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A New Approach for the Solution of 12th Order Boundary Value Problems using First-Kind Chebychev Polynomials

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Abstract

This paper proposes a new approach for finding the approximate solution of the 12th order boundary value problems. The method adopts the first-kind Chebychev polynomials as trial functions. The method is excessively simple and reliable as no linearization, discretization or perturbation is required. Also, round-off, truncation and computational errors are avoided. The performance of the method is critically examined in line with other existing methods available in the literature. All computations are performed with the help of maple 18 software.

Keywords: Boundary value problems, Chebychev polynomials, Trial solution, Approximate solution
MSC2010: 65L10

1.0 Introduction

Boundary Value problems are known generally for their role in a wide range of fields in science and engineering. As such obtaining the solution of these problems is of great importance in analyzing the performance of a given model. Existing analytic methods are often restricted due to singularity transformation, linearization, discretization or perturbation. The encountered anomalies in the analytic methodshave enable researchers in recent years to have successively developed numerical methods to solve these problems with much ease. Popular numerical methods developed include, power series approximation method (PSAM) [1], Tau-collocation method [2], weighted residual method (WRM) [3], homotopy perturbation method (HPM) [4], differential transform method (DTM) [5], optimal homotopy asymptotic method (OHAM) [6], variation iteration method (VIM) [7], homotopy method [8], orthogonal collocation method [9], Galerkin method [10], etc.

This paper proposes a new method for finding the approximate solution of the 12th order boundary value problems. The method assumes an approximate solution of the form

$y(x) = \sum_{i=0}^{n-1} a_i T_i(x)\, , x \in [a, b]$,

where $T_i(x)$, $i \geq 0$ are the first-kind Chebychev polynomials. In this method, the 12thorder boundary value problem is transformed into a system of ordinary differential equations (ODEs). Thereafter, the assumed approximate solution is introduced into this system of ODEs, and is evaluated at the boundaries $x \in [a, b]$, for the unique determination of the unknown parameters a_i , $i = 0(1)(n-1)$. Substitutingthese parameters in the assumed approximate solution yields the required approximate solution.

Systematically, the method is computationally simple and reliable. Also, linearization, discretization or perturbation is not required. Round-off, truncation and computational errors are also avoided. The performance of the method is examined with regard to other existing methods[3-5] available in the literature. Resulting numerical evidences show the method is accurate, reliable, efficient and effective.

2.0 Chebychev Polynomials of the First-Kind

The Chebychev polynomials of the first-kind $T_n(x)$ is defined as

$$T_n(x) = \cos\,(n cos^{-1} x)\,, \quad x \epsilon [-1,1], \tag{1}$$

where n is non-negative.

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Equation (1) can be rewritten as

$$T_n(x) = \cos n\Phi = \sum_{k=0}^{n} C_k^{(n)} x^k \ , \qquad (2)$$

where $\Phi = cos^{-1}x \ or \ x = cos\Phi$.

Similarly, Equation (2) can also be written as

$$T_n^*(x) = \cos\left[ncos^{-1}\left(\frac{2x-a-b}{b-a}\right)\right] = \sum_{k=0}^{n} C_k^{(n)} x^k \qquad (3)$$

is the nth degree shifted Chebychev polynomials valid in the interval $[a, b]$ [2].

The recurrence relations for the first-kind Chebychev polynomials is given as

$$T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0, \qquad (4)$$

with the initial conditions, $T_0(x) = 1$ and $T_1(x) = x$, respectively.

Similarly, the recurrence formulae for the shifted first-kind Chebychev polynomials is given as

$$T_{n+1}^*(x) - 2xT_n^*(x) + T_{n-1}^*(x) = 0, \qquad (5)$$

Where$T_n^*(x)$ is as stated in equation (3),with the initial conditions, $T_0(x) = 1$ and $T_1(x) = \frac{2x-a-b}{b-a}$, respectively.

