Transactions of the Nigerian Association of Mathematical Physics
***Volume 3, (January, 2017*), pp 111 – 118**
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On the Mathematical Analysis of a New Lassa Fever Model that Incorporates Quarantine as a Control Strategy

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Abstract

In this work we developed and analyze a Mathematical Model for the Spread and Control of Lassa Fever using Quarantine Technique. The model is a first order Ordinary Differential Equations, in which the human population is divided into six mutually- exclusive compartments namely; Susceptible Individuals (S_H) , Susceptible vector (S_V) , Infected Human (I_H) , Quarantine Human (Q_H) , Recovered Human (R_H) and Infected Vector (I_V) . The equilibrium states were obtained and their stabilities were analyzed by using Bellman and Cooke's theorem. The result shows that the endemic equilibrium state is stable and the criteria for stability of the disease free equilibrium state were established. Also, this is the first time a quarantine compartment will be incorporated into a vector borne dynamical model for Lassa fever.

Keywords: Disease Free Equilibrium, Endemic Equilibrium, Lassa Fever, Bellman and Cooke's theorem, Quarantine.

1.0 Introduction

Lassa fever is a viral disease, with possibility of transmission from an infected animal to a human; it can also be transmitted from individual to individual. The virus attacks the liver, kidney, nervous system and spleen, causing them to bleed. It was first discovered in the town of Lassa in 1969 in Borno State, Nigeria in the Yedseran river valley near south end of Lake Chad [1]. It takes between one to three weeks for the symptoms of Lassa fever to show up [2]. A rat that is common in endemic areas, known as Mastomys natalensis is the real host of the infection [3]. Human can be infected with this infection once they eat foods that contain saliva, excreta or urine of the hosted Lassa virus rat. Lassa fever is a disease that often occurs in the dry season because dust particles from dead rats that carry this virus are more mobile making it easy to inhale. The fever is caused by Lassa virus which belongs to the arena virus family [4]. People with greatest chance of acquiring the infection are the people living in the rural areas where the Mastomys are living [5].

Lassa fever is endemic in some countries like Nigeria, Sierra Leone, Liberia, Guinea etc. It affects nearly 2 to 3 million individuals with about 5000-10,000 fatalities yearly [6]. As at 22nd March 2012, approximately 623 cases including 70 deaths were recorded from 19 states out of the 36 states in Nigeria with Edo and Taraba having the highest number of deaths. In some patients neurological problems, including hearing loss (which may be transient or permanent) and tremors have been described [7].

Lassa fever can be transmitted from infected rat to human and also from human to human. Lassa fever is a deadly disease that is ravaging some West African countries including Nigeria with case fatality of approximately 5000-10,000 death occurring annually. It takes about 2 to 21 days for the symptoms to show up and some of the symptoms include cough, vomiting, facial swelling, diarrhea, chest pain and among others [2]. Lassa fever is a neglected tropical disease. Very few articles on mathematical modelling of Lassa fever can be found especially on modelling of its control. Hence, in this work we seek to describe the dynamics of the disease, formulate a mathematical model of the spread and control of the disease, obtain the stationary points of the model and perform stability analysis on the points.

Following the method adopted by Lagos State Government, it became very important to study the method mathematically when considering the success registered during the Ebola Saga. Since Ebola and Lassa are similar, applying a similar method

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of prevention is worth studying. Hence, it is hoped that the study of the disease mathematically will serve as information and planning template to government officials, public health officers, and administrators.
Mathematicians and Epidemiologists have tried over the years to describe infectious diseases mathematically. Several authors [8-13] have developed mathematical models to study dynamics or transmission of diseases, to access the disease's spread, and more importantly, to understand different ways to prevent epidemics, optimal control strategies via behavioural change, vaccination, treatment, quarantine and other measure.
Also, some authors have attempted to describe the dynamics of Lassa fever mathematical; for example a Susceptible-Infected-Susceptible model was developed and analyzed for the transmission of Lassa fever disease for stability [14], also another researchers developed and analyzed a SIR model for controlling Lassa fever transmission in Edo state, Nigeria [15] and lastly, a sensitivity analysis was carried out on a Lassa fever deterministic mathematical model by another group of researchers [16]. But none of the authors put into consideration the importance of quarantine as a control strategy.

