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Cooperative Advertising in a Manufacturer-Distributor-Retailer Supply Chain

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Abstract

Classical cooperative advertising models centre on manufacturer-retailer relationship. Although a lot of such models have been empirically validated and applied, none has categorically considered the distributor as an integral part of cooperative advertising model. This work deals with cooperative advertising in a manufacturer-distributor-retailer supply chain in which the manufacturer is the Stackelberg leader while the distributor and the retailer are the followers using optimal control and differential game theory. It models the dynamic effect of retail advertising on sale using Sethi sale-advertising model. The work addresses channel performance through the determination of the retailer's optimal advertising strategy, and the manufacturer and distributor's optimal participation rates. It compared two channel structures: subsidised and unsubsidised channels. It shows that the manufacturer and distributor should subsidise retail advertising only if the rate of increase of their individual payoffs are twice greater than that of the retailer. By comparing these structures using the advertising effort and payoffs, it observes that the subsidised channel structure's advertising effort is higher, and consequently generates higher individual player's payoff as well as higher channel payoff than the unsubsidised channel structure. It thus recommends that the manufacturer and distributor should subsidise retail advertising.

Keywords: Cooperative advertising, Sethi sale-advertising model, Optimal control, differential game theory, Supply chain

1.0 Introduction

Advertising can be considered as a kind of mass communication employed in the promotion of the sale of a manufacturer's product, service or a message for a person or organization. Cooperative advertising is a method of advertising in which the manufacturer of a product pays for a percentage of the retail advertising expenditure. It is a means through which manufacturers, distributors and retailers reach their desired market.

Generally the retail advertising expenditure is usually shared equally up to a limit by the manufacturer, distributor and/or retailer. In actual sense this is not ideal. The view in this work is that there should be optimal strategy that should be adopted in subsidizing retail advertising. This will ensure that none of the supply chain members, especially the manufacturer, is short changed in the process of participating in retail advertising.

In the classical cooperative advertising model which was first developed in [1], and subsequent considerations, only the manufacturer and retailer were involved. But it is a known fact that a lot of manufacturers do not deal directly with their retailers. The middle men between these two parties are the distributors. Thus the manufacturer sells to the distributor who in turn sells to the retailer. The retailer sells to the end-users.

2.0 Literature Review

Cooperative advertising models can be generally grouped into two types: static and dynamic. Berger [1] was the first static game model research on cooperative advertising. The work considered cooperative advertising as discount from the manufacturer to the retailer. It was viewed as advertising allowance. He concluded that by employing cooperative advertising

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both players are better off. In extending this model [2] considered cooperative advertising in a franchise with demand uncertainty and probabilistic sales function. In their model the manufacturer and his retailer have different opinion on the anticipated sales. Their result showed that the players participate more in cooperative advertising and the channel performs better. Bergen and John [3] studied how advertising spillover, differentiation across competing manufacturers, and differentiation across competing retailers affect the participation rate. They outlined three rules that should be employed in the determination of participation rate. Huang et al [4] considered manufacturer-as-leader and partnership advertising. They found that both types of advertising are higher under partnership than under the leader-follower form. Xie and Wei [5] discoursed channel coordination by finding optimal cooperative advertising strategies and equilibrium pricing in a manufacturer-retailer distribution channel. They used a square root based function which was similarly used in [6] and [7]. This has also been used in [8]. They concluded that better coordination can be achieved using cooperative game. Yang et al [9] considered a distribution channel consisting of a manufacturer and a retailer and investigated the effect of retailer's fairness concerns. He et al [10] considered a fashion and textile supply chain with a manufacturer and a retailer. They divided the selling season into two periods. They concluded that the manufacturer should provide different subsidy rates during the two periods, and that the two periods can be coordinated by a two-way subsidy contract.

