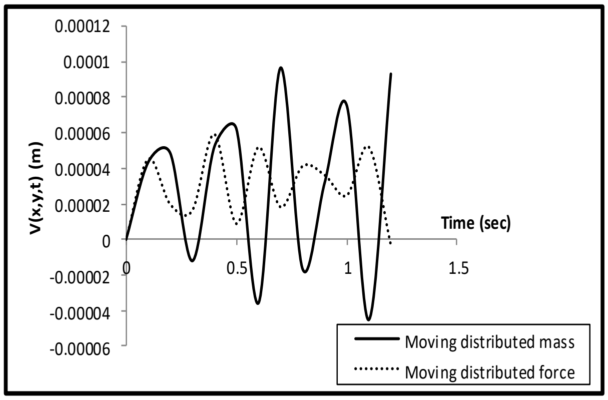

Fig. 7: Comparison of the displacement response of moving distributed force and moving distributed mass cases for simple-clamped rectangular plate on variable bi-parametric elastic foundation for So=20000, Ko=40000, and Ro=0.4.

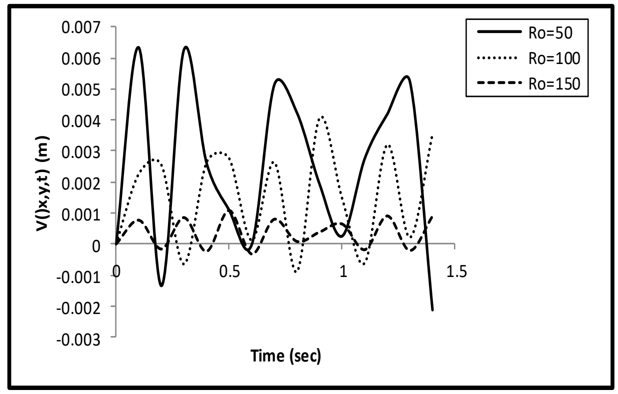

Fig. 8: Deflection profile of clamped-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed force for Ko=40000, So=20000 and various values of Ro.

Also, figure 7 shows the comparison of the traverse displacement response of moving distributed force and moving distributed mass cases of the simple-clamped rectangular plate on variable bi-parametric elastic subgrade for fixed values of Ro=0.4, Ko=20000 and So=40000.

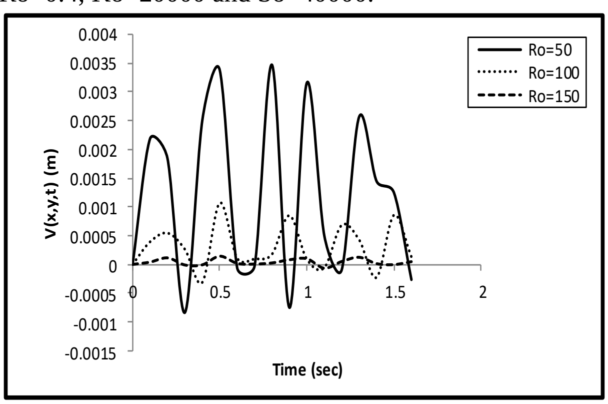

Fig. 9: Deflection profile of clamped-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed mass for Ko=40000, So=20000 and various values of Ro.

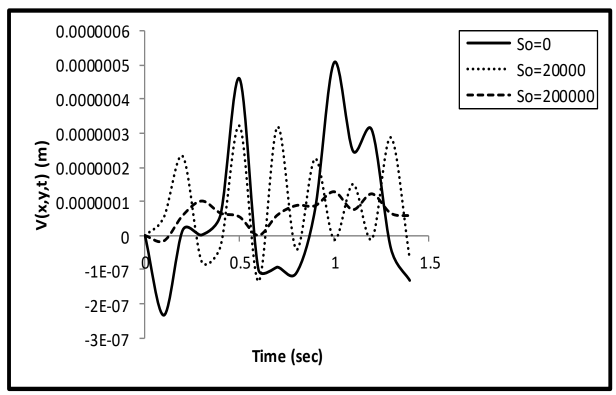

Fig. 10: Transverse displacement response of clamped-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed force for Ko=40000, Ro=0.4 and various values of So.

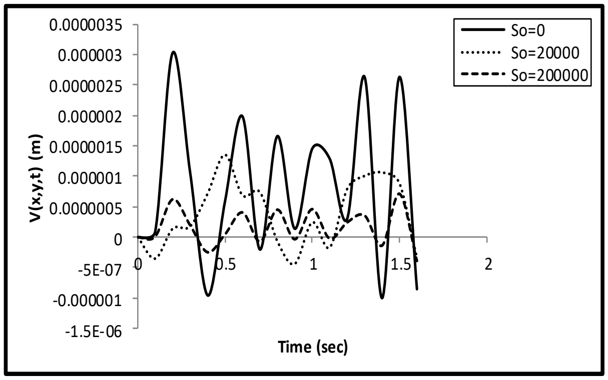

Fig. 11: Transverse displacement response of clamped-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed mass for Ko=40000, Ro=0.4 and various values of So.