2.0 Materials and Methods

2.1 Mathematical Formulation

Here, we divide the population into six mutually- exclusive compartments namely; Susceptible Human (S_H), Susceptible vector (S_V), Infected Human (I_H), Quarantine Human (Q_H), Recovered Human (R_H) and Infected Vector (I_V)
The model parameters to be incorporated include; recruitment rate of human due to natural birth and immigration (A_H), contact rate between infected vector and susceptible human (β_H), contact rate between infected human and susceptible vector (β_V). Natural death rate of human (μ), the rate at which quarantine human becomes susceptible (p_1), the rate at which recovered human becomes susceptible (p_2). The rate at which infectious individuals leave the compartment for the quarantine compartment (δ), removal rate from quarantine compartment (ε) to recovered compartment, removal rate from infectious compartment (γ) to recovered compartment, death due to infection for infection compartment (μ_1), death due to infection for quarantine compartment (μ_2), recruitment rate of vector (A_V) and death rate of vector due to natural causes or accident. The flow diagram is presented blow (fig1) in which the circles represent the different compartments and the arrows represent the transition between the compartments.

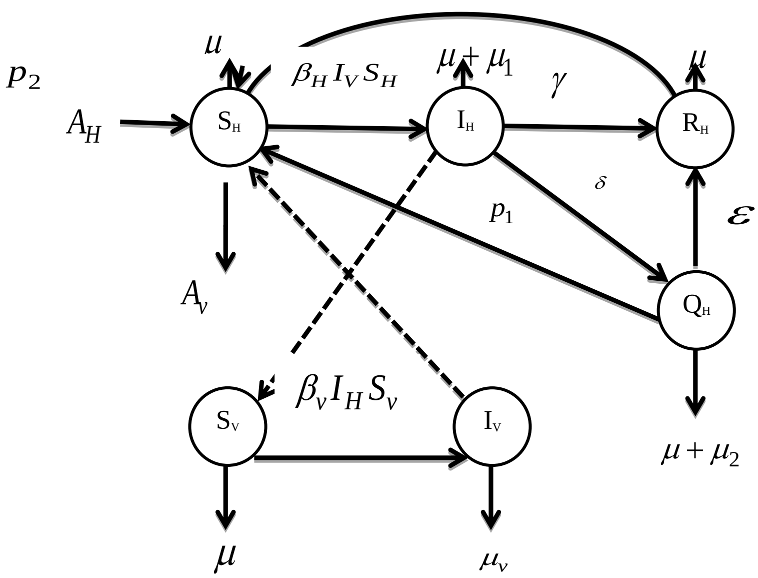

Fig 1: a schematic diagram of the model equations

2.2 Model Assumptions

The population of the susceptible human (S_H) increases through the recruitment of individuals into the population at a rate A_H, as recovered and quarantine humans become susceptible at a rate p_2 and p_1 respectively. The population decreases as susceptible human move into the infectious compartment (I_H) via interaction between the susceptible human (S_H) and infected vector (I_V) at a rate β_H and natural death of human at a rate μ .

The population of the infected human (I_H) increases as the susceptible human move into the infected human compartment via interaction between the susceptible human and infected vector I_V at a rate β_H ; and decreases as infected human move into the recovered class due to treatment at a rate γ, move into the quarantine class at a rate δ, natural death rate μ and death due to Lassa fever infection at a rate μ_1.

The population of the recovered humans (R_H) increases as infected and quarantine humans move into the recovered class due to treatment at a rate γ and ε respectively. The population decreases as the recovered humans become susceptible at a rate p_2 and natural death at a rate μ .

The population of the quarantine humans (Q_H) increases as infected humans leave the compartment for the quarantine compartment at a rate δ and the population decreases as members move into the recovered class (R_H) and susceptible human (S_H) class due to treatment at a rate ε and p_1 respectively, natural death rate μ and death due to the infection for quarantine compartment at a rate μ_2 .

The population of the susceptible vector S_V increases through the constant recruitment of vector into the population at a rate A_V and the population decreases as members of the susceptible vector S_V move into the infected vector compartment via interaction between the susceptible vector and infected human at a rate β_V and natural death at a rate μ .

