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Truncated Hybrid Chain Sampling Plan (THCSP) for Weibull Product Life Distribution

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Abstract

In this paper, an improved reliable plan (Truncated hybrid Chain Sampling Plan (THCSP)) is proposed for products life that follows Weibull distribution when the testing is truncated at a specified time(t). This type of sampling plan is used to save the testing time in real life situations. The optimal sample sizes (n) required for testing product quality to ascertain a true mean life is obtained under a given Maximum Allowable Percent Defective (β), test termination ratios$\left(\frac{t}{\mu_o}\right)$ and acceptance numbers(C). The operating characteristic (OC) values, Mean Life Ratios and curves of the plan are examined with varying ratio of the true mean life to the specified life. The advantage of this inspection plan is that it results in better economic reliability product quality testing that protects the producer from rejecting his good lots and consumers from accepting bad lots of finished products. It also guides the producer on how to improve on his product's quality. A numerical example is also discussed for illustrative purpose.

Keywords: Acceptance sampling, Reliability, Producer's risk, Consumer's risk, Quality control.
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1.0 Introduction

The purpose of acceptance sampling is illustrated in this paper. For instance, if a company receives a delivery of product from a merchant, this product is always a component or raw material used in the company's manufacturing process. A sample is taken from the lot and the relevant quality characteristic of the units in the sample is inspected [1]. Based on the information in this sample, a decision is made regarding lot outlook. Usually, when the life test indicates that the mean life (μ) of products exceeds the specified (μ_o) one, the lot of products is accepted, otherwise it is rejected. Accepted lots are put into production, while rejected lots may be returned to the merchant or may be subjected to some other lot disposition action. While it is usual to think of acceptance sampling as a receiving inspection activity, there are also other uses. Frequently, a manufacturer samples and inspects its own product at various stages of production. Lots that are accepted are sent forward for further processing, while rejected lots may be reworked or scrapped. For the purpose of reducing the test time and cost, a truncated life test may be conducted to determine the smallest sample size to ensure a certain mean life of products when the life test is terminated at a pre-assigned time t and the number of failures observed does not exceed a given acceptance number c.

Acceptance sampling is concerned with inspection and decision making regarding lots of product and constitutes one of the oldest techniques in quality assurance. Sampling plans is used to determine the acceptability of lots of items[2]. Life test refers to measurements of product life; product life can be measured in hours, miles, cycles or any other metric that applies to the period of successful operation of a particular product. Since time is a common measure of life, life data points are often called ***times-to-failure.*** There are different types of life products. Statistical distributions have been assumed by various authors (statisticians, mathematicians and engineers) to mathematically model or represent certain behaviour of products. The probability density function (pdf) and cumulative distribution function (cdf) are mathematical functions that explain the distribution of life of an item.

Epstein [3] and Sobel and Tischendrof [4] were the first to discuss Acceptance sampling based on truncated life tests for an exponential model. An extensionof their work was carried out in [5] by considering the Weibullmodel which includes the

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exponential distribution. Gupta[6 and 7] also considered the gamma and log-normal distributions respectively. Recently, [8-12] discussed Truncated Acceptance Sampling for various life distributions.

In this paper, we developed a Truncated Hybrid Chain Acceptance Sampling Plan by considering both the producers and consumers' risk, which has single sampling plan as attribute plan to obtain the test termination ratios, assuming that the life time of the product follows a Weibull distribution.

1.1 Proposed Truncated Hybrid Chain Sampling Plan (THCSP)

We attempt to propose aTruncated Hybrid Chain sampling Plan to determine the sample size as an improved ordinary chain sampling plan. We observed that use of ordinary chain sampling plan results in accepting only if either zero (0) or one (1) is contained in the tested lot. This proposed scheme is expected to be better than the ordinarychain sampling plan because the information of a present lot could lead to acceptance of a previous lot, thereby reducing cost and testing time and in turn also reduces wastages and scraps when products are essentially of high quality.

1.1.1 Assumptions for the application of Truncated Hybrid Chain Sampling Plan (THCSP)

i. Lots are expected to be of essentially high and the same quality**.**
ii. The production process is stable with the intention that results of past, present and future lots are generally indicative of the process.
iii. The inspection is said to be attributes (i.e, defective or non-defective) and the lot quality is defined as the proportion defective which is represented by the products' life cumulative distribution function.
iv. The lots contain at least one defective unit and variation may exist in the lot quality.