For details of the Chebychev polynomials of the second and third kind [9].

3.0 Proposed Method

Let the n[th] order boundary value problem be of the form [1]

$$y^{(n)}(x) = g(x) - f(x)y(x), \ a < x < b \qquad (6)$$

with the boundary conditions

$$y_1(a) = \lambda_1, \ y_2(a) = \lambda_1, \ y_3(a) = \lambda_2, \ldots, \ y_{2n}(a) = \lambda_{2n-1} \qquad (7)$$

and

$$y_1(b) = \beta_0, \ y_2(b) = \beta_1, \ y_3(b) = \beta_2, \ldots, \ y_{2n}(b) = \beta_{2n-1} \ . \qquad (8)$$

where $f(x), \ g(x)$ and $y(x)$ are real and continuous on $x \in [a,b]$, λ_i , β_i , $i = 0,1,2,3,\ldots,(n-1)$ are finite real constants.

Transforming equations (6) into system of ODEs such that we have

$$\frac{dy}{dx} = y_1, \frac{dy_1}{dx} = y_2, \ \frac{dy_2}{dx} = y_3, \ \frac{dy_3}{dx} = y_4 \ \ldots \frac{dy_{2n}}{dx} = g(x) - f(x)y(x), \qquad (9)$$

with the boundary conditions (7) and (8).

Let the approximate solution of (6), (7) and (8) be uniquely defined as

$$y(x) = \sum_{i=0}^{n-1} a_i T_i(x) \ , x \in [a,b], \qquad (10)$$

where a_i , $i = 0,1,2,\ldots,(n-1)$ are unknown parameters to be determined.

Using Equation (10) in (9), (8) and (7) in the interval $x \in [a,b]$, we obtain the matrix equation

$$Ax = b, \qquad (11)$$

where

$$A = \left[\sum_{i=0}^{n-1} T_i(a), \sum_{i=0}^{n-1} T_i^{'}(a), \sum_{i=0}^{n-1} T_i^{''}(a), \ldots, \sum_{i=0}^{n-1} T_i^{(2n-1)}(a), \ \sum_{i=0}^{n-1} T_i(b), \ \sum_{i=0}^{n-1} T_i^{'}(b), \ \sum_{i=0}^{n-1} T_i^{''}(b), \ldots, \sum_{i=0}^{n-1} T_i^{(2n-1)}(b)\right]^T,$$

(with $T_i(x), \ i \geq 1, \ , \ x \in [a,b]$ determined from the recurrence relation (4) or (5) and the initial conditions)

$$x = \left[a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, \ldots, a_{(n-1)},\right]^T,$$

$$b = \left[\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \ldots, \lambda_{(2n-1)}, \beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \ldots, \beta_{(2n-1)}\right]^T.$$

The unknown parameters are determined in (11) with a matrix solver, which in this case is the Gaussian elimination scheme, and substituting these parameters into (10) yields the approximate solution of (6) in the interval $[a,b]$.

The absolute error for this formulation is given as $|y(x) - y_n(x)|$, where $y(x)$ is the exact solution and $y_n(x)$ is the approximate solution.

4.0 Numerical Examples

Example 4.1

Consider the following problem in [3-5]

$$y^{(xii)}(x)+xy(x)=-(120+23x+x^3)e^x,\ \ 0<x<1, \tag{12}$$

Subject to the boundary conditions

$y(0)=0, y'(0)=1, y''(0)=0,\quad y'''(0)=-3, y^{(iv)}(0)=-8,$
$y^{(v)}(0)=-15\ y(1)=0, y'(1)=-e, y''(1)=-4e,\quad y'''(1)=-9e,$
$y^{(iv)}(1)=-16e,\ \ y^{(iv)}(1)=-25e.$

The exact solution is $y(x)=x(1-x)\exp(x)$.

