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Equivalence Relations on Fuzzy Bi-Group

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Abstract

In this paper, we extend the concept of equivalence relations to fuzzy bi-group.We introduce the idea of a support of a fuzzy bi- group and discuss its fundamental properties.We restructure the definition of t-level subbi-group as it exists in literature and useit to study equivalence relations on distinct fuzzy sub bi-groups of the samebi-group.We define bi-level subset of a fuzzy bi-group and fuzzy power level sub bi-group of a bi-group andstudy the effect of equivalence relations on distinct fuzzy subbi-groupon these concepts. We establish a one to one correspondence between two distinct t-level supportsof γ_G and η_G of G and show that t-level support of γ_G is equivalent to t-level support of η_G .

Keywords: Bi-groups, support of fuzzy bi-group, t- level sub bi-group, fuzzy power level sub bi-group.
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1.0 Introduction

1.1 Linear Regression Model

Fuzzy set was introduced by Zadeh[1]. Rosenfeld[2]introduced of fuzzy subgroups. The notion of-group was first introduced by Maggu[3]. This idea was extendedby Vasantha and Meiyappan[4]. These authors gavemodification of some results earlier established by Maggu. Meiyappan[5]introduced and characterized fuzzy sub-bigroup of a bigroup. Akinola and Agboola[6] modified the definition of fuzzy bi-group given by Meiyappan and used the modified definition to study the concept of permutable and mutually permutable fuzzybi-group. Akinola et al.[7] studied further properties of fuzzy bi-group in the aspect of fuzzy bi-group homomorphism .

Michiro[8] defined fuzzy congruence on group and investigated its properties.Jain[9] studied equivalence relation on set of fuzzy subgroups of an arbitrary group G and gave equivalent conditions that characterize this relation. Some other researchershave advanced the work of Jain. Recent one whose ideas are relevant to our work can befound in Zhaowen el al.[10].

In this work, we extend the concept of equivalence relations to fuzzy bi-group.We introduce the idea of support of a fuzzy bi-group and show how it behaves in eachcompartment of a bi-group. We modify the definition of t-level sub bi-group asit exists in literature and use these ideas to study equivalence relations on distinctfuzzy subgroups of the same fuzzy bi-group.We establish the conditions to be satisfied by fuzzy bi-groups of the same bi-group tobe equivalent. We study how these conditions are applicable to different compartments of thesame bi-group. The results of these investigation are presented in Propositions(3.5)and (3.6). We define bi-level subset and fuzzy power level sub bi-group of a bi-group and study theeffect of equivalence relations on distinct fuzzy subgroups of these concepts.The outcomes of the study are presented in Theorem(3.9), Theorem(3.11) and corollary(3.12).

2.0 Preliminaries

For the sake of completeness, some of the results in literature that are sequel to establishing our results are mentioned in this section.