1.1.2 Operating procedure of the Proposed Truncated Hybrid-Chain Sampling Plan (THCSP)

The operating procedure for the proposedTruncated Hybrid-Chain Sampling Plan (THCSP) is as following:

i. Select a sample of size **n**from each of the submitted lots and observe the number defectives (d)during a pre-fixed time **t**.
ii. Accept the present lot if the number of defective is less than the first acceptance number c_1 (i.e, if $d<c_1$).
iii. Reject the lot if the number of defective exceed the first acceptance number c_2 (i.e, if d>c2).
iv. If the number of defective is between the first and second acceptance number c_2 (i.e, if $c_1< d \leq c_2$), make use of the information of the next in-coming lot, i.e., the current lot is accepted if the number of defectives of the proceeding lot result in $d \leq c_1$in the sample.
v. If the proceeding lot result in $c_1< d \leq c_2$, then make use of next proceeding lot and check whether $d \leq c_1$ or $d > c_2$ for acceptance or rejection respectively.
vi. The tester continue utilizing the proceeding lot information until the lots are accepted or rejected (i.e, d ≤ c1 or d > c2).

1.1.3 Advantages of the proposed truncated Hybrid Chain Sampling Plan (THCSP)

i. It is also very easy to design, administer and explain compare to ordinary chain sampling plan.
ii. The sample size from lots to lots is reduced compared to ordinary chain sampling plan.
iii. Psychologically, the Producer will be more satisfied because prior information of a current inspected lot could lead to the acceptance of previous lots.

The flow chart of the sampling scheme is as shown in figure 1.

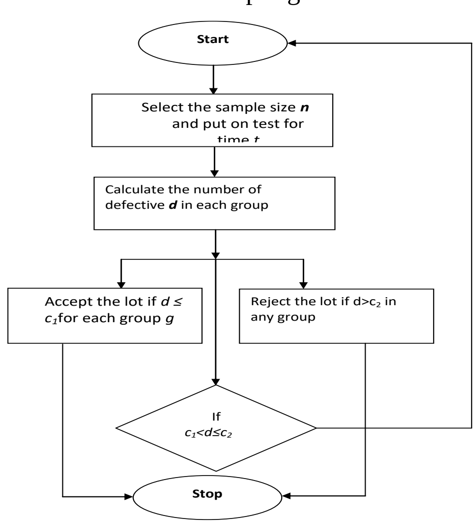

Fig. 1: Flow chart process for truncated hybrid double acceptance sampling plan.

Table 1: Nomenclature

S/No.	Notation	Definition	S/No.	Notation	Definition
1.	t	Termination time/Maximum test duration	**7.**	$\frac{t}{\mu_0}$	Test termination ratio
2.	P	Failure probability	**8.**	α	Producer's risk
3.	β	Maximum Allowable percent Defectives (MAPD)	**9.**	β	Consumer's risk,
4.	μ	True Mean life	**10.**	n	Sample size
5.	μ_0	Specified Mean life	**11.**	C	Acceptance number
6.	Pa	Probability of Acceptance	**12.**	d	Number of defectives

1.2 Weibull Distribution

The Weibull distribution is the generalization of the exponential distribution. The distribution was proposed by Weibull in 1939 and is widely applied in failure situations. The Weibull is one of the most popular distributions for analyzing lifetime data. This distribution has been studied in the several literatures and has applications in fields other than lifetime distributions[5]. The Weibull analysis has an advantage in that it has the ability to provide reasonably accurate failure analysis and failure forecast with extremely small samples. Another advantage of this distribution is that it provides a simple and useful graphical plot of failure data[2]. Solutions are possible at the earliest indications of a problem. Small samples also allow cost effective component testing. For example, *sudden death Weibull tests* are completed when the first failure occurs in each lot of components (for example, lots of ball bearings). If all the bearings are tested to failure, the cost and time required is much greater.

Another advantage of Weibull distribution is that it provides a simple and useful graphical plot of the failure data. The data plot is extremely important to the engineer and to the manager. Marshall and Olkin[13] stated that the Weibull distribution is applicable to many survival and reliability analysis with decreasing, increasing and hazard rate. The three parameters distribution, represent location, scale and shape, and because of them, it is quite a bit of flexible for analyzing skewed data. The Weibull distribution has become one of the most commonly used lifetime distributions in reliability engineering and elsewhere due to its versatility and relative simplicity. It is a flexible distribution that can take on the characteristics of other types of distributions, based on the value of the shape parameter. The Weibull distribution is very popular among engineers. One reason for this is that the Weibull cumulative distribution function (cdf) has a closed form. The Weibull distribution function is defined as:

$$F(\alpha,\mu,x) = 1 - exp\left[-\left(\frac{x}{\alpha}\right)^{\mu}\right] \quad \textbf{(1)}$$

