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Feasible Generalized Ridge Estimators as Alternatives to Ridge and Feasible Generalized Least Squares Estimators

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Abstract

The assumptions of the classical linear regression model are hardly satisfied in real life situation. Violation of the assumption of independent explanatory variables and error terms in classical linear regression model lead to the problems of multicollinearity and autocorrelation respectively. Estimators to handle each problem have been separately developed. Moreover, in practice these two problems do exist together but estimators for parameters' estimation when both exist are not common in existence. Consequently, this research work proposes estimators, Feasible Generalized Ridge Estimators to handle the two problems when they exist jointly in a data set, and examines the performances of the proposed estimators as alternative to the Ridge and Feasible Generalized Least Square Estimators for handling multicollinearity or autocorrelation problem.

The existing and proposed estimators were categorized into five (5) groups namely; One–Stage Estimators (OSE), Two–Stage Estimators (TSE), Feasible Generalised Least Square Estimators (FGLSE), Two-Process Estimators (TPE) and Modified Ridge Estimators (MRE). Monte Carlo experiments were conducted one thousand times (1000) on a linear regesssion model exhibiting different degree of multicollinearity ($\lambda =$ 0.4, 0.6, 0.8, 0.95 and 0.99) and autocorrelation ($\rho = 0.4, 0.8, 0.95$ and 0.99) at six sample sizes (n =10, 20, 30, 50, 100 and 250). Finite sampling properties of the estimators namely; Bias closest to zero (BAS), Absolute Bias (ABAS), Variance (VAR) and Mean Square Error (MSE) of the estimators were evaluated, examined and compared at each level of multicollinearity, autocorrelation and sample sizes. Results of the investigation when only multicollinearity is in the model revealed that the proposed TPE, FGLSELO-CORC and FGLSEAO-ML, often result into smaller bias than the existing ridge estimators; and the results are the same with the existing FGLSE. Moreover, the best estimator is an OSE, Ordinary Ridge Estimator with emperical Bayesian K ridge parameter (OREKBAY). Furthermore when autocorrelation alone is present in the model, results showed that the proposed MRE, FGREAO-ML, is best when the sample size is small, n=10. At other sample sizes the best estimators are generally the proposed TPE, FGLSELO-CORC and FGLSEAO-ML which give the same results as the FGLSE, CORC and ML estimators respectively. In conclusion, some of the proposed estimators can therefore be used as alternatives to FGLSE.

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1.0 Introduction

1.1 Linear Regression Model

A regression model is commonly used for studying the relationship between a response variable Y and a set of explanatory variables (the X variables).The response variable is also called the dependent variable or the outcome variable; and the explanatory variables are also called the independent variable or predictors or the covariates. If we have a regression model with response variable and one explanatory variable it is called a simple linear regression i.e.

$$Y = \beta_0 + \beta X + u \tag{1.1}$$

A regression model that involves more than one explanatory variable is called a multiple regression model. In other words it is a linear relationship between a dependent variable and a group of independent variables. Multiple regressions fit a model to predict a dependent (Y) variable from two or more independent (X) variables [1].The general single-equation linear regression model may be represented as:

$$Y = \beta_0 + \sum_{j=1}^{k} \beta_j X_{ij} + U \tag{1.2}$$

Where Y is the dependent variable; $X_1, X_2, X_3, \ldots, X_k$ are independent variables; β_0 and β_j are regression coefficients, representing the parameters of the model for a specific population; and u is a stochastic disturbance- term which may be interpreted as resulting from the effect of unspecified independent variables and/or a totally random element in the relationship specified.

For a sample of n observations:

The general form is

$$Y = X\beta + U \tag{1.3}$$

Where,

X matrix is an n x k+1 matrix of observable and fixed values and of full rank

$\beta$is a (k +1 x 1) vector of unknown parameters to be estimated.

U is (n x 1) vector of random error.

The use of multiple regression models often depends on the estimates of the individual regression coefficients. Some examples of inferences that are frequently made include:

1. Identifying the relative effects of the regressors variables,
2. Prediction and/or estimation, and
3. Selection of an appropriate set of variables for model building.

In an attempt to explain the dependent variable, various assumptions have to be made on some components of the model [2]. When all the assumptions of the classical linear regression model outlined are satisfied, the Ordinary Least Squares (OLS) estimator is said to be Best Linear unbiased Estimator (BLUE) [3].