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Definition (2.1): A function $\mu: A \rightarrow [0,1]$ in $\Re$ is called a fuzzy set.
Definition (2.2): Let γ be a fuzzy set in a set G. Then, the t-levelsubset of γ, denoted by γ_t is defined as: $\gamma_t = \{x \in G: \gamma(x) \geq t\}fort \in [0,1]$.
Theorem (2.3): Let G be a group and μ be a fuzzy subgroup of G.Then the level subsets μ_t, for $t \in [0,1], t = \mu(e)$ is a subgroup of G, where e is the identity of G.
Proof: Omitted.
Definition (2.4): If A is a group and $\mu: A \rightarrow [0,1]$ a fuzzy set,$\mu$is called a fuzzy subgroup of a group G is the following are satisfied
(i) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$
(ii) $\mu(x^{-1}) \geq \mu(x)$ for $x, y \in A$.
Theorem (2.5): Let μ be a fuzzy subset of a group G. Then μ is a fuzzy subgroup of G if and only ifG_μ^t isa subgroup (called level subgroup) of the group G for every $t \in [0, \mu(e)]$, where e is the identity element of the group G.
Proof: Omitted.
Definition (2.6): A fuzzy normal subgroup μ of G is defined as afuzzy subgroup satisfying the condition$\mu(xy) \geq \mu(yx)$.
The concept of a relation has a natural extension to fuzzy sets and playsan important role in the theory of such sets and their applications justas it does in the case of conventional sets.
A fuzzy binary relation R_λ on a set X is defined as a fuzzysubset of $X \times X$. The composition of two fuzzy relations $R_\lambda$and R_μ is defined as
$(R_\lambda \circ R_\mu)(x, y) = Sup_{t \in X}\{\min[R_\lambda(x, t), R_\mu(t, y)]\}, \forall x, y \in X$.
Definition (2.7): A fuzzy binary relation R_λ on a set X is said tobe a similarity relation on the set X if it is reflexive, symmetric and transitive that is,for every $x, y, z \in X$.
(i) $R_\lambda(x, x) = 1$
(ii) $R_\lambda(x, y) = R_\lambda(y, x)$
(iii) $\min\{R_\lambda(x, y), R_\lambda(y, z)\} = R_\lambda(x, z)$.
Definition (2.8): Let γ_1 be a fuzzy subset of a setX_1 and γ_2 be a fuzzy subset of a set X_2, then thefuzzy union of the sets γ_1 and $\gamma_2$is defined as a function $\gamma_1 \cup \gamma_2: X_1 \cup X_2 \rightarrow [0,1]$ given by:

$$(\gamma_1 \cup \gamma_2)(x) = \begin{cases} \max(\gamma_1(x), \gamma_2(x)) \text{ if } x \in X_1 \cap X_2, \\ \gamma_1(x) \text{ if } x \in X_1 \ \& \ x \notin X_2, \\ \gamma_2(x) \text{ if } x \in X_2 \ \& \ x \notin X_1. \end{cases}$$

Definition (2.9): A set $(G, +, \cdot)$ with two binaryoperations " $+$ " and " $\cdot$ " is called a bi-group if there existtwo proper subsets G_1 and G_2 of G such that
(i) $G = G_1 \cup G_2$,
(ii) $(G_1, +)$ is a group,
(iii) $(G_2, \cdot)$ is a group.
Definition (2.10): Let $G = G_1 \cup G_2$ be a bi-group. Then $\mu = \mu : G \rightarrow [0,1]$is said to be a fuzzy sub bi-group of G if there exist two fuzzy subsets of $\mu_1\,(of\ G_1)$and $\mu_2\,(of\ G_2)$ such that ;
(i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$
(ii) $(\mu_2, \cdot)$ is a fuzzy subgroup of $(G_2, \cdot)$
(iii) $\mu = \mu_1 \cup \mu_2$
Definition (2.11): Let $G = (G_1 \cup G_2, +, \cdot)$ be a bi-group and $\mu = (\mu_1 \cup \mu_2)$ be a fuzzy sub bi-group of the bi-group G. The bilevel subset of the fuzzy sub bi-group μ of the bi-group G is defined as $G_\mu{}^t = G^t{}_{1_{\mu_1}} \cup G^t{}_{2_{\mu_2}}$ for $t \in [0, \min\{\mu_1(e_1), \mu_2(e_2)\}]$ where e_1 and e_2 are the identities of G_1 and G_2 respectively.

Definition (2.12): Let G be a group and μ and $\nu \in F(G)$, the set of all fuzzy subgroups of G.Three equivalence relations are defined respectively as follow:
(i) We say that μ is equivalent toν, written as $\mu \approx \nu$ if $F_\mu = F_\nu$.
(ii) We say that μ is equivalent to ν, written as $\mu \sim \nu$, if we have $\mu(x) > \mu(y) \Leftrightarrow \nu(x) > \nu(y)$, for all $x, y \in G$,and $\mu(x) = 0 \Leftrightarrow \nu(x) = 0$, for all $x \in G$.
Note that the condition $\mu(x) = 0$holds if and only if $\nu(x) = 0$simply says thatthe supports of μ and ν are equal.