The parameters $\mu > 0$ and $\alpha > 0$ are referred to as scale and shape parameter, respectively. The Weibull density has the following form:

$$f(\alpha,\mu,x) = F'(\alpha,\mu,x) = \frac{d}{dx}F(\alpha,\mu,x) = \frac{\mu}{\alpha}\left(\frac{x}{\alpha}\right)^{\mu-1} \exp\left[-\left(\frac{x}{\alpha}\right)^{\mu}\right] \quad \textbf{(2)}$$

If$\mu = 1$, the Weibull distribution coincides with the exponential distribution with mean α densities.

The mean and variance of a Weibull distributionis given as:

$$E(X) = \alpha\Gamma\left(1+\frac{1}{\mu}\right) \quad \textbf{(3)}$$

and

$$\sigma^2 = \alpha^2\left[\Gamma\left(1+\frac{2}{\mu}\right) - \{\Gamma\left(1+\frac{1}{\mu}\right)\}^2\right] \quad \textbf{(4)}$$

When the shape parameter μ in a Weibull distribution is given any fixed value, it reduces to one parameter weibull distribution. If $\mu = 2,$ it reduces to Rayleigh distribution and if the scale parameter $\alpha = 1$, the Weibull distribution reduces to exponential distribution. In Time Truncated Acceptance Sampling Plan, the cumulative distribution function as recently used in[14] is given as:

$$F(t,\mu) = 1 - e^{-(\frac{t}{\mu_0})^{\alpha}} \quad \textbf{(5)}$$

Where μ is the scale parameter (quality parameter) and α is the shape parameter.

2.0 Materials and Methods

2.1 Development of Operating Characteristics for Proposed Hybrid Chain Sampling Plan (THCSP)

An acceptance sampling plan is best described in graphical terms on an operating characteristic curve (OC curve). An OC curve is a plot of the actual number of nonconforming units in a lot (expressed as a percentage) against the probability that the lot will be accepted when sampled according to the plan. The shape of an OC curve is determined primarily by sample size, n, and acceptance number, c, although there is a small effect of lot size(N). The OC function of the sampling plan $(n, c, \frac{t}{\mu_0})$ is the probability ofaccepting a lot and is given by

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-1} \quad (6)$$

The proposed Hybrid chain sampling plan is also described by three parameters, *n*, c_1 and c_2. It is of a note that the proposed Hybrid chain Sampling Plan is a combination of the truncated double and chain acceptance sampling plan. The probability of lot acceptance (Pa) is obtained by adopting and modification using the Operating Characteristic (OC) function of double sampling plan, which is derived as follows:

Suppose the probability of acceptance of a submitted lot with fraction defective p based on a given sample with the first acceptance number is:

$$P_a(p_1) = \sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i} \quad (7)$$

The corresponding probability of lot rejection is:

$$\Pr(p_1) = 1 - \sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i} \quad (8)$$

Let the probability of acceptance of a submitted lot with fraction defective p based on a given sample with the second acceptance number be given as:

$$P_a(p_2) = \sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i} \quad (9)$$

These probabilities are therefore given by:

$$P(a) = \Pr(d \le c_1 : p) = \sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i} \quad (10)$$

and the corresponding probability of lot rejection

$$\Pr(p_1) = \Pr(d > c_2 : p) = 1 - \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i} \quad (11)$$

The probability of rejection of a submitted lot with fraction defective *p* based on a given sample with the first and second acceptance number is given as:

$$\Pr(p) = \sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i} - \sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i} \quad (12)$$

Using the idea of conditional probability of accepting lot with C1, given sample n, the resulting probability of lot acceptance or Operating Characteristic (OC) function is given as:

$$Pa(p) = \frac{\sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i}}{1 - \left[\sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i} - \sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i}\right] * \left[\sum_{i=0}^{c_1} \binom{n}{i} (1-p)^{n-i}\right]} \quad (13)$$

Therefore, the operating characteristic formula for our proposed plan, where *p* is the failure probability of an item in the lot is given as:

$$Pa = \frac{\mathrm{Pa}}{1 - [\mathrm{Pa}(P_2) - \mathrm{Pa}(P_1)] * P_a(p_1)} \quad (14)$$

The failure probabilities are represented by the cumulative distribution function (cdf) of the life time distributions.