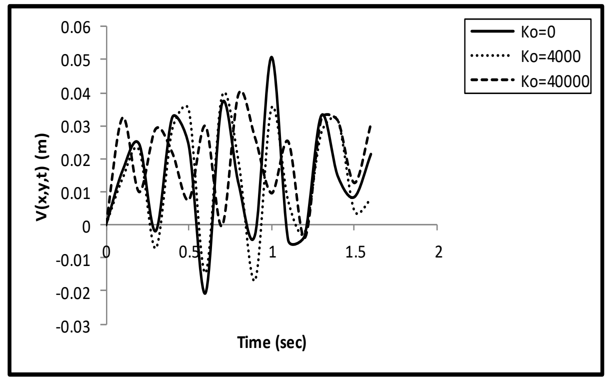

Fig. 12: Displacement response of clamped-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed force for So=20000, Ro=0.4 and various values of Ko.

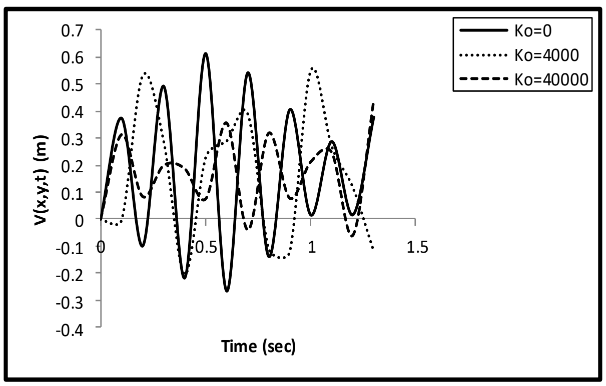

Fig. 13: Displacement response of clamped-clamped rectangular plate on variable bi-parametric elastic foundation and traversed by moving distributed mass for So=20000, Ro=0.4 and various values of Ko.

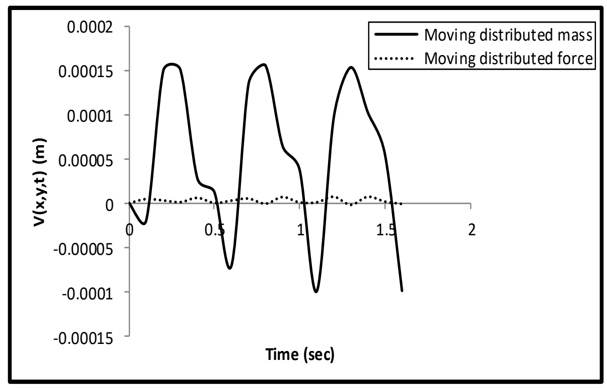

Fig. 14: Comparison of the displacement response of moving distributed force and moving distributed mass cases for clamped-clamped rectangular plate on variable bi-parametric elastic foundation for So=20000, Ko=40000, and Ro=0.4.

The displacement response of clamped-clamped rectangular plate resting on variable bi-parametric elastic subgrade under the action of moving distributed force is given in figure 8 for various values of rotatory inertia correction factor and fixed values of foundation stiffness So=20000 and shear modulus Ko=40000. The result shows that as Ro increases, the deflection profile of the isotropic rectangular plate with constant velocity decreases. Similar results are obtained when the clamped -clamped plate is subjected to a moving distributed mass as shown in figure9.

In figure 10, the displacement response of a clamped -clamped rectangular plate traversed by moving distributed force for various values of shear modulus Ko and for fixed values of rotatory inertia correction factor Ro=0.4 and foundation stiffness So=40000. It is observed that higher values of foundation stiffness So reduce the deflection profile of the rectangular plate. While similar results are obtained for the clamped-clamped rectangular plate under moving distributed mass at constant velocity for various value of shear modulus Ko as shown in figure 11.

Figures 12 and 13 show the transverse displacement response of the clamped-clamped rectangular plate to moving distributed forces and masses respectively for various values of foundation stiffness So and for fixed values of rotatory inertia correction factor Ro=0.4 and shear modulus Ko=40000. From the figures, it is observed that as the values of foundation stiffness increase, the deflection profile of the clamped-clamped plate under the action of moving distributed forces and moving distributed masses decreases.

Also, figure 14 shows the comparison of the traverse displacement response of moving distributed force and moving distributed mass cases of the clamped-clamped rectangular plate on variable bi-parametric elastic subgrade for fixed values of Ro=0.4, Ko=20000 and So=40000.

6.0 Conclusion

The problem of the dynamic behavior of a rectangular plate resting on variable bi-parametric elastic foundation and traversed by moving distributed masses moving at uniform velocity has been investigated. In this work, closed form solutions of the fourth order partial differential equation governing the system of two-dimensional structure are presented. The solution technique is based on the shadnam et al method [12],the expansion of Heaviside function in series form, a modification of Struble's asymptotic technique and the use of Laplace transformation.

For the illustrative examples considered, the solutions obtained are analyzed and the resonance conditions for all the problems investigated show that resonance is reached earlier in a system traversed by moving distributed mass than that under the action of a moving distributed force. Also, an increase in the values of rectangular plate parameters namely, foundation stiffness, shear modulus, rotatory inertia correction factor decrease the response amplitude of the plate. It is established that for all the illustrative examples considered, the moving distributed force solution is not an upper bound for the accurate solution of the moving distributed mass cases. Thereby established the non-reliability of the moving distributed force solution as a safe approximation to the moving distributed mass problem is confirmed.

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