The population of the infected vector S_V increases as members of the susceptible vector move into the infected vector compartment via interaction between the susceptible vector and infected human at a rate β_V and decreases due to natural death of the vector.

From figure 1 and the assumptions stated above, we have the following differential equations;

$$\frac{dS_H(t)}{dt} = A_H + P_2R_H(t) + P_1Q_H(t) - (\mu + \beta_H I_V(t))S_H(t) \tag{1}$$

$$\frac{dI_H(t)}{dt} = \beta_H I_V(t)S_H(t) - (\mu + \mu_1 + \delta + \gamma)I_H(t) \tag{2}$$

$$\frac{dR_H(t)}{dt} = \gamma I_H(t) + \varepsilon Q_H(t) - (\mu + P_2)R_H(t) \tag{3}$$

$$\frac{dQ_H(t)}{dt} = \delta I_H(t) - (\mu + \mu_2 + P_1 + \varepsilon)Q_H(t) \tag{4}$$

$$\frac{dS_V(t)}{dt} = A_V - (\mu + \beta_V I_H(t))S_V(t) \tag{5}$$

$$\frac{dI_V(t)}{dt} = \beta_V I_H(t)S_V(t) - \mu_V I_V(t) \tag{6}$$

2.3 Mathematical Analysis of the Model

Equilibrium Points

At equilibrium, $\frac{dS_H(t)}{dt} = \frac{dI_H(t)}{dt} = \frac{dQ_H(t)}{dt} = \frac{dR_H(t)}{dt} = \frac{dS_V(t)}{dt} = \frac{dV(t)}{dt} = 0$ (7)

For convenience, Let $S_H = x_1,\ I_H = x_2,\ Q_H = x_3,\ R_H = x_4,\ S_V = x_5$ *and* $I_V = x_6$. Then,

$$A_H + P_2x_3 + P_1x_4 - (\mu + \beta_H x_6)x_1 = 0 \tag{8}$$

$$\beta_H x_6 x_1 - (\mu + \mu_1 + \delta + \gamma)x_2 = 0 \tag{9}$$

$$\gamma x_2 + \varepsilon x_4 - (\mu + P_2)x_3 = 0 \tag{10}$$

$$\delta x_2 - (\mu + \mu_2 + P_1 + \varepsilon)x_4 = 0 \tag{11}$$

$$A_V - (\mu + \beta_V x_2)x_5 = 0 \tag{12}$$

$$\beta_V x_2 x_5 - \mu_V x_6 = 0 \quad (13)$$

From equation (13)

$$\beta_V x_2 x_5 = \mu_V x_6 \quad (14)$$

$$x_6 = \frac{\beta_V x_2 x_5}{\mu_V} \quad (15)$$

Substitute equation (15) into equation (9)

$$\frac{\beta_H \beta_V x_2 x_5 x_1}{\mu_V} - (\mu + \mu_1 + \delta + \gamma) x_2 = 0 \quad (16)$$

$$\left(\frac{\beta_H \beta_V x_5 x_1}{\mu_V} - (\mu + \mu_1 + \delta + \gamma) \right) x_2 = 0 \quad (17)$$

$$\frac{\beta_H \beta_V x_5 x_1}{\mu_V} - (\mu + \mu_1 + \delta + \gamma) = 0 \;\; or \;\; x_2 = 0 \quad (18)$$

Suppose $x_0 = 0$

Substitute $x_2 = 0$ into equations (9), (10), (11), (12) and (13)

$$\beta_H x_6 x_1 = 0 \quad (19)$$

$$\varepsilon x_4 - (\mu + P_2) x_3 = 0 \quad (20)$$

$$-(\mu + \mu_2 + P_1 + \varepsilon) x_4 = 0 \quad (21)$$

$$A_V - \mu x_5 = 0 \quad (22)$$

$$-\mu_V x_6 = 0 \quad (23)$$

From equation (22)

$$x_6 = \frac{0}{-\mu_V} \quad (24)$$

$$x_6 = 0 \quad (25)$$

From equation (22)