2.2 Mean Life Ratio Value

In order to calculate the minimum required ratio values, the producer's risk is been considered. The producer's risk is the probability of rejection of the lot when$\mu \ge \mu_0$, it can be computed as follows;

$$Pr(R) = \mathrm{P(Rejecting\ a\ lot)} = 1 - P(AcceptingtheLot/\mu \ge \mu_0)$$
$$= \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-1} \quad (15)$$

For the given sampling plan and for a given value of the producer's risk, say γ, one may beinterested in knowing the minimum value of$\frac{\mu}{\mu_0}$, that will ensure the producer's risk to beat most γ. The$\frac{\mu}{\mu_0}$,is the smallest quantity for which Psatisfies the inequality:

3.0 Results and Analysis

Suppose the lifetime of the testing items follow a Weibull distribution with known shape parameter, the numerical values that will serve as guide to the tester is presented in the in tables 3 to 6.

Figures 2 is the OC plots of probability of acceptance (Pa) against mean ratio ($\frac{\mu}{\mu_0}$) for fixed $\frac{t}{\mu_0} = 0.942, 1.257, 1.571, 2.358, 3.141$ *and* $3.972$and varying β= 0.25, 0.10, 0.05, 0.01. For fixed β = 0.05 and varying $\frac{t}{\mu_0}$, figure 3depicts the OC curves.

In order to compare our results with existing researches, we used existing combined parameters to simulate our results using ***R*** Software.

3.1 Sensitivity Analysis

In statistical quality control, Operating Characteristic (OC) Curves plays an important role in determining the probability of accepting manufactured lots when using different sampling plans. It shows the relationship between a designed parameter and lot acceptance when we conduct a lifetime experiment. The OC curves helps in the selection of acceptance sampling plans and also help in reducing risks. The different behavior of OC values and combined parameters are presented in figure 2 to 5.Thus, after analyzing the trends of the results given in Tables 3 to 5, one can make the trade-off between the required minimum sample size, confidence level, acceptance number and experimental time ratio to achieve the best sampling plan.

3.2 Real Life Example

The data used in this study were collected from the Quality Assurance and Assembly Plant of Machine and Tool department, Udofe Metal Industries, KM 3, Igarra, Okpe Road, Edo State, Nigeria. The data are the approximate number of revolutions (millions) of Oil Palm Milling Machine Ball-Bearings before failure. The data as retrieved from the record file of the Quality Assurance Department of the Industry are: 28.44, 28.16, 29.22, 32.56, 30.83, 27.44, 26.64, 34.88, 29.02, 30.42, 29.61, 30.02, 28.94, 31.94, 30.04, 29.79, 27.20, 33.54, 31.45 and 29.23.

Suppose a manufacturer want to develop a Sampling Plan and know whether the life of his products(ball bearing) are above the specified mean life revolution of 30 million revolutions per hour with Maximum Allowable Percent Defective ($\beta = 0.10$) and the life test would be ended at 25 million revolutions, which should have led to the ratio $\frac{t}{\mu_o} = 0.833$. Consider that the lifetime of products follows a Weibull distribution. Thus, from table 3[15], for an acceptance number C=2, the designed parameters of the Sampling Plan are $\left(n, C, \frac{t}{\mu_o}\right) = 4, 2\ and\ 0.833$ for $\beta = 0.10$. That is the manufacturer needs to select a sample of 4 products and put on test, the lot is rejected if more than 2 failures occur during 25 million revolution test per hour, otherwise accept it.

The OC values for the acceptance sampling plan $\left(n, C, \frac{t}{\mu_o}\right) = 4, 2\ and\ 0.833$ for $\beta = 0.10$as extracted from table 5for a Weibull product life distribution with $\frac{\mu}{\mu_0} = 2$ is as shown in table 2 below.

Table 2: OC values for the acceptance sampling plan $\left(n, C, \frac{t}{\mu_o}\right) = 4, 2\ and\ 0.833$ for $\beta = 0.10$ under Weibull distribution with $\frac{\mu}{\mu_0} = 2$

$\boldsymbol{\frac{\mu}{\mu_0}}$	**2**	**4**	**6**	**8**	**10**	**12**
0.628	0.68035	0.87380	0.96868	0.96317	0.97595	0.98311

It can deduce from the above table that if the true mean life is twice the required mean life ($\boldsymbol{\frac{\mu}{\mu_0} = 2}$**),** the producer's risk is approximately 1- 0.68035= 0.31965.