Using Stackelberg differential game [11] modeled the interaction between the manufacturer (the channel leader) and the retailer (the follower) in a decentralised channel. They resolved that it is better for the manufacturer to provide partial support than no support, and that the relative magnitude of the margins of the players determine who benefits more in a cooperative advertising relationship. Jorgensen et al [12] improved on the above model. They introduced diminishing marginal returns to goodwill into the model, and studied a Nash game where the manufacturer does not support retail advertising, and a Stackelberg game where the manufacturer as the leader supports retail advertising. Jorgensen et al [13] considered the workability of cooperative advertising program when the retailer's promotion can negatively affect the brand image. Karray and Zaccour [14] extended the work in [12]. They considered retailer's local promotional effort and the manufacturer's national advertising effort. He et al [15] considered a cooperative advertising relationship in a retail competition where the manufacturer as the Stackelberg leader supports his retailer who is competing with another retailer. The competing retailers play a Nash game. Chutani and Sethi [16] extended [15] to analyse a cooperative situation where a manufacturer sells his product through one or two independent competing retailers.

From the above considerations we observe that all currently existing cooperative advertising models are on manufacturer(s)-retailer(s) relationship. The assumption has been that the cooperative advertising supply chain is made up of only these two parties. But it is a known fact that manufacturers do not usually deal directly with their retailers. Thus there must be a third party who serves as bridge between the manufacturer of the product and the retailer who finally sells to the end-users. This third party is the distributor. Thus it is more realistic to consider a manufacturer-distributor-retailer supply chain than a manufacturer-retailer relationship. This is the centre of this work.

Now, with the consideration of the distributor as an integral part of the supply chain on cooperative advertising, this paper proposes for the first time a dynamic Stackelberg differential game model on cooperative advertising in a manufacturer-distributor-retailer supply chain. In this model only the retailer is involved in advertising the product while both the manufacturer and distributor subsidise the retail advertising effort. This work will obtain the optimal advertising and participation policies. Further, we will consider the effect of subsidy on the awareness share, players' margins and players' payoffs, and the effect of the players' margins on retail advertising effort. From these we will compare the subsidised channel and subsidised channel performance.

3.0 The Model

3.1 The Advertising Expenditures

To increase the players' payoffs the retailer engages in advertising. This increases the market share of the channel, which directly affects the payoffs. The retailer decides the advertising effort $a_R(t)$, at time t. While the distributor decides the participation rate $S_D(x)$, the manufacturer decides the participation rate $S_M(x)$, where x is the market share (awareness).

In the cooperative advertising literature the advertising cost function is usually assumed to be quadratic [11,15,16,17,18]. This means increasing marginal cost of advertising. Thus we let the advertising cost to be quadratic in the retail advertising effort $a_R(t)$. As such the retailer, distributor and manufacturer's advertising expenditures are $\left(1 - S_D(t) - S_M(t)\right)a_R(t)^2$, $S_D(t)a_R(t)^2$ and $S_M(t)a_R(t)^2$ respectively.

3.2 The Dynamics of the Awareness Share

To model the effect of advertising on sale we will use Sethi Model [19] which is a modified version of the original Vidale-Wolfe advertising model [20]. Different versions of this model (Sethi model) have been developed and extended in the cooperative advertising literature [15,21,22,23,24,25,26]. The model is given by

$$x'(t) = \beta a_R(t)\sqrt{1 - x(t)} - \delta x(t), \qquad x(0) = x_0 \in [0,1], \ t \geq 0 \qquad (1)$$

Where
$x(t)$ = market share
x_0 = initial proportion of the market share
β = advertising effectiveness parameter
δ = decay rate
Observe that $a_R(t)$ is the square root of the retail advertising expenditure $a_R(t)^2$, thus making it a concave function of the advertising expenditure.

3.3 The Leader-Followers Model Order of Events

The manufacturer who is the channel leader first announces his feedback participation rate $S_M \in [0,1]$. Next, the distributor who is the first follower also discloses his feedback participation rate $S_D \in [0,1]$. In response to these the retailer decides on his advertising effort $a_R(t)$ by solving the control problem

$$V_R(x) = \max_{\substack{a_R(t)\geq 0 \\ t\geq 0}} \int_0^\infty e^{-rt}\left\{m_R x(t) - \big(1 - S_D(t) - S_M(t)\big)\big(a_R(t)\big)^2\right\}dt \qquad (2)$$

subject to (1), where $V_R(x)$ is the retailer's payoff; r is the discount rate; $m_R$is the retailer's margin.
The distributor anticipates the retailer's reactions and incorporates them into his control problem, and solves for his participation rate $S_D(x)$. Thus the distributor's optimal control problem is given by