$$A_V = \mu x_5 \quad (26)$$

$$x_5 = \frac{A_V}{\mu} \quad (27)$$

From equation (20)

$$x_4 = \frac{0}{-(\mu + \mu_2 + P_1 + \varepsilon)} \quad (28)$$

$$x_4 = 0 \quad (29)$$

Now, substitute equation (29) into equation (19)

$$-(\mu + P_2) x_3 = 0 \quad (30)$$

$$x_3 = 0 \quad (31)$$

Now, substitute equations (19), (29) and (31) into equation (8)

$$A_H - \mu x_1 = 0 \quad (32)$$

$$x_1 = \frac{A_H}{\mu} \tag{33}$$

2.4 Stability Analysis of Disease Free Equilibrium (DFE)

We recall that the system of the equation of the model at equilibrium is

$$A_H + P_2 x_3 + P_1 x_4 - (\mu + \beta_H x_6) x_1 = 0 \tag{34}$$

$$\beta_H x_6 x_1 - (\mu + \mu_1 + \delta + \gamma) x_2 = 0 \tag{35}$$

$$\gamma x_2 + \varepsilon x_4 - (\mu + P_2) x_3 = 0 \tag{36}$$

$$\delta x_2 - (\mu + \mu_2 + P_1 + \varepsilon) x_4 = 0 \tag{37}$$

$$A_V - (\mu + \beta_V x_2) x_5 = 0 \tag{38}$$

$$\beta_V x_2 x_5 - \mu_V x_6 = 0 \tag{39}$$

The Jacobian matrix at disease free equilibrium is given by J $\left(\frac{A_v}{\mu}, 0, 0, 0, \frac{A_v}{\mu_v}, 0\right) =$

$$\begin{bmatrix} -\mu & 0 & p_3 & p_1 & 0 & -\frac{A_H \beta_H}{\mu} \\ 0 & -(\delta+\mu+\gamma+\mu_1) & 0 & 0 & 0 & \frac{A_H \beta_H}{\mu} \\ 0 & \gamma & -(\mu+p_2) & \varepsilon & 0 & 0 \\ 0 & \delta & 0 & -(\mu+\mu_2+p_1+\varepsilon) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu \\ 0 & \frac{A_V \beta_V}{\mu} & 0 & 0 & 0 & -\mu_V \end{bmatrix}$$

The characteristics equation becomes

$$\begin{bmatrix} -\alpha_1-\lambda & 0 & p_3 & p_1 & 0 & -\alpha_2 \\ \alpha_3 & -\alpha_7-\lambda & 0 & 0 & 0 & \alpha_2 \\ 0 & \gamma & -\alpha_8-\lambda & \varepsilon & 0 & 0 \\ 0 & \delta & 0 & -\alpha_9-\lambda & 0 & 0 \\ 0 & -\alpha_4 & 0 & 0 & -\alpha_6-\lambda & 0 \\ 0 & \alpha_4 & 0 & 0 & \alpha_6 & -\mu_V-\lambda \end{bmatrix}$$

The determinant is

$$(-\mu-\lambda_1)\,(-(\delta+\mu+\gamma+\mu_1)-\lambda_2)\,(-(\mu+p_2)-\lambda_3)(-(\mu+\mu_2+p_1+\varepsilon)-\lambda_4)$$
$$(-\mu-\lambda_5)\,(-\mu_V-\lambda_6) = 0 \tag{40}$$

This implies

$$\lambda_1 = -\mu,\ \lambda_2 = -(\delta+\mu+\gamma+\mu_1),\ \lambda_3 = -(\mu+p_2),\ \lambda_4 = -(\mu+\mu_2+p_1+\varepsilon)$$
$$\lambda_5 = -\mu \quad \lambda_6 = -\mu_V \tag{41}$$

Since all the eigen-values are all negatives, it means the diseases free equilibrium state is stable.