(iii)We say that μ is equivalent to ν, written as $\mu \simeq_t \nu$, if there exists an isomorphism f from $supp\mu$ to $supp\nu$, such that for all $x, y \in supp\mu$ we have $\mu(x) > \mu(y) \Leftrightarrow \nu(f(x)) > \nu(f(y))$.

Theorem (2.13): Let G be a group and $\mu, \nu \in F(G)$. Then $\mu \sim \nu$ if and only if $F_\mu = F_\nu$ and $supp\mu = supp\nu$.

Proof: Omitted.

Definition (2.14): Let G be a group and $\mu, \nu \in F(G)$. Then μ isequivalent to ν written as $\mu \simeq_k \nu$, if there exist a one to one and onto function f from $F_\mu = F_\nu$ such that for all $\mu_t \in F_\mu$, $\mu_t \cong f(\mu_t)$.

Definition (2.15): Let $(G,+,\cdot)$ and $(H,\oplus,\circ)$ be bi-groups. Let $\gamma_G = \gamma_{G_1} \cup \gamma_{G_2}$ and $\rho_H = \rho_{H_1} \cup \rho_{H_2}$ be separate fuzzy sub bi-group of G and H respectively. If $\theta : G \to H$ is a bi-grouphomomorphism then the mapping $\phi : \gamma_G \to \rho_H$ is said to be weakly fuzzy homomorphicif for any $x, y \in G, \theta(\gamma_G(xy)) = \rho_H(\phi(x)\phi(y))$. It is denoted as $\gamma_G (\frac{\theta,\phi}{\sim}) \rho_H$.

3. 0 The Results

Let G be a bi-group. In what follow, $\Phi(G)$ is the set of all fuzzy sub bi-groups of G, $<$stands for the conventional less than as well as sub bi-group of a bi-group. The meaning, ineither case,is easily distinguishable from the context.

Definition (3.1): Let $G = G_1 \cup G_2$ be a bi-group G and let $\gamma_G \in \Phi(G)$, the set $\{x \in G : \min\{\gamma_{G_1}(x), \gamma_{G_2}(x)\} > 0\}$ is called the support of the fuzzy bi-group λ_G and is denoted by $supp\ \gamma_G$.

For the fuzzy bi-group $\gamma_G = \gamma_{G_1} \cup \gamma_{G_2}$ of the bi-group G. For any $x \in G$, $\gamma_G(x) = (\gamma_{G_1} \cup \gamma_{G_2})(x) = \gamma_{G_1}(x)$, if $x \in G | G_2$. Hence, $Supp\ \gamma_G = Supp\ \gamma_{G_1}$ for all $x \in G | G_2$. If $x \in G | G_1$, $\gamma_G(x) = (\gamma_{G_1} \cup \gamma_{G_2})(x) = \gamma_{G2}(x)$, and $\gamma_G = Supp\ \gamma_{G_2}$. If $x \in G_1 \cap G_2$,

$$\gamma_G(x) = (\gamma_{G_1} \cup \gamma_{G_2})(x) = \max\{\gamma_{G_1}(x), \gamma_{G_2}(x)\} \geq \min\{\gamma_{G_1}(x), \gamma_{G_2}(x)\} > 0.$$

Hence, $Supp\,\gamma_G = \min\{Supp\,\gamma_{G_1}, Supp\,\gamma_{G_2}\}$ for all $x \in G_1 \cap G_2$.

The following result is therefore established.

Proposition (3.2): Suppose that $\gamma_G = \gamma_{G_1} \cup \gamma_{G_2}$ is a fuzzy bi-group of the bi-group G. For $x \in G$,

(i) $Supp\ \gamma_G = Supp\ \gamma_{G_1}$ for all $x \in G | G_2$.

(ii) $Supp\ \gamma_G = Supp\ \gamma_{G_2}$ for all $x \in G | G_1$.

(iii) $Supp\ \gamma_G = \min\{Supp\ \gamma_{G_1}, Supp\ \gamma_{G_2}\}$ for all $x \in G_1 \cap G_2$.