$$V_D(x) = \max_{0\leq S_D(t)\leq 0} \int_0^\infty e^{-rt}\left\{m_D x(t) - S_D(t)\left(a_R\big(x(t)\big|m_D, S_D(t)\big)\right)^2\right\}dt \qquad (3)$$

$$x'(t) = \beta a_R\big(x(t)\big|m_D, S_D(t)\big)\sqrt{1-x(t)} - \delta x(t), \qquad x(0) = x_0 \in [0,1],\ t\geq 0 \qquad (4)$$

Further, the manufacturer anticipates the retailer and distributor's reaction and incorporates them into his optimal control problem and solves for the participation rate $S_M(x)$. As such his optimal control problem is

$$V_M(x) = \max_{0\leq S_M(t)\leq 0} \int_0^\infty e^{-rt}\left\{m_M x(t) - S_M(t)\left(a_R\big(x(t)\big|m_M, S_M(t)\big)\right)^2\right\}dt \qquad (5)$$

$$x'(t) = \beta a_R\big(x(t)\big|m_M, S_M(t)\big)\sqrt{1-x(t)} - \delta x(t), \qquad x(0) = x_0 \in [0,1],\ t\geq 0. \ (6)$$

4.0 The Players' Strategies

The next result gives the retail advertising effort, and the participation rates of the distributor and manufacturer.
Theorem 3.1 Suppose the distributor and manufacturer's margins are given. Then the advertising effort is given by

$$a_R\big(x(t)\big|m_D, m_M, S_D(t), S_M(t)\big) = \frac{V_{Rx}\beta\sqrt{1-x(t)}}{2\big(1 - S_D(t) - S_M(t)\big)}\ , \qquad (7)$$

the distributor's subsidy is given by

$$S_D = \begin{cases} \dfrac{(2V_{Dx} - V_{Rx})\big(1 - S_M(t)\big)}{2V_{Dx} + V_{Rx}} \\ 0 \ \ \text{otherwise}\end{cases} = \begin{cases} \dfrac{2V_{Dx} - V_{Rx}}{2(V_{Dx} + V_{Mx})},\ \ 2V_{Dx} > V_{Rx},\ \ S_M(t) \neq 1 \\ 0 \ \ \text{otherwise}\end{cases}; \qquad (8)$$

and the manufacturer's subsidy is given by

$$S_M = \begin{cases} \dfrac{(2V_{Mx} - V_{Rx})\big(1 - S_D(t)\big)}{2V_{Mx} + V_{Rx}} \\ 0 \ \ \text{otherwise}\end{cases} = \begin{cases} \dfrac{2V_{Mx} - V_{Rx}}{2(V_{Dx} + V_{Mx})}\ ,\ \ 2V_{Mx} > V_{Rx},\ \ S_D(t) \neq 1 \\ 0 \ \ \text{otherwise}\end{cases}. \qquad (9)$$

Proof: From (1) and (2) we have the HJB equation.

$$rV_R(x) = \max_{\substack{a_R(t)\geq 0 \\ t\geq 0}} \left\{m_R x(t) - \big(1 - S_D(t) - S_M(t)\big)\big(a_R(t)\big)^2 \right.$$
$$\left. + V_{Rx}\left[\beta a_R(t)\sqrt{1-x(t)} - \delta x(t)\right]\right\}. \qquad (10)$$

Differentiating wrt a_R and equating to 0 we have that

$$-2\big(1 - S_D(t) - S_M(t)\big)a_R(t) = -V_{Rx}\beta\sqrt{1-x(t)}$$

which leads to (7).
Now, putting (7) into (10) we

$$rV_R(x) = m_R x(t) - \big(1 - S_D(t) - S_M(t)\big)\left[\frac{V_{Rx}\beta\sqrt{1-x(t)}}{2\big(1 - S_D(t) - S_M(t)\big)}\right]^2$$

$$+V_{Rx}\left\{\beta\frac{V_{Rx}\beta\sqrt{1-x(t)}}{2\big(1-S_D(t)-S_M(t)\big)}\sqrt{1-x(t)}-\delta x(t)\right\}$$
$$=m_Rx(t)+\frac{(V_{Rx})^2\beta^2\big(1-x(t)\big)}{4\big(1-S_D(t)-S_M(t)\big)}+V_{Rx}\delta x(t) \qquad (11)$$