Using Equation (11), the coefficient matrix A for (12) is given as

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 0 & 2 & -8 & 18 & -32 & 50 & -72 & 98 & -128 & 162 & 200 & 242 \\ 0 & 0 & 16 & -96 & 320 & -800 & 1680 & -3136 & 5376 & -8640 & 13200 & -19360 \\ 0 & 0 & 0 & 192 & -1536 & 6720 & -21504 & 56448 & -129024 & 266112 & -506880 & 906048 \\ 0 & 0 & 0 & 0 & 3072 & -30720 & 165888 & -645120 & 2027520 & -5474304 & 13178880 & -289935336 \\ 0 & 0 & 0 & 0 & 0 & 61440 & -737280 & 4730880 & -21626880 & 79073280 & -246005760 & 676515840 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 8 & 18 & 32 & 50 & 72 & 98 & 128 & 162 & 200 & 242 \\ 0 & 0 & 16 & 96 & 320 & 800 & 1680 & 3136 & 5376 & 8640 & 13200 & 19360 \\ 0 & 0 & 0 & 192 & 1536 & 6720 & 21504 & 56448 & 129024 & 266112 & 506880 & 906048 \\ 0 & 0 & 0 & 0 & 3072 & 30720 & 165888 & 645120 & 2027520 & 5474304 & 13178880 & 28993536 \\ 0 & 0 & 0 & 0 & 0 & 61440 & 737280 & 4730880 & 21626880 & 79073280 & 246005760 & 676515840 \end{bmatrix}$$

$$x=\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} \end{bmatrix}^T$$

$$b=\begin{bmatrix} 0 & 1 & 0 & -3 & -8 & -15 & 0 & -e & -4e & -9e & -16e & -25e \end{bmatrix}^T$$

Solving the above system we have

$$y(x)=x-\frac{1}{2}x^3-\frac{1}{3}x^4-\frac{1}{8}x^5-\frac{865}{4}x^6+\frac{1909}{24}ex^6+\frac{22075}{24}x^7-\frac{2707}{8}ex^7-\frac{3189}{2}x^8+\frac{7039}{12}ex^8+\frac{11241}{8}x^9 -\frac{6203}{12}ex^9+\frac{1847}{8}ex^{10}-\frac{7531}{12}x^{10}+\frac{907}{8}x^{11}-\frac{1001}{24}ex^{11}.$$

Numerical results obtained with the proposed method are given in**Table 1**. The results of the proposed method given in the **Table 1** are far superior to that obtained with WRM, HPM and DTM [3-5].

Table 1: Shows the numerical results of the proposed method and error obtained forexample 4.1 compared with the results obtained with WRM[3], HPM[4] and DTM[5].

X	Exact Solution	Approximate Solution by proposed method	Present error	WRM Error	HPM Error	DTM error
0.1	0.099465382620	0.099465382620	0	$1.*10^{-16}$	$3.00*10^{-11}$	-1.6376E-015
0.2	0.195424441300	0.195424441300	0	$1.*10^{-15}$	0	-2.0797E-013
0.3	0.283470349700	0.283470349700	0	0	$-1.00*10^{-10}$	-3.4360E-12
0.4	0.358037927500	0.358037927500	0	$2.*10^{-15}$	$2.00*10^{-10}$	-2.4556E-11
0.5	0.412180317800	0.412180317800	0	$1.*10^{-15}$	$1.10*10^{-9}$	-1.1021E-010
0.6	0.437308512000	0.437308512200	$2.*10^{-10}$	$1.*10^{-15}$	$4.40*10^{-9}$	-3.6677E-010
0.7	0.422888068500	0.422888068500	0	$1.*10^{-15}$	$1.35*10^{-8}$	-9.8945E-010
0.8	0.356086548500	0.356086548400	$1.*10^{-10}$	$1.*10^{-15}$	$3.68*10^{-8}$	-2.2839E-009
0.9	0.221364280000	0.221364280000	0	$1.*10^{-15}$	$9.01*10^{-8}$	-4.6760E-009
1.0	0.000000000000	0.000000000000	0	$6.33029146870*10^{-17}$	$2.02700*10^{-07}$	-8.7157E-009

Example 4.2

Let us solve the following problem[4,5].