Definition (3.3): Let $G = G_1 \cup G_2$ be a bi-group G and let $\gamma_G, \eta_G \in \Phi(G)$, we say that γ_G is equivalent to η_G written as $\gamma_G \simeq \eta_G$ if there exists an isomorphism f from $supp\ \gamma_G$ to $supp\ \eta_G$ such that for all $x, y \in supp\ \gamma_G$, we have $\gamma_G(x) > \gamma_G(y) \Leftrightarrow \eta_G(f(x)) > \eta_G(f(y))$.

Definition (3.4): Let $G = G_1 \cup G_2$ be a bi-group. Let $\Phi(G)$ be the setof all fuzzy bi-group of the bi-group G. For $\gamma_G, \eta_G \in \Phi(G)$, wesay that γ_G is equivalent to η_G written as $\gamma_G \simeq \eta_G$ if for $x, y \in G$,

(i) $\gamma_G(x) = \gamma_{G_1}(x) > \gamma_{G_1}(y) = \gamma_G(y) \Leftrightarrow \eta_G(x) = \eta_{G_1}(x) > \eta_{G_1}(y) = \eta_G(y)$

or

$$\gamma_G(x) = \gamma_{G_2}(x) > \gamma_{G_2}(y) = \gamma_G(y) \Leftrightarrow \eta_G(x) = \eta_{G_2}(x) > \eta_{G_2}(y) = \eta_G(y)$$

(ii) $\gamma_G(x) = 0 \Leftrightarrow \gamma_G(y) = 0 \ \ or \ \ \eta_G(x) = 0 \Leftrightarrow \eta_G(y) = 0$.

The following propositions are direct consequences of definition (3.4).

Proposition (3.5): If for all $x, y \in G_1 \cap G$, $\gamma_G \simeq \eta_G$, then

$$\gamma_{G_1}(x) > \gamma_{G_1}(y) \Leftrightarrow \eta_G(x) = \eta_{G_1}(x) > \eta_{G_1}(y),$$

and

$$\gamma_G(x) = 0 \Leftrightarrow \gamma_G(y) = 0 \ \ or \ \ \eta_G(x) = 0 \Leftrightarrow \eta_G(y) = 0.$$

Proof:
Suppose that $\gamma_G \simeq \eta_G$, then for $x, y \in G_1 \cap G$,
$\gamma_G(x) > \gamma_G(y) \Rightarrow \gamma_{G_1} \cup \gamma_{G_2}(x) > \gamma_{G_1} \cup \gamma_{G_2}(y)$
$\Rightarrow \max\{\gamma_{G_1}(x), \gamma_{G_2}(x)\} > \max\{\gamma_{G_1}(y), \gamma_{G_2}(y)\}$
$\Rightarrow \gamma_{G_1}(x) > \gamma_{G_1}(y)$ (i)
similarly,
$\eta_G(x) > \eta_G(y) \Rightarrow \eta_{G_1} \cup \eta_{G_2}(x) > \eta_{G_1} \cup \eta_{G_2}(y)$
$\Rightarrow \max\{\eta_{G_1}(x),\ \eta_{G_2}(x)\} > \max\{\eta_{G_1}(y),\ \eta_{G_2}(y)\}$
$\Rightarrow \eta_{G_1}(x) > \eta_{G_1}(y)$ (ii)
Combining (i) and (ii), it is easy to deduce that
$\gamma_{G_1}(x) > \gamma_{G_1}(y) \Leftrightarrow \eta_{G_1}(x) > \eta_{G_1}(y).$
The fact that
$\eta_{G_1}(x) > \eta_{G_1}(y) \Leftrightarrow \gamma_{G_1}(x) > \gamma_{G_1}(y)$
follows the same approach.
Now, suppose that $\gamma_{G_1}(x) = 0$ and $\gamma_G \simeq \eta_G$, then $0 = \gamma_{G_1}(x) = \gamma_G(x),$ there is definitely $y \in G$ such that $\eta_G(y) > \eta_{G_1}(y) = 0.$
Hence, $\gamma_{G_1}(x) > 0 \Rightarrow \eta_{G_1}(y) = 0$, and that concludes the proof.