1.2 Aim and Objectives

The aim of the research work is to propose some Feasible Generalized Ridge Regression Estimators and examine their performances when used to handle multicollinearity or autocorrelation problem in linear regression model and the objectives are:

1. To develop/provide estimators that can handle the problem of multicollinearity and autocorrelation jointly.
2. To determine the best estimator among the proposed and existing ones when multicollinearity alone is a problem.
3. To determine the best estimator among the proposed and existing ones whenautocorrelation alone is a problem.
4. To identify situation where the performances of the proposed estimators are the same or better than the existing ones.

2.0 Literature Review

2.1 Literature review on Multicollinearity

A high degree of multicollinearity among the explanatory variables, X, of a linear regression model, Y = Xβ + U, has a disastrous effect on estimation of the coefficient,β by the Ordinary Least Squares (OLS). In the presence of multcollinearity, the OLSE is inefficient [4, 5]. Several works have been done in literature on handling multicollinearity problem. These are summarized in the table 1

Table 1: Summary of work done on multicollinearity problem.

Author(s)	**Year**	**Focus**
[6]	1956	Introduced the stein estimator as method for solving multicollinearity.
[7]	1965	Principal Components Regression was introduced to handle the problem of multicollinearity by eliminating the model instability and reducing the variances of the regression coefficients. The sample correlation for any pair of components is observed to be zero.
[8]	1966	Introduced the Partial Least Squares Regression into handling multicollinearity problem. This method is similar to the method of the Principal Components Analysis. However, it utilizes the dependent variable.
[9]	1970	Proposed the ridge estimator for dealing with multicollinearity in a regression model. It is the modification of the OLS that allows biased estimation of the regression coefficients.
[10]	1975	Proposed ridge estimator for dealing with multicollinearity and provided optimal value K of the ridge parameter given as $\widehat{K}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$; where$\hat{\sigma}^2$ is an unbiased estimator of error-variance from OLS estimation and $\hat{\alpha}_i^2$ is also regression coefficient from OLS estimation.
[11]	1977	They suggested deleting predictor variables that are highly collinear.
[12]	1992	Grafted Ordinary Ridge Regression (ORR) estimation into Restricted Least Squares (RLS) estimation procedure and obtain his Restricted Ridge Regression (RRR).
[13]	1993	Introduce a family of estimators for any parameter d ϵ (-∞ , +∞) given by $\hat{\beta}_d = (S + I)^{-1} (X^I Y + d\hat{\beta})$. The Liu estimator is identical to OLS estimator of $\hat{\beta}$
[14]	1996	Generalized Maximum Entropy(GME):The estimator requires a number of support values supplied subjectively and exogenously used in solving severe multicollinearity problems
[15]	1999	A new biased estimator. Grafted the Liu estimator into restricted least squares estimation and obtain a new family of estimator.
[16]	2001a and 2001b	Proposed Maximum Entropy Leuven (MEL) estimator. It employs information available in the sample data more efficiently than the OLS does. Unlike the RLS or GME estimator, they do not require any constraint or additional information to be supplied.
[17]	2002	Obtain the principal components which are a linear combination of the explanatory variables and regressed the selected components on the response variable.
[18]	2008	Introduced the K-d class estimator which combines the following estimator: OLS, Ordinary ridge and Liu estimators.

2.2 The Ridge Estimators for Solving Multicollinearity in Linear Model

Ridge regression is a method of biased linear estimation which has been shown to be more efficient than the OLS estimator when data exhibit Multicollinearity. It reduces multicollinearity by adding a ridge parameter, K, to the main diagonal elements of X'X, the correlation matrix. ItSuggested the addition of a small positive constant K≥ 0 to the$X'X$ matrix in the OLS to reduce the ill-conditioned effect when there is collinearity among the explanatory variables [9]. The resulting estimator of β is defined as:

$$\hat{\beta}(k) = (X'X + KI)^{-1} X'y \quad (2.1)$$

The constant K is known as bias or ridge parameter. As K increases from zero and continues up to infinity, the regression estimates tend toward zero. Though this estimator results are bias, for certain value of K, they yield minimum MSE compared to the OLS estimator [10].

2.3 Literature Review on Autocorrelation of Error Terms

The inefficiency of Ordinary Least Square to estimate the parameters of linear regression model in the presence of autocorrelation led to the development of Generalized Least Squares (GLS) estimator. It requires that the true autocorrelation value to be known which is not often so. Using the estimated autocorrelation value leads to the development of Feasible Generalized Least Squares (FGLS) estimators. Thus, studying the finite sample properties of these estimators becomes very imperative. This seems to be very difficult analytically. However, Monte Carlo approach is often being utilized to accomplish this task. An economist[19],observed that the presence of autocorrelated error terms requires some modifications for the OLS estimator to be used. His suggestion involved an autoregressive transformation of the series involved and that the quasi first differences of such series should be used. This necessitated [20] to observe that the transformation suggested in [19] can lead to a less efficient estimator. He therefore suggested that the addition of one weighted observation to CORC procedure may give a better estimator practically without any extra cost. One of the earliest Monte Carlo investigations on this study involve the work in [21]. The estimators they examined include OLS, CORC, Two-Step estimators based on Durbin $\hat{\rho}$ and Prais–

Winsten. Their major conclusions were:The OLS estimator is less efficient than all other methods considered for moderate and high values of ($|\rho|>0.3$) and there is a definite gain from using feasible generalized least squares when $|\rho|\geq 0.3$ and little loss from using such methods otherwise.