Given

$$y^{(12)}(x)-y^{(3)}(x)=2e^x y^2(x),\ \ 0<x<1, \tag{13}$$

Subject to the boundary conditions

(a) $y^{(0)}(0)=1, y^{(2)}(0)=1, y^4(0)=1,\ y^6(0)=1,\ y^{(8)}=1,\ y^{(10)}(0)=1,\ y^{(0)}(1)=y^{(2)}(1)=y^{(4)}(1)=y^{(6)}(1)=y^{(8)}(1)=y^{(10)}(1)=\frac{1}{e}.$

(b) $y^{(k)}(0)=(-1)^{(k)},=1,\ y^{(k)}(1)=(-1)^{(k)}e^{-1}, k=0(1)5.$

The exact solution is $y(x) = e^{-x}$.
Using Equation (11), the coefficient matrix A for (13) is given as

$$
\begin{bmatrix}
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
0 & 0 & 16 & -96 & 320 & -800 & 1680 & -3136 & 5376 & -8640 & 13200 & -19360 \\
0 & 0 & 0 & 0 & 3072 & -30720 & 16588 & -645120 & 2027520 & -5474304 & 13178880 & -28993536 \\
0 & 0 & 0 & 0 & 0 & 0 & 1474560 & -206438840 & 153354240 & -805109760 & 3354624000 & -1180827648\;0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1321205760 & -2378170368\;0 & 2246049792\;00 & -1482392862\;720 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1902536294\;400 & -4185579847\;6800 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 16 & 96 & 320 & 800 & 1680 & 3136 & 5376 & 8640 & 13200 & 19360 \\
0 & 0 & 0 & 0 & 3072 & 30720 & 165888 & 645120 & 2027520 & 5474304 & 13178880 & 28993536 \\
0 & 0 & 0 & 0 & 0 & 0 & 1474560 & -206438840 & 153354240 & -805109760 & 3354624000 & -1180827648\;0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1321205760 & -2378170368\;0 & 2246049792\;00 & -1482392862\;720 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1902536294\;400 & -4185579847\;6800
\end{bmatrix}
$$

$$x = [a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8 \quad a_9 \quad a_{10} \quad a_{11}]^T$$

$$b = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0.367879441 \quad 0.367879441 \quad 0.367879441 \quad 0.367879441 \quad 0.367879441 \quad 0.367879441]^T$$

Solving the above system we have

$$y(x) = 1 - 1.000002689x + 0.5000000001x^2 - 0.1666622435x^3 + 0.04166666668x^4 - 0.008335519593x^5 + 0.001388888889x^6 - 0.0001978951784x^7 + 0.00002480158730x^8 - 0.00000289494963x^9 + 2.755731922\ 10^{-7}x^{10} - 1.583595276\ 10^{-8}x^{11}.$$

Numerical results obtained with the proposed method are given **Table 2**. The results of the proposed method given in the **Table 2** are far superior to that obtained with HPM and DTM[4,5].

Table 2: Shows the numerical results of the proposed method and error obtained for example 4.2 compared with the results obtained with HPM[4] and DTM [5]