From table 6, the experimenter can get the values of mean life ratio for different choices of c and $\frac{t}{\mu_o}$in order to assert that the producer's risk was less than 0.05. In this example, the mean life ratiovalue of his product should be 3.049 for c = 2, $\boldsymbol{\frac{t}{\mu_0}} = 0.942$ and $\beta = 0.10$. This means the product can have a mean life of 3.049 times the required mean lifetime in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.90.

4.0 Discussion of Results

This section interprets our observation from the simulated results and operating characteristics (OC) plots using R software as shown in table 3 to 5 and figure 2 to 5 below.

4.1 Interpretation of the Behaviour of Operating Characteristics (OC)

From table 4, the following can be deduced:

i. On fixing the experiment time ratio and varying mean ratio, the probability of acceptance is decreasing with an increase in the confidence level. We also observed the same trend in respect of experiment time ratio for a fixed confidence level.
ii. On fixing the confidence level and experiment time ratio, the probability of acceptance increases as the mean ratio increases.

4.2 Interpretation for the minimum required mean ratio at fixed producer's risk

From table 5, the following can be deduced:

i. It was observed that the minimum mean ratios required for smaller acceptance number in order that the lot will be accepted with the probability $(1 - \alpha)$ are very high as compared to higher acceptance number for any combination of confidence level and experiment time ratio.
ii. On fixing the acceptance number, the required minimum means ratio increases as the confidence level increases.

4.3 Interpretation of Operating Characteristics (OC) Curves

i. From figure 2, for any fixed value of Maximum Allowable Percent Defectives (β) and experiment time ratio, the OC values of Weibill product life distribution increases as the mean life ratio increases.
ii. From figure 3, for any fixed value of Maximum Allowable Percent Defectives (β) and experiment time ratio, the OC values of Weibill product life distribution also increases as the experimental ratio increases. This may happen due to the incorporation of the past parametric fluctuations with the experimental data.
iii. From figure 4 to 5 our proposed plan resulted to smaller consumers' risk than the when compares with the work of Sundamani and Jayasri [16].

4.4 Comparison of our Proposed Truncated Hybrid Chain Sampling Plan with Existing Results using Producers' Risk

We also compare the producers' risk of the four studied distributions with the existing ones. Figure 4 and 5 compares our results with those of Sundamany and Jayasri [16] for Weibull distribution.

5.0 Conclusion

In this study, a Truncated Hybrid Chain Acceptance Sampling Plan for the Weibull product life distribution was proposed. It is assumed that the shape parameter is known and we have presented the results in tables for the developed minimum sample size required to guarantee a certain mean life of the test units.

Conclusively, our results can also serve for other product life distributions that belong to the family of Weibull distribution. Therefore, our tables can be used to develop the acceptance sampling plan for these product life distributions that will reduce testing time, cost and minimize the producer and consumers' risk and on the other hand, guide the producer in improving his product life.

The design parameters for this proposed plan were determined by the two point approach considering both the producer's (α) and the consumer's risks (β) simultaneously. The quality level of an item (product) was then considered in terms of the mean life ratio to the specified life.

Our simulated results are presented in table 3 to 5 with real life example to illustrate the results.

Table 3: Developed minimum sample size for Weibull distribution and the corresponding acceptance number c when the shape parameter α = 2.

		$\frac{t}{\mu_0}$							
β	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.713
0.25	0	2	2	2	2	2	2	1	1
	1	3	3	3	2	2	2	2	2
	2	4	4	3	3	3	3	3	2
	3	5	5	4	4	3	3	3	3
	4	6	6	4	4	4	4	4	4
	5	7	7	5	5	4	4	4	4
	6	8	8	5	5	5	5	5	5
	7	8	8	6	6	5	5	5	5
	8	9	9	7	7	6	6	6	6
	9	10	10	7	7	6	6	6	6
	10	11	11	8	8	7	7	7	7
0.10	0	2	2	2	2	2	2	2	1
	1	3	3	3	3	2	2	3	3
	2	4	4	3	3	3	3	3	3
	3	5	5	5	4	4	4	4	4
	4	6	6	5	4	4	4	4	4
	5	7	7	5	5	4	4	4	4
	6	8	8	5	5	5	5	5	5
	7	8	8	6	6	5	5	5	5
	8	9	9	8	7	6	6	6	6
	9	11	10	9	7	6	6	6	6
	10	11	11	9	8	7	6	6	6
0.05	0	3	3	3	3	3	3	2	2
	1	5	5	5	5	4	4	4	3
	2	5	5	5	5	5	5	4	4
	3	7	7	6	6	6	6	6	4
	4	7	7	7	6	6	6	6	4
	5	7	7	7	7	6	6	6	5
	6	7	7	7	7	6	6	6	6
	7	8	8	8	7	6	6	6	6
	8	8	8	8	7	7	7	7	7
	9	9	8	8	7	7	7	7	7
	10	9	8	8	8	8	7	7	8