The work in [21] was revisited in [22]; but in addition to the estimators considered by Rao and Grilliches, he examined the performance of ML estimator. Some of the results obtained did not agree with that of Rao and Grilliches. For example unlike Rao and Grilliches, the ML and the non–linear estimator are superior to the two stage methods of CORC, Durbin and the Prais–Winstern while the Durbin–Prais–Winsten is superior to Durbin for large $|\rho|$.

The worked on parameter estimation and hypothesis testing in a linear regression model with autocorrelated errors in [23]. The small-sample properties of feasible GLS estimators and tests of individual regression coefficients were examined. Bayesian and non-Bayesian estimators for an autocorrelation coefficient (denoted by ρ) were examined by Monte Carlo experiments. It was shown that when the value of ρ is large, the Bayesian estimator for ρ performs better than non-Bayesian estimators for ρ. When the value of ρ is large and the sample size is 20, none of the feasible GLS estimators performs well in hypothesis testing. However, when the sample size is 40, the feasible Bayesian GLS estimator performs much better that the feasible non-Bayesian GLS estimators in hypothesis testing.

2.4 The Generalized Least Squares Model

If the classical linear regression model (1.3) with all its assumptions but

$E(UU') \neq \sigma^2 I_n$ is considered, the resulting model is called the Generalized Least Squares Model (GLSM).

Pre-multiplying both sides of equation (1.3) by an $n \times n$ non-singular matrix P, we obtain

$$PY = PX\beta + PU \quad (2.2)$$

The error term becomes PU with $E(PU) = 0$ and $E(PU'UP)' = \sigma^2 P\Omega P'$. Thus, if it is possible to specify P such that $P\Omega P' = I$ implying that $P'P = \Omega^{-\prime}$, then the OLS estimates of the transformed variable PY and PX in equation (2.2) have all the optimal properties of OLS and so the usual inferences could be valid. Re-defining equation (2.2) as

$$Y^* = X^*\beta + U^* \quad (2.3)$$

Where $Y^* = PY$, $X^* = PX$ and $U^* = PU$.

By Gauss–Markov theorem [3], the best linear unbiased estimator of βvia the transformed model

2.5 Estimators for Solving Autocorrelation

Consider the Linear Regression Model with Autoregressive of order 1, AR (1) given as

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \dots\dots\dots + \beta_k x_{k\text{-}t} + u_t \quad (2.4)$$

Where $u_t = \rho u_{t-1} + \varepsilon_t$

Therefore, the variance – covariance matrix becomes:

$$E(UU') = \sigma^2\Omega = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{n-2} & \rho^{n-1} \\ \rho & 1 & \rho & \dots & \rho^{n-3} & \rho^{n-2} \\ \rho^2 & \rho & 1 & \dots & \rho^{n-4} & \rho^{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho^{n-2} & \rho^{n-3} & \rho^{n-4} & \vdots & 1 & \rho \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \dots & \rho & 1 \end{bmatrix} \quad (2.5)$$

and $\sigma^2 = \sigma_u^2 = \frac{\sigma_v^2}{(1-\rho^2)}$,

and the inverse of Ω is

$$\Omega^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho & 0 & \dots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \dots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \vdots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix} \quad (2.6)$$

We now search for a suitable transformation matrix P^*.

The existing estimators used are corchraneorcutt(CORC)and Maximum Likelihood Estimators (ML)

3.0 Methodology

3.1 Model Formulation for Simulation Study

3.1.1 Model Formulation

Consider the linear regression model given as:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \mu_t \quad (3.1)$$

Where $\mu_t = \rho\mu_{t-1+}\varepsilon_{t,}|\rho| < 1, \quad t = 1,2,,\dots\dots\dots\dots n, \varepsilon_t \sim N\ (0,\sigma^2)$.
The regressors are fixed and exhibit different degree of multicollinearity.

3.2 The Monte Carlo Experiments

The experiment were replicated (R) one thousand time (1000) and at sample sizes of
n = 10,20,30,50,100 and 250.