2.5 Stability analysis of the Endemic Equilibrium State (EES)

The characteristics equation becomes

$$\begin{bmatrix} -\alpha_1-\lambda & 0 & p_3 & p_1 & 0 & -\alpha_2 \\ \alpha_3 & -\alpha_7-\lambda & 0 & 0 & 0 & \alpha_2 \\ 0 & \gamma & -\alpha_8-\lambda & \varepsilon & 0 & 0 \\ 0 & \delta & 0 & -\alpha_9-\lambda & 0 & 0 \\ 0 & -\alpha_4 & 0 & 0 & -\alpha_5-\lambda & 0 \\ 0 & \alpha_4 & 0 & 0 & \alpha_6 & -\mu_V-\lambda \end{bmatrix}$$

The characteristics equations obtained from the Jacobian determinant with the Eigen value λ

$$\begin{aligned}
&\lambda^6+(\alpha_8+\alpha_7+\alpha_1+\alpha_9+\alpha_5+\mu_V)\lambda^5+((\alpha_1+\alpha_7+\alpha_9+\alpha_8+\mu_V)\alpha_5+(\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8\\
&+(\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\lambda^4+(((\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8+(\alpha_7+\alpha_1+\mu_V)\alpha_9\\
&+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_5+((\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_8+(\alpha_7+\mu_V)\alpha_1\\
&-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3(\delta P_1+P_2\gamma)\lambda^3+((((\alpha_7+\alpha_1+\mu_V)\alpha_9\\
&+(\mu_V+\alpha_7)\alpha_1-\alpha_3(\delta P_1+P_2\gamma-\alpha_4\alpha_2))\alpha_5+(((\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1\\
&+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1\\
&+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\gamma P_1)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(\delta P_1+P_2\gamma)\mu_V-\alpha_4\alpha_2\alpha_6+\delta P_2E))\lambda^2+((((\mu_V+\alpha_7)\alpha_1\\
&-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(\delta P_1-\alpha_4\alpha_2))\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(P_2\gamma-\alpha_2\alpha_4)\alpha_9\\
&-((\delta P_1+P_2\gamma)\mu_V+\delta P_2E)\alpha_3)\alpha_5+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1-\\
&\alpha_3(\delta P_1\mu_V-\alpha_4\alpha_2\alpha_6))\alpha_8+(-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(P_2\gamma\mu_V-\alpha_2\alpha_4\alpha_6))\alpha_9-\delta P_2E\alpha_3\mu_V)\lambda+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1\\
&+\alpha_4\alpha_2\alpha_3)\alpha_9-\alpha_3\delta P_1\mu_V)\alpha_8-\alpha_3\delta P_2E\mu_V(\delta E+\alpha_9\gamma))-\alpha_2\alpha_4\alpha_6\alpha_9\alpha_8(\alpha_3-\alpha_3)=0
\end{aligned} \tag{42}$$

Where $\alpha_1=\dfrac{A_H\beta_H}{\mu}, \alpha_2=\dfrac{A_H\beta_v}{\mu}, \alpha_3=\dfrac{A_v\beta_v}{\mu_v}, \alpha_4=\dfrac{A_v\beta_v}{\mu_v}-\mu_v$

We can now apply the result of Bellman and Cooke's theorem of stability [17], H (Z) =P (z, e^2) where P (z, w) is a polynomial with principal term.

Suppose H (y) = F (y) + i G(y) (43)

If all zeros of H (y) have negatives real parts, then zeros of F (y) and G(y) are real, simple and alternate and

F (0) G' (0) – F' (0) G (0) >0 for all y belongs to real numbers (44)

Conversely, all zeros of H (z) will be in the left hand plane provided that it is in either of the following conditions

1. All zeros of F (y) and G (y) are real, simple and the inequality (44) is satisfied for at least one y
2. All the zeros of F (y) are real and for each zero the relation (43) is satisfied
3. All the zeros of G (y) are real and for each zero, the relation (43) is satisfied.