Continuation of Table 3

	0	c	4	4	3	3	3	3	2
	1	4	4	4	4	4	3	3	3
	2	5	5	5	5	4	4	3	3
	3	5	5	4	4	4	4	4	4
	4	6	6	4	4	4	4	4	4
	5	7	7	5	5	5	5	5	5
	6	8	8	5	5	5	5	5	5
	7	8	8	6	6	5	5	5	5
	8	9	9	7	7	6	6	6	6
	9	10	10	7	7	6	6	6	6
0.01	10	11	11	8	8	6	6	7	7

Table 4: Design parameters of the proposed THCSP with $c_1 = 0$ and $c_2 = 2$ for Weibull distribution with $\alpha = 2$ and Maximum Allowable Percent Defectives

β	$\frac{t}{\mu_o}$	n	$\frac{\mu}{\mu_0}$					
			2	4	6	8	10	12
	0.628	**3**	0.79596	0.92891	0.96771	0.98170	0.98824	0.99182
	0.912	**3**	0.68035	0.87380	0.96868	0.96317	0.97595	0.98311
	1.257	**3**	0.56396	0.79574	0.89017	0.93337	0.95575	0.96863
	1.571	**3**	0.42251	0.72877	0.84320	0.90151	0.93339	0.95227
	2.356	**3**	0.03483	0.59069	0.72884	0.81349	0.86698	0.90154
	3.141	**3**	0.00080	0.42281	0.63368	0.72887	0.79587	0.84327
	3.927	**2**	0.50011	0.53923	0.63474	0.72338	0.79027	0.83836
0.25	**4.712**	**2**	0.50000	0.51608	0.58523	0.66651	0.73607	0.79029
	0.628	**3**	0.79596	0.92891	0.96771	0.98170	0.98824	0.99182
	0.912	**3**	0.68035	0.87380	0.96868	0.96317	0.97595	0.98311
	1.257	**3**	0.56396	0.79574	0.89017	0.93337	0.95575	0.96863
	1.571	**3**	0.42251	0.72877	0.84320	0.90151	0.93339	0.95227
	2.356	**3**	0.03483	0.59069	0.72884	0.81349	0.86698	0.90154
	3.141	**3**	0.00080	0.42281	0.63368	0.72887	0.79587	0.84327
	3.927	**3**	0.00001	0.16742	0.54526	0.65571	0.72881	0.78421
0.10	**4.712**	**2**	0.50000	0.51608	0.58523	0.66651	0.73607	0.79029
	0.628	**4**	0.75336	0.90666	0.95724	0.97569	0.98436	0.98911
	0.912	**4**	0.62727	0.84185	0.95889	0.95179	0.96831	0.97767
	1.257	**4**	0.45480	0.75311	0.86127	0.91404	0.94229	0.95883
	1.571	**4**	0.21386	0.68057	0.80633	0.87491	0.91407	0.93786
	2.356	**4**	0.00522	0.50337	0.68065	0.77278	0.83385	0.87495
	3.141	**4**	0.00005	0.21423	0.56950	0.68068	0.75326	0.80642
	3.927	**3**	0.00001	0.16742	0.54526	0.65571	0.72881	0.78421
0.05	**4.712**	**3**	0.00000	0.03483	0.42271	0.59069	0.66945	0.72884
	0.628	**5**	0.71749	0.88511	0.94692	0.96973	0.98050	0.98641
	0.912	**5**	0.57607	0.81351	0.94939	0.94081	0.96086	0.97232
	1.257	**5**	0.32256	0.71722	0.83527	0.89594	0.92940	0.94932
	1.571	**4**	0.21386	0.68057	0.80633	0.87491	0.91407	0.93786
	2.356	**4**	0.00522	0.50337	0.68065	0.77278	0.83385	0.87495
	3.141	**4**	0.00005	0.21423	0.56950	0.68068	0.75326	0.80642
	3.927	**3**	0.00001	0.16742	0.54526	0.65571	0.72881	0.78421
0.01	**4.712**	**3**	0.00000	0.03483	0.42271	0.59069	0.66945	0.72884

