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On Optimal Strategy for the Control of Syphilis Disease

A.A. Abdullahi

Department of Mathematics and Statistics, The Federal Polytechnic, Ado-Ekiti, Nigeria.

Abstract

We formulate and analyze the dynamics of a Syphilis model with the introduction of two controls: $u_1(t)$ and $u_2(t)$. The two time independent controls represent strategies for the improvement of the treatment and cure of the Syphilis disease. Optimal control theory allows us to find the optimal way to implement the strategies, reducing the number of infectious person and minimizing the cost of implementation as much as possible. This is aimed to assist the policy maker in making a right decision.

Keywords: Epidemics, Optimal control, Pontryagin's Maximum Principle.

1.0 Introduction

Efforts have been made since the late 1950s, by the public health officials making effort to control and eliminate the organisms that causes infectious diseases. The introduction of antibiotics sanitation and vaccination brought a positive perspective of disease eradication[1]. Hence, factors such as resistance to the medicine by the micro-organisms, demographic evolution, accelerated urbanization and increased travelling, led to new infectious diseases and the reemergence of existing diseases. Some of newly identified diseases are Lyme disease(1975), Legionneires disease(1976), HepatiticE(1989),etc[1]. The evolution of Human Immuno deficiency Virus (HIV) in 1981 suddenly became a significant sexually transmitted disease throught the world. Malaria, dengue fever, Syphilis among others have also reemerged and are spreading.

Syphilis is resurgent in many high-income countries, disproportionately affecting urban men who have sex with men (MSM) into new regions because of climate change[2]. Syphilis is a multistage disease that progresses, when untreated, from primary to secondary, latent and finally to tertiary infection. The primary stage symptoms of syphilis involves the presents of a single chancre (a firm, painless, non-itchy ulceration). The primary mode of transmission is by direct sexual contact with lesions of individuals with primary or secondary syphilis. Infection rates patterns of known cases ranged from 20-85% in contact tracing studies. Secondary syphilis with a diffuse rash which involves the palms of the hands and soles of the feet. Latent syphilis with a Plittle to no symptoms and the tertiary syphilis with gummas, neurological or cardiac symptoms. As its name implies, latent syphilis has no clinical manifestations. Early latent syphilis is infection of less than two years duration. An infection of more than two years duration without clinical evidence of treponemal infection is referred to as last latent syphilis. WHO has based this division on the infectiousness of syphilis and its response to therapy. Syphilis is thought to have infected 12 million additional people worldwide in 1999, with greater than 90% of cases in the developing world. After decreasing dramatically since the widespread availability of penicillin in the 1940s, rates of infection have increased since the turn of the millennium in many countries, often in combination with human immunodeficiency virus (HIV). This has been attributed partly to increased promiscuity, prostitution, decreasing use of condoms, and unsafe sexual practices among men who have sex with men.

Modeling of epidemiological phenomenon has a very long story with the first model for small pox formulated by Daniel Bernoulli in 1760. Mathematical modeling of the population models continues to provide vital insights into population behaviour and control[3]. Over the years, it has also become an important tool in understanding the dynamics of diseases, and the decision making process regarding intervention programs for controlling population and disease problems in many countries[4].

In the past years this has become an essential tool in understanding the dynamics of diseases, and the decision which has to do with process regarding intervention programs for controlling population and disease problems .

Similarly, different strategies and techniques have been employed to study vital optimal control problem related to dynamical systems[5] applied optimal control theory to model malaria disease that include vaccinations and treatment[6].

Corresponding author: A.A. Abdullahi, E-mail: Adedeji.abdullahi6263@yahoo.com, Tel.: +2347063379115, 08029023836

Hence, the successful eradication of any diseases does not depend only on the ability to understand the transmission dynamics of a particular disease and the application of optimal control strategies and the implementation of logistic policies[7].