X	Exact Solution	Approximate Solution by Proposed method	Present error	HPM error	DTM error
0.1	0.9048374180	0.9048371536	2.6440e-07	$-1.61*10^{-7}$	$-1.61*10^{-7}$
0.2	0.8187307531	0.8187302502	5.0290e-07	$-3.07*10^{-7}$	$-3.07*10^{-7}$
0.3	0.7408182207	0.7408175285	6.9220e-07	$-4.22*10^{-7}$	$-4.22*10^{-7}$
0.4	0.6703200460	0.6703192324	8.1360e-07	$-4.97*10^{-7}$	$-4.97*10^{-7}$
0.5	0.6065306597	0.6065298043	8.5540e-07	$-5.21*10^{-7}$	$-5.21*10^{-7}$
0.6	0.5488116361	0.5488108225	8.1360e-07	$-4.98*10^{-7}$	$-4.98*10^{-7}$
0.7	0.4965853038	0.4965846117	6.9210e-07	$-4.22*10^{-7}$	$-4.22*10^{-7}$
0.8	0.4493289641	0.4493284614	5.0270e-07	$-3.07*10^{-7}$	$-3.07*10^{-7}$
0.9	0.4065696597	0.4065693954	2.6430e-07	$-1.61*10^{-7}$	$-1.61*10^{-7}$
1.0	0.3678794412	0.3678794412	0.0000e+00	$3.00*10^{-10}$	$1.11*10^{-16}$

Now, we consider the same problem (13) but with different set of boundary conditions i.e.,
$y^{(k)}(0) = (-1)^{(k)}, = 1,\ y^{(k)}(1) = (-1)^{(k)}e^{-1}, k = 0(1)5.$
Using Equation (11), the coefficient matrix A for (13) is given as

$$y(x) = 1 - x + \frac{1}{2}x^3 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 - \frac{31739}{120}x^6 + \frac{21569}{30}e^{-1}x^6 + \frac{27595}{24}x^7 - \frac{75011}{24}e^{-1}x^7 - \frac{48797}{24}x^8 + \frac{33161}{6}e^{-1}x^8 + \frac{43721}{24}x^9 - \frac{59423}{24}e^{-1}x^9 + \frac{6727}{3}x^{10}e^{-1} - \frac{98989}{120}x^{10} + \frac{18089}{120}x^{11} - \frac{18089}{120}ex^{11}.$$

Numerical results obtained with the proposed method are given **Table 3**. The results of the proposed method given in the **Table 3** are far superior to that obtained with DTM[5].

Table 3: Shows the numerical results of the proposed method and error obtained for example 4.2 compared with the results obtained with DTM [5].

X	Exact Solution	Approximate Solution by proposed method	Present Error	DTM error
0.1	0.9048374180	0.9048374180	0.0000e+00	-4.1078e-015
0.2	0.8187307531	0.8187307530	1.0000e-10	-1.3023e-013
0.3	0.7408182207	0.7408182207	0.0000e+00	-6.7535e-013
0.4	0.6703200460	0.6703200461	0.0000e+00	-1.5278e-012
0.5	0.6065306597	0.6065306597	0.0000e+00	-1.9817e-012
0.6	0.5488116361	0.5488116360	1.0000e-10	-1.5745e-012
0.7	0.4965853038	0.4965853040	2.0000e-10	-7.1704e-013
0.8	0.4493289641	0.4493289640	1.0000e-10	-1.4222e-013
0.9	0.4065696597	0.4065696597	0.0000e+00	-4.1633e-015
1.0	0.3678794412	0.3678794412	0.0000e+00	1.2212e-015

Example 4.3

Given the following problem [5],

$$y^{(12)}(x) = y(x) - 24\cos x - 132\sin x, \ -1 < x < 1, \tag{14}$$

Subject to the boundary conditions: $y(-1) = 0, y'(-1) = 2\sin(1), y''(-1) = -4\cos(1) - 2\sin(1), \ y'''(-1) = 6\cos(1) - 6\sin(1), \ y^{iv}(-1) = 8\cos(1) + 12\sin(1), \ y^{(v)}(-1) = -20\cos(1) + 10\sin(1), y(1) = 0, y'(1) = 2\sin(1), y''(1) = 4\cos(1) + 2\sin(1), \ y'''(1) = 6\cos(1) - 6\sin(1), \ y^{(iv)}(1) = -8\cos(1) - 12\sin(1), \ y^{(iv)}(1) = -20\cos(1) + 10\sin(1)$.