Table 5: Minimum ratio of true mean life to specified mean for acceptance of lot of when the life time of a product follows a Weibull distribution

		$\frac{t}{\mu_0}$							
β	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.713
0.25	0	6.188	7.177	7.797	9.746	10.271	13.694	17.118	20.541
	1	3.038	3.276	3.821	3.971	5.956	5.854	7.318	8.781
	2	2.363	2.666	2.955	3.252	4.097	5.462	5.142	6.171
	3	2.067	2.244	2.571	2.575	3.264	4.352	4.147	4.976
	4	1.898	2.116	2.169	2.463	2.780	3.706	4.632	4.269
	5	1.787	1.941	2.060	2.165	2.884	3.275	4.094	3.793
	6	1.709	1.818	1.981	2.142	2.603	2.963	3.704	4.444
	7	2.394	2.381	2.362	2.338	2.350	2.291	2.314	2.644
	8	2.257	2.251	2.218	2.229	2.197	2.186	2.171	2.478
	9	2.150	2.150	2.140	2.110	2.076	2.100	2.058	2.356
	10	2.081	2.067	2.048	2.044	2.026	1.986	1.969	2.251
0.10	0	7.327	8.297	9.570	9.746	14.619	13.694	17.118	20.541
	1	3.699	3.970	4.368	4.777	5.956	7.941	9.926	8.781
	2	2.829	3.049	3.269	3.694	4.877	5.462	6.828	8.193
	3	2.391	2.645	2.790	3.214	3.862	4.352	5.440	6.528
	4	2.192	2.320	2.517	2.711	3.274	3.706	4.632	5.559
	5	2.026	2.190	2.340	2.575	3.247	3.846	4.094	4.912
	6	1.909	2.031	2.214	2.476	2.926	3.471	3.704	4.444
	7	1.750	1.850	1.950	1.970	2.400	2.740	2.730	3.280
	8	1.670	1.760	1.800	1.970	2.230	2.560	2.580	3.100
	9	1.630	1.690	1.770	1.840	2.090	2.420	2.450	2.940
	10	1.600	1.680	1.740	1.731	1.980	2.300	2.340	2.810
0.05	0	8.762	9.282	11.062	11.962	14.619	19.492	24.365	20.541
	1	3.988	4.557	4.853	5.460	7.165	7.941	9.926	11.911
	2	2.995	3.222	3.555	4.087	4.877	5.462	6.828	8.193
	3	2.567	2.766	2.992	3.214	3.862	5.149	5.440	6.528
	4	6.188	7.177	7.797	9.746	10.271	13.694	17.118	20.541
	5	3.038	3.276	3.821	3.971	5.956	5.854	7.318	8.781
	6	2.363	2.666	2.955	3.252	4.097	5.462	5.142	6.171
	7	2.067	2.244	2.571	2.575	3.264	4.352	4.147	4.976
	8	1.898	2.116	2.169	2.463	2.780	3.706	4.632	4.269
	9	1.787	1.941	2.060	2.165	2.884	3.275	4.094	3.793
	10	1.709	1.818	1.981	2.142	2.603	2.963	3.704	4.444
0.01	0	2.394	2.381	2.362	2.338	2.350	2.291	2.314	2.644
	1	2.257	2.251	2.218	2.229	2.197	2.186	2.171	2.478
	2	2.150	2.150	2.140	2.110	2.076	2.100	2.058	2.356
	3	2.081	2.067	2.048	2.044	2.026	1.986	1.969	2.251
	4	7.327	8.297	9.570	9.746	14.619	13.694	17.118	20.541
	5	3.699	3.970	4.368	4.777	5.956	7.941	9.926	8.781
	6	2.829	3.049	3.269	3.694	4.877	5.462	6.828	8.193
	7	2.391	2.645	2.790	3.214	3.862	4.352	5.440	6.528
	8	2.192	2.320	2.517	2.711	3.274	3.706	4.632	5.559
	9	2.026	2.190	2.340	2.575	3.247	3.846	4.094	4.912
	10	1.909	2.031	2.214	2.476	2.926	3.471	3.704	4.444