2.0 The Model and its Analysis

The model sub-divides the total human population at time t denoted by N(t) into six compartments of susceptible male $S_m(t)$, susceptible female $S_f(t)$, infected male $I_m(t)$, infected female $I_f(t)$, complications C(t) and Treated T(t), where N(t) is given as

$$N(t) = S_m(t) + S_f(t) + I_m(t) + I_f(t) + C(t) + T(t) \quad (1)$$

The susceptibles are individuals that have not contracted the infection but may be infected through sexual contacts. The population recruits into the susceptible classes at the rate π_m for susceptible male and π_f for the susceptible female. Infected individuals are those with the infection and can transmit the infection by sexual act to the susceptibles, $\alpha_1$represent contact rate at which susceptible male move to infected male, similarly α_2 is the contact rate of movement of susceptible female into infected female class. The complications are individuals in the population with the infection at the latent stage that can leads to other diseases or death, β_1, β_2 are the rate of progression of infected male and female into the complications class respectively. Treated are people in the population that have recovered due to treatment,r_1, r_2 reprsent the recovery / treated rate of infected male and infected female while v is the treated rate of complications class. We assume that the death rate is not negligible and so the nature death rate is represented by µ and due to untreated syphilis which can lead to death, we represent the syphilis induced death rate by δ.

The model equation is given as:

$$\frac{dS_m}{dt} = \pi_m - \alpha_1 I_f S_m - \mu S_m \quad (2a)$$

$$\frac{dS_f}{dt} = \pi_f - \alpha_2 I_m S_f - \mu S_f \quad (2b)$$

$$\frac{dI_m}{dt} = \alpha_1 I_f S_m - (r_1 + \beta_1 + \mu) I_m \quad (2c)$$

$$\frac{dI_f}{dt} = \alpha_2 I_m S_f - (r_2 + \beta_2 + \mu) I_f \quad (2d)$$

$$\frac{dC}{dt} = \beta_1 I_m + \beta_2 I_f - (v + \mu + \delta) C \quad (2e)$$

$$\frac{dT}{dt} = r_1 I_m + r_2 I_f + vC + \mu T \quad (2f)$$

3.0 Analysis of Optimal Control

We extend model (2a) – (2f) to include some controls. Our state system is the following system of six ordinary differential equations

$$\frac{dS_m}{dt} = \pi_m - (1-u_1)\alpha_1 I_f S_m - \mu S_m \quad (2g)$$

$$\frac{dS_f}{dt} = \pi_f - (1-u_2)\alpha_2 I_m S_f - \mu S_f \quad (2h)$$

$$\frac{dI_m}{dt} = (1-u_1)\alpha_1 I_f S_m - (r_1 + \beta_1 + \mu) I_m \quad (2i)$$

$$\frac{dI_f}{dt} = (1-u_2)\alpha_2 I_m S_f - (r_2 + \beta_2 + \mu) I_f \quad (2j)$$

$$\frac{dC}{dt} = \beta_1 I_m + \beta_2 I_f - (v + \mu + \delta) C \quad (2k)$$

$$\frac{dT}{dt} = r_1 I_m + r_2 I_f + vC + \mu T \quad (2l)$$

The control u_1,where $0 \le u_1 \le 1$ deals with reducing the exposure of susceptible (male and female) humans to those infected. T he control $u_2$for $0 \le u_2 \le 1$, models the efforts needed in bringing down the infection.

The objective is to minimize the number of infected humans in the population while maintaining the cost associated to contro l u_1 and u_2 as much as possible. Therefore, we seek to minimize the number of infected individual hosts and cost of employi ng mass treatment . Our optimal control problem with objectives is expressed as

$$J(u_1, u_2) = \int_0^{t_f} \left[I_m + I_f + \frac{B_1}{2} u_1^2 + \frac{B_2^2}{2} \right] dt \quad (3)$$

Where B_1 and $B_2$are the weighing constants for the treatment of human host and mass treatment for infectious individuals.

We therefore seek an optimal control $u_1^* and\ u_2^*$ such that

$$J(u_1^*, u_2^*) = \min_{\Omega} J(u_1, u_2) \quad (4)$$

$$(I_m, I_f, U)\Re_0$$

Where $\Omega = \{(u_1^*, u_2^*) \in L^1(0, t_f) | d_1 \le u_j \le e_{j,} i = 1,2\}$ (5)