Let the equation (43) take the form

$$H(\lambda)=\begin{aligned}
&\lambda^6+(\alpha_8+\alpha_7+\alpha_1+\alpha_9+\alpha_5+\mu_V)\lambda^5+((\alpha_1+\alpha_7+\alpha_9+\alpha_8+\mu_V)\alpha_5+(\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8\\
&+(\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\lambda^4+(((\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8+(\alpha_7+\alpha_1+\mu_V)\alpha_9\\
&+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_5+((\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_8+(\alpha_7+\mu_V)\alpha_1\\
&-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3(\delta P_1+P_2\gamma)\lambda^3+((((\alpha_7+\alpha_1+\mu_V)\alpha_9\\
&+(\mu_V+\alpha_7)\alpha_1-\alpha_3(\delta P_1+P_2\gamma-\alpha_4\alpha_2))\alpha_5+(((\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1\\
&+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1\\
&+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\gamma P_1)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(\delta P_1+P_2\gamma)\mu_V-\alpha_4\alpha_2\alpha_6+\delta P_2E))\lambda^2+((((\mu_V+\alpha_7)\alpha_1\\
&-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(\delta P_1-\alpha_4\alpha_2))\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(P_2\gamma-\alpha_2\alpha_4)\alpha_9\\
&-((\delta P_1+P_2\gamma)\mu_V+\delta P_2E)\alpha_3)\alpha_5+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1-\\
&\alpha_3(\delta P_1\mu_V-\alpha_4\alpha_2\alpha_6))\alpha_8+(-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(P_2\gamma\mu_V-\alpha_2\alpha_4\alpha_6))\alpha_9-\delta P_2E\alpha_3\mu_V)\lambda+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1\\
&+\alpha_4\alpha_2\alpha_3)\alpha_9-\alpha_3\delta P_1\mu_V)\alpha_8-\alpha_3\delta P_2E\mu_V(\delta E+\alpha_9\gamma))-\alpha_2\alpha_4\alpha_6\alpha_9\alpha_8(\alpha_3-\alpha_3)=0
\end{aligned} \tag{45}$$

Now set λ = iw in to equation (45) and apply the result of Bellman and Cooke's theorem [17]

$$
\begin{aligned}
H(iw) = {} & -w^6+i(\alpha_8+\alpha_7+\alpha_1+\alpha_9+\alpha_5+\mu_V)w^5+((\alpha_1+\alpha_7+\alpha_9+\alpha_8+\mu_V)\alpha_5+(\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8 \\
& +(\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)w^4-i(((\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8+(\alpha_7+\alpha_1+\mu_V)\alpha_9 \\
& +(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_5+((\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_8+(\alpha_7+\mu_V)\alpha_1 \\
& -\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3(\delta P_1+P_2\gamma)w^3-((((\alpha_7+\alpha_1+\mu_V)\alpha_9 \\
& +(\mu_V+\alpha_7)\alpha_1-\alpha_3(\delta P_1+P_2\gamma-\alpha_4\alpha_2))\alpha_5+(((\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1 \\
& +\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1 \\
& +\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\gamma P_1)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(\delta P_1+P_2\gamma)\mu_V-\alpha_4\alpha_2\alpha_6+\delta P_2E))w^2+i((((\mu_V+\alpha_7)\alpha_1 \\
& -\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(\delta P_1-\alpha_4\alpha_2))\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(P_2\gamma-\alpha_2\alpha_4)\alpha_9 \\
& -((\delta P_1+P_2\gamma)\mu_V+\delta P_2E)\alpha_3)\alpha_5+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1- \\
& \alpha_3(\delta P_1\mu_V-\alpha_4\alpha_2\alpha_6))\alpha_8+(-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(P_2\gamma\mu_V-\alpha_2\alpha_4\alpha_6))\alpha_9-\delta P_2E\alpha_3\mu_V)w+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1 \\
& +\alpha_4\alpha_2\alpha_3)\alpha_9-\alpha_3\delta P_1\mu_V)\alpha_8-\alpha_3\delta P_2E\mu_V(\delta E+\alpha_9\gamma))-\alpha_2\alpha_4\alpha_6\alpha_9\alpha_8(\alpha_3-\alpha_3)=0
\end{aligned}
\tag{46}
$$