5.1 Operating Characteristics Curves

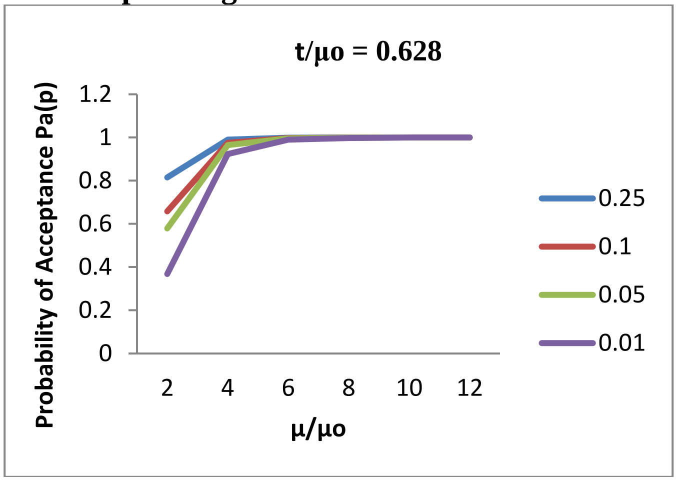

Fig. 2: Operating characteristics curve of probability of acceptance against mean life ratios forvarious Maximum Allowable Percent Defective (MAPD) for Weibull distribution

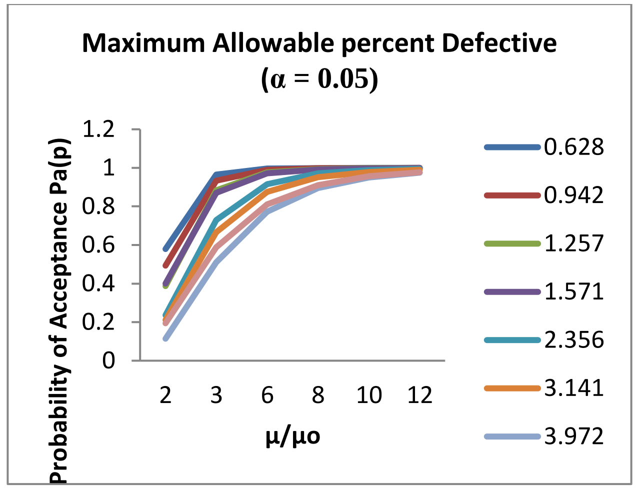

Fig. 3: Operating characteristics curve of probability of acceptance against experimental mean ratios at various experimental time ratios for Weibull distribution.

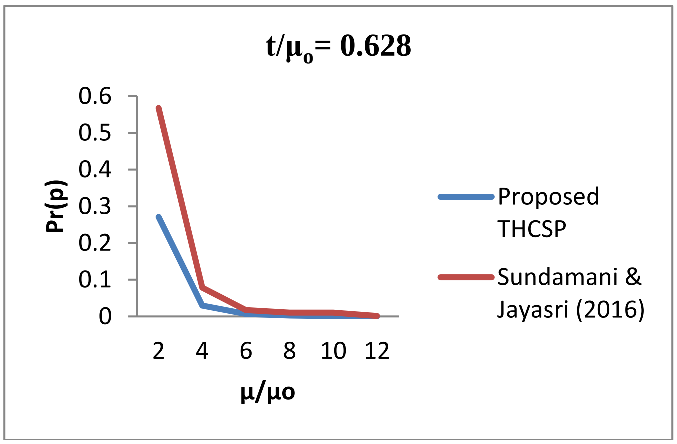

Fig. 4: Consumer's risk plot with respect to experiment time ratio for chain sampling (β= 0.05) for Weibull Distribution when $\frac{t}{\mu_o}$ = 0.628

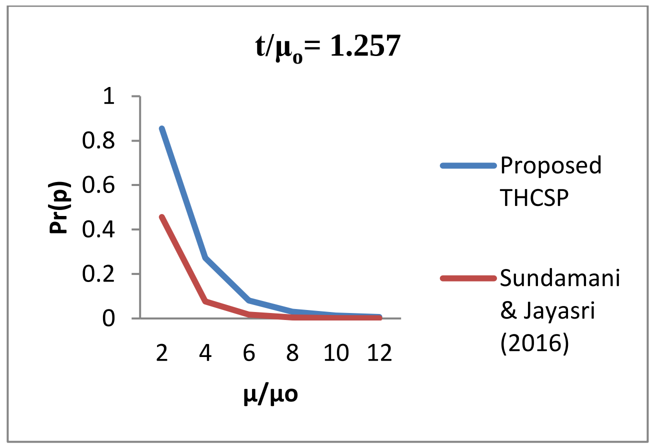

Fig. 5: Consumer's risk plot with respect to experiment time ratio for chain sampling (β = 0.05) for Weibull Distribution when $\frac{t}{}$ = 1.257

6.0 References

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