From (3) and (4) we have the HJB equation

$$rV_D(x)=\max_{0\le S_D(t)\le 1}\left\{m_Dx(t)-S_D(t)\left(a_R\big(x(t)\big|m_D,S_D(t)\big)\right)^2\right.$$
$$\left.+V_{Dx}\left[\beta\left(a_R\big(x(t)\big|m_D,S_D(t)\big)\right)\sqrt{1-x(t)}-\delta x(t)\right]\right\}. \qquad (12)$$

Using (7) in (12) we have

$$rV_D(x)=\max_{0\le S_D(t)\le 1}\left\{m_Dx(t)-S_D(t)\frac{(V_{Rx})^2\beta^2\big(1-x(t)\big)}{4\big(1-S_D(t)-S_M(t)\big)^2}\right.$$
$$\left.+\frac{V_{Dx}V_{Rx}\beta^2\big(1-x(t)\big)}{2\big(1-S_D(t)-S_M(t)\big)}-V_{Dx}\delta x(t)\right\} \qquad (13)$$

Maximizing (13) wrt $S_D(t)$ we have

$$\frac{(V_{Rx})^2\beta^2\big(1-x(t)\big)}{4}\frac{\big(1-S_D(t)-S_M(t)\big)^2-2S_D(t)\big(1-S_D(t)-S_M(t)\big)(-1)}{\left(\big(1-S_D(t)-S_M(t)\big)^2\right)^2}$$
$$+\frac{V_{Dx}V_{Rx}\big(1-x(t)\big)}{2}\frac{1}{\big(1-S_D(t)-S_M(t)\big)^2}=0$$
$$\Rightarrow \quad S_D=\begin{cases}\dfrac{(2V_{Dx}-V_{Rx})\big(1-S_M(t)\big)}{2V_{Dx}+V_{Rx}}, & 2V_{Dx}>V_{Rx},\quad S_M(t)\neq 1\\ 0 & \text{otherwise}\end{cases} \qquad (14)$$

Using (7) in (5)-(6) we have the HJB

$$rV_M(x)=\max_{0\le S_M(t)\le 1}\left\{m_Mx(t)-S_M(t)\frac{(V_{Rx})^2\beta^2\big(1-x(t)\big)}{4\big(1-S_D(t)-S_M(t)\big)^2}\right.$$
$$\left.+\frac{V_{Mx}V_{Rx}\beta^2\big(1-x(t)\big)}{2\big(1-S_D(t)-S_M(t)\big)}-V_{Mx}\delta x(t)\right\} \qquad (15)$$

Maximizing (15) wrt S_M we have

$$\Rightarrow \quad S_M=\begin{cases}\dfrac{(2V_{Mx}-V_{Rx})\big(1-S_D(t)\big)}{2V_{Mx}+V_{Rx}}, & 2V_{Mx}>V_{Rx},\quad S_D(t)\neq 1\\ 0 & \text{otherwise}\end{cases} \qquad (16)$$

Thus from (14) and (16) we have

$$S_D=\left(\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\right)(1-S_M)=\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}-\left(\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\right)S_M \qquad (17)$$
$$S_M=\left(\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}\right)(1-S_D)=\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}-\left(\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}\right)S_D \qquad (18)$$

Using (18) in (19) we have

$$S_D=\frac{\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}-\left(\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\right)\left(\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}\right)}{1-\left(\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\right)\left(\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}\right)} \qquad (19)$$
$$=\frac{2V_{Dx}-V_{Rx}}{2(V_{Dx}+V_{Mx})} \qquad (20)$$

Thus for $S_M>0$, we have that

$2V_{Dx}>V_{Rx}$ and $V_{Dx}+V_{Mx}>0.$ (21)

Now using (19) in (18) we

$$S_M = \frac{\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}} - \left(\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\right)\left(\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}\right)}{1-\left(\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\right)\left(\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}\right)} \tag{22}$$

$$= \frac{2V_{Dx}-V_{Rx}}{2(V_{Dx}+V_{Mx})} \tag{23}$$

Therefore, for $S_D > 0$ we have

$$2V_{Mx} > V_{Rx} \quad \text{and} \quad V_{Dx}+V_{Mx} > 0. \tag{24}$$

From (7) we observe that the retail advertising effort is influenced by a lot of factors. First, we observe that as the awareness share increases the advertising effort reduces, and the retailer does not need to advertise when everyone is aware of the product (that is when $x = 1$). Thus advertising effort is inversely proportional to market awareness.