Generation of the Explanatory Variables

The equations provided and used in [24] and [25] were used to generate normally distributed random variables with specified inter-correlation. With p = 3, the equations are:

$$X_1 = \mu_1 + \sigma_1 z_1 \qquad (3.2)$$

$$X_2 = \mu_2 + \lambda_{12}\sigma_{12} \quad + \sqrt{m_{22}z_2} \qquad (3.3)$$

$$X_3 = \mu_3 + \lambda_{13}\sigma_3 z_1 + \frac{m_{23}}{\sqrt{m_{22}}} + \sqrt{n_{33}z_3} \qquad (3.4)$$

Where $m_{22} = \sigma^2{}_2(1-\lambda_{12}^2)$
$m_{23} = \sigma_2\sigma_3(\lambda_{23} - \lambda_{12}\lambda_{13})$
and $\quad n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$; and $z_i \sim N(0,1), i = 1\ ,2\ ,3$

By these equations, the inter-correlation matrix has to be positive definite among theindependent variables. In this study ,

$\lambda_{12} = \lambda_{13} = \lambda_{23} = \lambda = 0.4, 0.6, 0.8, 0.95\ and\ 0.99; and x_i \sim N(0,1)$, $i = 1\ ,2\ ,3$

3.3 Generation of the Error Term

The error terms were generated by using the distributional properties of the autocorrelation error terms of AR (1) model given as:

$$u_t \sim N\left(0, \frac{\sigma_e^2}{(1-\rho^2)}\right) \qquad (3.5)$$

Thus, assuming the model start from infinite past, the error terms were generated as follows:

$$U_1 = \frac{\varepsilon_1}{\sqrt{1-\rho^2}} \qquad (3.6)$$

$$u_t = \rho u_{t-1} + \varepsilon_{t,} t = 2\ ,3,4\ \dots\dots\dots.n \qquad (3.7)$$

In this study, autocorrelation value (ρ) is varied from 0.4, 0.8, 0.95, and 0.99.

3.4 Generation of Dependent Variable

The model parameter values were taken as $\beta_0 = 0, \beta_1$= 0.8,$\beta_2 = 0.1$ and
$\beta_3 = 0.6$ **.** Thus, the dependent variable was also generated.

Evaluation, examination and comparison of the estimators were done based on their finite sampling properties. These include Bias closest to zero (BAS), Absolute Bias(ABAS), Variance (VAR) and Mean Square Error (MSE) .Time series processor TSP 5.0 was used to write the program[26].

4.0 Results and Discussion

4.1 Results when Multicollinearity Alone is in the Model

Having ranked the proposed and existing estimators on the basis of each criterion at each category of estimators, the results of the investigation when multicollinearity alone is in the model are presented in Table 2.

Table 2: Summary of the best estimators based on the criteria at different sample sizes

N	BIAS	ABS	VAR	MSE	SR
10	FGLSELO-CORC/CORC (2) **FGLSEAO-ML**/ML **(3)**	GRE (2) **OREKBAY (3)**	GRE (2) **OREKBAY (3)**	GRE (2) **OREKBAY (3)**	GRE (2) **OREKBAY (3)**
20	**FGLSEAO-ML**/ML **(3)** OLSE (2)	**OREKBAY (5)**	**OREKBAY (5)**	**OREKBAY (5)**	**OREKBAY (5)**
30	**FGLSEAO-ML**/ML **(5)**	OREKLA (1) **OREKBAY (4)**	OREKLA (1) **OREKBAY (4)**	**OREKBAY (5)**	GRE(1) **OREKBAY(4)**
50	**FGLSELO-CORC**/ CORC **(5)**	GRE (2) **OREKBAY (3)**	GRE (1) **OREKBAY (4)**	GRE (2) **OREKBAY (3)**	GRE (2) **OREKBAY (3)**
100	FGLSELO-CORC/ CORC (1) **FGLSEAO-ML/** ML (3) OLSE(1)	**OREKBAY (5)**	GRE (2) **OREKBAY (3)**	**OREKBAY (4)** FGREAOKBAY (1)	**OREKBAY (5)**
250	**OLSE (4)** FGLSELO-CORC/ CORC (1)	**OREKBAY (4)** GRE (1)	**OREKBAY (5)**	**OREKBAY (5)**	**OREKBAY (5)**

Notes: i Number in the parenthesis is the number of counts of choice estimators over the levels of Multicollinearity.
ii The best estimator with the highest number of counts is in bold form.

Interpretation

It can be observed that the best estimator under all the criteria except bias closest to zero is OREKBAY. Under the bias criterion, the proposed estimator FGLSEAO-ML(ML) is best when n≤30 and n=100. When n=50, the proposed estimator FGLSELO-CORC is the best and when n is very large (n=250), OLSE is best. In the overall OREKBAY is best.