We analyze model (2g) – (2l), in other words model of the spread of Syphilis in population applying optimal perspectives. W e take into account the objective function (3) to model (2g) – (2l). Pontryagins Maximum Principle will be employed to deter mine the optimal control $u_1^*\ and\ u_2^*$ with necessary conditions. The necessary conditions to establish optimal control u_1^*, u_2^* t hat meet condition (4) and its constraints model (2g) – (2l) will be determined by applying Pontryagins Maximum Principle[8]. The Principle changes (2.2), (3) and (4) into a problem of minimizing point wise a Hamiltoniaan, H, with respect to $((u_1, u_2)$, simply

$$H(S_m, S_f, I_m, I_f, C, T, u_1, u_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) = I_f(t) + I_m(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) + \sum_{i=1}^{6} \lambda_i g_i \quad (6)$$

Where g_i is the right hand side of the difference equations of the ith state variable. By using Pontryagin's Maximum Principle and the existence of results obtained for optimal control, we have

Theorem 1: There exists an optimal control u_1^*, u_2^* and corresponding solution, $S_m^*, S_f^*, I_m^*, I_f^*, C, T$, that minimizes $J(u_1, u_2)$ over Ω. Moreso, there exist adjoint function, $\lambda_1(t) \dots, \lambda_6(t)$, such that

$$\frac{d\lambda_1}{dt} = \lambda_1\left((1-u_1)\alpha_1 I_f - \mu\right) - \lambda_3((1-u_2)\alpha_1 I_f)$$

$$\frac{d\lambda_2}{dt} = \lambda_2((1\text{-}u_1)\ \alpha_2 I_m - \mu)) - \lambda_4((1-u_2)\alpha_2 I_m)$$

$$\frac{d\lambda_3}{dt} = \lambda_2\left((1-u_1)\alpha_2 S_f\right) + \lambda_3(r_1 + \beta_1 + \mu)) - \lambda_4((1-u_2)\alpha_2 S_f - \lambda_5\beta_1 - \lambda_6 r_1 - 1$$

$$\frac{d\lambda_4}{dt} = -1 + \lambda_1\left((1-u_1)\alpha_1 S_m\right) - \lambda_3(1-u_2)\alpha_1 S_m + \lambda_4(r_2 + \beta_2 + \mu) - \lambda_5\beta_2 - \lambda_6 r_2$$

$$\frac{d\lambda_5}{dt} = \lambda_5(\upsilon + \mu + \delta)) - \lambda_6\upsilon$$

$$\frac{d\lambda_6}{dt} = -\lambda_6 T \quad (7)$$

With tranversality con

$$\lambda_i(t_f) = 0, i = 1 \dots, 6 \quad (8)$$

And $N = S_m^* + S_f^* + I_m^* + I_f^* + C^* + T^*$ (9)

Theorem2: The optimal control (u_1^*, u_2^*) that minimizes $J(u_1^*, u_2^*)$ over Ω is expressed as

$$u_1^* = max\left\{0, min\left\{1, -\frac{(\alpha_1 I_f S_m\lambda_1 + \alpha_2 I_m S_f\lambda_2)}{B_1}\right\}\right\}$$

And

$$u_2^* = max\left\{0, min\left\{1, \frac{\alpha_1 I_f S_m\lambda_3 + \alpha_2 I_m S_f\lambda_4}{B_2}\right\}\right\} \quad (10)$$

Proof: (8) gives the existence of an optimal control due to the convexity of integrand J with respect to (u_1^*, u_2^*), *a priori* boundedness of the state solutions, and the Lipschitz property of the state system with respect to the state variables. Employing Pontryagin's Maximum Principle, we have

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{S_m}, \lambda_1(t_f) = 0,$$

$$\frac{d\lambda_6}{dt} = -\frac{\partial H}{T}, \lambda_6(t_f) = 0. \quad (11)$$

Computed at the optimal control pair and respective corresponding states, which leads to the stated adjoint system (7) and (8). Considering the optimality conditions

$$\frac{\partial H}{u_1} = 0, \frac{\partial H}{u_2} = 0$$

And determine the values for u_1^*, u_2^*, subject to the constraints, the characterization (10) can be obtained.

4.0 Conclusion

The optimality system is the state and adjoint systems coupled with the optimal control characterization. In this study, we derived and analyzed a deterministic model for the spread of Syphilis that include mass treatment and mass education. Applying optimal control strategy, we find a solution to the eradication of Syphilis disease in a finite time. We hope to carry out the numerical analysis in future work for proper analysis and advice.

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