The exact solution is $y(x) = (x^2 - 1)\sin(x)$.

Using Equation (11), the coefficient matrix A for (14) is given as

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 0 & 1 & -4 & 9 & -16 & 25 & -36 & 49 & -64 & 81 & -100 & 121 \\ 0 & 0 & 4 & -24 & 80 & -200 & 420 & -784 & 1344 & -2160 & 3300 & -4840 \\ 0 & 0 & 0 & 24 & -192 & 840 & -2688 & 7056 & -16128 & 33264 & -63360 & 113256 \\ 0 & 0 & 0 & 0 & 192 & -1920 & 10368 & -40320 & 126720 & -342144 & 823680 & -1812096 \\ 0 & 0 & 0 & 0 & 0 & 1920 & -23040 & 147840 & -675840 & 2471040 & -7687680 & 21141120 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 9 & 16 & 25 & 36 & 49 & 64 & 81 & 100 & 121 \\ 0 & 0 & 4 & 24 & 80 & 200 & 420 & 784 & 1344 & 2160 & 3300 & 4840 \\ 0 & 0 & 0 & 24 & 192 & 840 & 2688 & 7056 & 16128 & 33264 & 63360 & 113256 \\ 0 & 0 & 0 & 0 & 192 & 1920 & 10368 & 40320 & 126720 & 342144 & 823680 & 1812096 \\ 0 & 0 & 0 & 0 & 0 & 1920 & 23040 & 147840 & 675840 & 2471040 & 7687680 & 21141120 \end{bmatrix}$$

,

$$x = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} \end{bmatrix}^T$$

$$b = \begin{bmatrix} 0 & 2\sin(1) & -4\cos(1) - 2\sin(1) & 6\cos(1) - 6\sin(1) & 8\cos(1) + 12\sin(1) & -20\cos(1) + 10\sin(1) \\ 0 & 2\sin(1) & 4\cos(1) + 2\sin(1) & 6\cos(1) - 6\sin(1) & -8\cos(1) - 12\sin(1) & -20\cos(1) + 10\sin(1) \end{bmatrix}$$

Solving the above system we have

$$y(x) = -0.9999999759x + 1.166666521x^3 - 0.1749996354x^5 + 0.008531258334x^7 - 0.0002008013021x^9 + 0.000002632838542x^{11}.$$

Numerical results obtained with the proposed method are given **Table 4**. The results of the proposed method given in the **Table 4** are far superior to that obtained with DTM [5].

Table 4: Shows the numerical results of the proposed method and error obtained for example 4.3 compared with the results obtained with DTM[5].

X	Exact Solution	Approximate Solution by proposed method	Present Error	DTM error
0.1	-0.0988350825	-0.0988350802	2.2500e-09	-1.6376E-015
0.2	-0.1907225576	-0.1907225538	3.8000e-09	-2.0797E-013
0.3	-0.2689233881	-0.2689233840	4.1000e-09	-3.4360E-12
0.4	-0.3271114075	-0.3271114041	3.4000e-09	-2.4556E-11
0.5	-0.3595691540	-0.3595691518	2.2000e-09	-1.1021E-010
0.6	-0.3613711830	-0.3613711820	1.0000e-09	-3.6677E-010
0.7	-0.3285510205	-0.3285510202	3.0000e-10	-9.8945E-010
0.8	-0.2582481927	-0.2582481927	0.0000e+00	-2.2839E-009
0.9	-0.1488321128	-0.1488321128	0.0000e+00	-4.6760E-009
1.0	0.0000000000	0.0000000000	0.0000e+00	-8.7157E-009

5.0 Conclusion

This paper has effectively considered finding the approximate solution of the 12^{th} order boundary value problem where the first-kind Chebychev polynomials have been employed as basis or trial functions. The method has been experimented on some numerical examples andit proved to be more effective, accurate andefficient when compared with the other existing methods [3-5] available in literature.

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