Proposition (3.6): If for all $x, y \in G_2 \cap G$, $\gamma_G \simeq \eta_G$,then $\gamma_{G_2}(x) > \gamma_{G_2}(y) \Leftrightarrow \eta_{G_2}(x) > \eta_{G_2}(y),$ and $\gamma_G(x) = 0 \Leftrightarrow> \gamma_G(y) = 0 \ \ or \ \ \eta_G(x) = 0 \Leftrightarrow \eta_G(y) = 0.$

Proof: Similar to that of Proposition (3.5).
We now redefine the concept of bi-level subset of Definition(2.11) to suite our purpose.

Definition (3.7): Let $G = G_1 \cup G_2$ be a bi-group, let $\gamma_G, \eta_G \in \Phi(G)$ be fuzzy bi-groups of G. For $x \in G,$ we define $[\gamma_G]^t(x) = \{x \in G : \gamma_G(x) > t\},$ for $t \in [0, \min\{\mu_1(e_1), \mu_2(e_2)\}]$ where e_1 and e_2 are the identities of G_1 and G_2 respectively. $[\gamma_G]^t$ is called the t- level subbi-group of the bi-group G.

Definition (3.8): Let $G = G_1 \cup G_2$ be a bi-group, let $\gamma_G, \eta_G \in \Phi(G)$. γ_G is t- equivalent to η_G written as $\gamma_G \simeq_t \eta_G$ if there exists isomorphism $\lambda : \gamma_G \to \eta_G$ such that for all $[\gamma_G]_t \epsilon\ \Phi(G),\ [\gamma_G]_t \simeq_t \lambda([\gamma_G]_t)$.

Theorem (3.9): Let $G = G_1 \cup G_2$ be a bi-group, let $\gamma_G, \eta_G \in \Phi(G)$ be such that $\gamma_G \simeq_t \eta_G$, then there exists a one to one and onto correspondence $\theta^*: [supp\ \gamma_G]_t \to [supp\ \eta_G]_t$ such that for all $[\gamma_G]_t \in \Phi^*(G),$ we have $[\gamma_G]_t \simeq \theta^*([\gamma_G]_t)$.

Proof:
For any $x \in [\gamma_G]_t$, we have that $\gamma_{G_1}(x) \geq t$ and $\gamma_{G_2}(x) \geq t$. [since $[\gamma_G]_t \leq G \Rightarrow [\gamma_{G_1}]_t \leq G_1$ and $[\gamma_{G_2}]_t \leq G_2$]. $Supp\ \gamma_G = \{x\epsilon G: \min\{\gamma_{G_1}(x), \gamma_{G_2}(x)\} > 0$
and $[\gamma_G]_t = \{x \in G : \gamma_G(x) > t\} = \{x \in G : \max[\gamma_{G_1}(x), \gamma_{G_2}(x) > t]\}.$
For $t \neq 0, \max[\{\gamma_{G_1}(x), \gamma_{G_2}(x)\} > t] > \min\{\gamma_{G_1}(x), \gamma_{G_2}(x)\}$ therefore $Supp\ \gamma_G \leq [\gamma_G]_t$ for $t \in (0, \min\{\mu_1(e_1), \mu_2(e_2)\}]$ where e_1 and e_2 are the identities of G_1 and G_2 respectively. Similarly, $Supp\ \eta_G \leq [\eta_G]_t$ and this applies to all $t_1, t_2, t_3, \ldots \in (0, \min\{\mu_1(e_1), \mu_2(e_2)\}]$.
Let $\theta^*: [supp\ \gamma_G]_t \to [supp\ \eta_G]_t$ be a function. Since $\gamma_G \simeq_t \eta_G$, it follows by definition(3.8) that for any y in $[\eta_G]_t$,there must be an x in $\theta^{*-1}(y)$ such that $\gamma_{G_1}(x) = \eta_{G_1}(x) \geq t$ and so x is in $[\gamma_{G_1}]_t$. Similarly, x is in $[\gamma_{G_2}]_t$, and therefore x is in $[\gamma_G]_t$.