Resolving in to real and imaginary parts we have

H (iw) = F (w) + iG (w)

$$
\begin{aligned}
F(w) = {} & -w^6++((\alpha_1+\alpha_7+\alpha_9+\alpha_8+\mu_V)\alpha_5+(\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8 \\
& +(\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)w^4- \\
& ((((\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_3(\delta P_1+P_2\gamma-\alpha_4\alpha_2))\alpha_5 \\
& +(((\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1 \\
& +\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\delta P_1)\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2 \\
& -\alpha_3\delta P_1)\alpha_8+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3\gamma P_1)\alpha_9 \\
& -\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(\delta P_1+P_2\gamma)\mu_V-\alpha_4\alpha_2\alpha_6+\delta P_2E))w^2 \\
& +(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4\alpha_2\alpha_3)\alpha_9-\alpha_3\delta P_1\mu_V)\alpha_8 \\
& -\alpha_3\delta P_2E\mu_V(\delta E+\alpha_9\gamma))-\alpha_2\alpha_4\alpha_6\alpha_9\alpha_8(\alpha_3-\alpha_3)=0
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
G(w) = {} & (\alpha_8+\alpha_7+\alpha_1+\alpha_9+\alpha_5+\mu_V)w^5-(((\alpha_7+\mu_V+\alpha_1+\alpha_9)\alpha_8+(\alpha_7+\alpha_1+\mu_V)\alpha_9 \\
& +(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_5+((\alpha_7+\alpha_1+\mu_V)\alpha_9+(\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_8+(\alpha_7+\mu_V)\alpha_1 \\
& -\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_9+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2-\alpha_3(\delta P_1+P_2\gamma)w^3+((((\mu_V+\alpha_7)\alpha_1 \\
& -\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(\delta P_1-\alpha_4\alpha_2))\alpha_8+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(P_2\gamma-\alpha_2\alpha_4)\alpha_9 \\
& -((\delta P_1+P_2\gamma)\mu_V+\delta P_2E)\alpha_3)\alpha_5+(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1- \\
& \alpha_3(\delta P_1\mu_V-\alpha_4\alpha_2\alpha_6))\alpha_8+(-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(P_2\gamma\mu_V-\alpha_2\alpha_4\alpha_6))\alpha_9-\delta P_2E\alpha_3\mu_V)w
\end{aligned}
\tag{48}
$$

Differentiating equation (47) and (48) with respect to w and setting w = 0 we have

$$F^{`}(0) = 0 \tag{49}$$

$$
\begin{aligned}
G^{`}(0) = {} & ((((\mu_V+\alpha_7)\alpha_1-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(\delta P_1-\alpha_4\alpha_2))\alpha_8 \\
& +(-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1-\alpha_3(P_2\gamma-\alpha_2\alpha_4)\alpha_9-((\delta P_1+P_2\gamma)\mu_V+\delta P_2E)\alpha_3)\alpha_5 \\
& +(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4(\alpha_3-\alpha_6)\alpha_2)\alpha_9-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(\delta P_1\mu_V-\alpha_4\alpha_2\alpha_6))\alpha_8 \\
& +(-\alpha_4\alpha_2\alpha_6\alpha_1-\alpha_3(P_2\gamma\mu_V-\alpha_2\alpha_4\alpha_6))\alpha_9-\delta P_2E\alpha_3\mu_V)
\end{aligned}
\tag{50}
$$

Also set w = 0 in to equations (47) and (48)

$$
\begin{aligned}
F(0) = {} & +(((-\alpha_4\alpha_2+\alpha_7\mu_V)\alpha_1+\alpha_4\alpha_2\alpha_3)\alpha_9-\alpha_3\delta P_1\mu_V)\alpha_8 \\
& -\alpha_3\delta P_2E\mu_V(\delta E+\alpha_9\gamma))-\alpha_2\alpha_4\alpha_6\alpha_9\alpha_8(\alpha_3-\alpha_3)=0
\end{aligned}
\tag{51}
$$

$$G(0) = 0 \tag{52}$$

Since

$$F'(0) = 0, \; F(0) \neq 0, \; G'(0) \neq 0 \text{ and } G(0) = 0 \tag{53}$$

Hence, F (0) G' (0) – F' (0) G (0) $\neq 0$.
Therefore, the nonzero equilibrium state is stable

3.0 Conclusion

A Mathematical model for Lassa fever was developed in this study. The Disease Free Equilibrium State (DFE) was found to be stable.

The Endemic Equilibrium State (EE) was analyzed and the result indicates it will be stable. These results show that the disease will continue to experience peaks and troughs which are in conformity with what is obtainable in the real world.

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