4.2 Results when Autocorrelation alone is in the model

Having ranked the proposed and existing estimators on the basis of each criterion at each category of estimators, the results of the investigation when autocorrelation alone is in the model are presented in table 3.

Table 3: Summary of the best estimators based on criteria at different sample sizes

N	**BIAS**	**ABS**	**VAR**	**MSE**	**SR**
10	**FGLSELO-CORC**/CORC (1) **FGMLREKLA** (1) **FGCORCREKLA** (1) **FGLSEAO-ML/ML** (1)	GRE (1) **FGREAO-ML** **(3)**	GRE (1) **FGREAO-ML** **(3)**	GRE (1) **FGREAO-ML** **(3)**	FGMLRE (1) **FGREAO-ML** **(3)**
20	OLSE (1) FGLSEAO-ML (1) **FGLSEAO-ML/ML** **(2)**	OREKBAY **(2)** **FGLSELO-CORC/CORC** **(2)**	OREKBAY (1) **FGRELO-CORC** **(2)** FGLSELO-CORC/CORC (1)	**OREKBAY** **(2)** **FGLSELO-CORC/CORC** **(2)**	OREKBAY (1) FGREAO-ML/ML (1) **FGLSELO-CORC/CORC** **(2)**
30	**FGLSEAO-ML**/ML **(2)** **FGLSELO-CORC/CORC** **(2)**	OREKBAY (1) **FGLSEAO-ML/ML** **(2)** FGLSELO-CORC/CORC (1)	OREKLA (1) **FGREAO-ML** **(2)** FGLSEAO-ML/ML (1)	OREKBAY (1) **FGLSEAO-ML/ML** **(2)** FGLSELO-CORC/CORC (1)	OREKBAY (1) **FGLSEAO-ML/ML** **(2)** FGLSELO-CORC/CORC (1)
50	**FGLSELO-CORC**/CORC **(3)** FGLSEAO-ML (1)	FGREAOKBAY (1) **FGLSEAO-ML**/ML **(1)** **FGLSELO-CORC**/CORC **(2)**	FGREOKBAY (1) FGLSEAO-ML/ML (1) **FGRELO-CORC** **(2)**	OREKBAY (1) **FGLSEAO-ML/ML** **(2)** FGLSELO-CORC/CORC (1)	OREKBAY (1) **FGLSEAO-ML/ML** **(2)** FGLSELO-CORC/CORC (1)
100	**FGLSELO-CORC/CORC** **(2)** FGLSEAO-ML/CORC (1) FGLSEAO-ML/ML (1)	**FGLSEAO-ML/ML** **(3)** FGLSELO-CORC/CORC (1)	FGMLRE (1) **FGLSEAO-ML/ML** **(2)** FGRELO-CORC/CORC (1)	OREKBAY (1) **FGLSEAO-ML/ML** **(2)** FGLSELO-CORC/CORC (1)	**FGLSEAO-ML/ML** **(3)** FGLSELO-CORC/CORC (1)
250	FGLSEAO-ML/ML (1) **FGLSELO-CORC/CORC** **(2)** ML (1)	**FGLSEAO-ML/ML** **(3)** FGLSELO-CORC/CORC (1)	**FGREAO-ML** **(2)** FGLSEAO-ML/ML (1) FGLSELO-CORC/CORC (1)	**FGLSEAO-ML/ML** **(2)** FGLSEAO-ML (1) FGLSELO-COC/CORC (1)	**FGLSEAO-ML/ML** **(2)** FGLSEAO-ML (1) FGLSELO-CORC/CORC (1)

Notes: i Number in the parenthesis is the number of counts of choice estimators over the levels of Multicollinearity.

ii. The best estimator with the highest number of counts is in bold form.

5.0 Conclusions

Results of the investigation when only multicollinearity is in the model revealed that the proposed TPE, FGLSELO-CORC and FGLSEAO-ML, result into smaller bias than the existing ridge estimators; and the results are the same with the existing FGLSE. Moreover, the best estimator is an OSE, Ordinary Ridge Estimator with Bayesian K ridge parameters (OREKBAY). Furthermore when autocorrelation alone is present in the model, results shows that the proposed TPE, FGREAO-ML, is best when the sample size is small, n=10. At other sample sizes the best estimators are generally the proposed TPE, FGLSELO-CORC and FGLSEAO-ML which give the same results as the FGLSE, CORC and ML estimators respectively.
In conclusion, some of the proposed estimators can therefore be used as alternatives to FGLSE

6.0 References

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