Let $x,y,z,w,...\in G$ and $t_1,t_2,t_3,t_4,...\in(0,\min\{\mu_1(e_1),\mu_2(e_2)\}]$ such that $\gamma_G(x)>t_1,\gamma_G(y)>t_2,\gamma_G(z)>t_3,\gamma_G(w)>t_4,...$. Let $t_1<t_2<t_3<t_4<\cdot<\cdot<\cdots<0$, then $[\gamma_G]_{t_1}>[\gamma_G]_{t_2}>[\gamma_G]_{t_3}>[\gamma_G]_{t_4}>\cdot>\cdot$ Similarly for such $x,y,z,w,...\in G$ and $t_1,t_2,t_3,t_4,...\in(0,\min\{\mu_1(e_1),\mu_2(e_2)\}]$, $[\eta_G]_{t_1}>[\eta_G]_{t_2}>[\eta_G]_{t_3}>[\eta_G]_{t_4}>\cdot>\cdot$ For a finite bi-group, It is easy to establish a one to one correspondence θ^* between $[\gamma_G]_t$ and $[\eta_G]_t$ such that $[\gamma_G]_t\simeq\theta^*([\gamma_G]_t)$.

Definition (3.10): Let $\gamma_G=\gamma_{G_1}\cup\gamma_{G_2}$ be afuzzy sub bi-group of a bi-group G. For $t\in(0,\min\{\mu_1(e_1),\mu_2(e_2)\}]$, $[\gamma_G]_t$ is called t- level sub bi-group of a bi-group G. The set $P([\gamma_G]_t)$ of all possible fuzzy sub bi-group of a bi-group G is called a fuzzy power level sub-bigroup of a bigroup G.

The number of element in $P([\gamma_G]_t)$, denoted by $o[P([\gamma_G]_t)]$ is called the order ofthe fuzzy power level subbi-group of a bi-group G.

Theorem (3.11): Let $P([\gamma_G]_t)$ and $P([\eta_G]_t)$ be two fuzzy power level sub bi-group of a bi-group G. Suppose that $\gamma_G,\eta_G\in\Phi(G)$ be such that$\gamma_G\simeq_t\eta_G$, then there exists a one to one correspondence $P([\gamma_G]_t)$ and $P([\eta_G]_t)$.

Proof:

By Theorem (3.9), it has been shown that if $\gamma_G,\eta_G\in\Phi(G)$ be such that$\gamma_G\simeq_t\eta_G$, then there exists a one to one and onto correspondencebetween γ_G and η_G . For $t_1,t_2,t_3,t_4,...,t_n\in(0,\min\{\mu_1(e_1),\mu_2(e_2)\}]$ it is easy to construct $[\gamma_G]_{t_i}$ and $[\eta_G]_{t_i}$ in such a way that for $t_1<t_2<t_3<t_4<\cdots<t_n$, then $[\gamma_G]_{t_1}>[\gamma_G]_{t_2}>[\gamma_G]_{t_3}>\cdots>[\gamma_G]_{t_n}$ and $[\eta_G]_{t_1}>[\eta_G]_{t_2}>[\eta_G]_{t_3}>\cdots>[\eta_G]_{t_n}$. Since $\theta^*:[\gamma_G]_t\to[\eta_G]_t$ is an injectivefunction, by implication $\theta^*:P([\gamma_G]_t)\to P([\eta_G]_t)$ is also an injective function.

Corollary (3.12): Let $P([\gamma_G]_t)$ and $P([\eta_G]_t)$ be two fuzzy power level sub bi-group of a bi-group G. Suppose that $\gamma_G,\eta_G\in\Phi(G)$ be such that$\gamma_G\simeq_t\eta_G$and there exists a one to one correspondence $P([\gamma_G]_t)$ and $P([\eta_G]_t)$, then $o[P([\gamma_G]_t)]=o[P([\eta_G]_t)]$.

Proof:

Follows directly from Theorem (3.9) and Theorem (3.11).

4.0 References

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