Another factor affecting the advertising effort is the subsidy from the distributor and manufacturer. As the subsidy increases, the advertising effort increases. It is important to note that $0 \le S_D + S_M \le 1$. In fact neither of them nor both should totally subsidise retail advertising, because that would amount to $S_D = 1$ (with $S_M = 0$), $S_M = 1$ (with $S_D = 0$) or $S_D + S_M = 1$, which will lead to a_R becoming unbounded. This is unrealistic. Thus it is ideal to have $0 \le S_D + S_M < 1$.

The other factors are the advertising effectiveness and the rate of increase of the retailer's payoff. They are in direct proportion with advertising. These greatly motivate the retailer to advertise the product.

From (8) and (9) we infer that the distributor and manufacturer's subsidy to the retailer are only possible if the rates of increase of their payoffs are twice greater than the rate of increase of the retailer's payoff. Further we also observe that the manufacturer's subsidy influences the distributor's subsidy, and vice versa. Simply put, neither the distributor nor the manufacturer should totally subsidise retail advertising. That is $S_D,\ S_M \neq 1$. It is important to observe that total subsidy by either the distributor or the manufacturer would leave no room for the other player to participate in retail advertising. Thus if both must participate then none must totally subsidise retail advertising.

5.0 Stackelberg Equilibrium for Unsubsidised Retail Advertising

Theorem 5.1 Suppose that neither the distributor nor the manufacturer supports retail advertising. Then the retailer's advertising effort is given by

$$a_R(x(t)|m_R, m_D, m_M) = \frac{B_R\beta\sqrt{1-x(t)}}{2}, \tag{25}$$

and the players' payoffs are

$$V_R(x) = A_R + B_R x, \tag{26}$$

$$V_D(x) = A_D + B_D x, \tag{27}$$

$$V_M(x) = A_M + B_M x, \tag{28}$$

where

$$B_R = \frac{2\left[-(r+\delta)+\sqrt{(r+\delta)^2+\beta^2 m_R}\right]}{\beta^2}, \tag{29}$$

$$B_D = \frac{2m_D}{2(r+\delta)+\beta^2 B_R}, \tag{30}$$

$$B_M = \frac{2m_M}{2(r+\delta)+\beta^2 B_R}, \tag{31}$$

$$A_R = \frac{\beta^2 B_R^2}{4r}, \tag{32}$$

$$A_D = \frac{\beta^2 B_D B_R}{2r}, \tag{33}$$

$$A_M = \frac{\beta^2 B_M B_R}{2r} \tag{34}$$

Proof: Since $S_D(t) = S_M(t) = 0$ we have that (11) becomes

$$rV_R(x) = m_R x(t) + \frac{(V_{Rx})^2\beta^2(1-x(t))}{4} - V_{Rx}\delta x(t); \tag{35}$$

(13) becomes

$$rV_D(x) = m_D x(t) + \frac{\beta^2 V_{Dx}V_{Rx}(1-x(t))}{2} - V_{Dx}\delta x(t); \tag{36}$$

and (15) becomes

$$rV_M(x) = m_M x(t) + \frac{\beta^2 V_{Mx}V_{Rx}(1-x(t))}{2} - V_{Mx}\delta x(t). \tag{37}$$

Let

$V_R(x) = A_R + B_R x$, (38)

$V_D(x) = A_D + B_D x$, (39)

$V_M(x) = A_M + B_M x$, (40)

$\Longrightarrow \quad V_{Rx} = B_R , \quad V_{Dx} = B_D, \quad V_{Mx} = B_M$. (41)

Since $S_D(t) = S_M(t) = 0$, using (41) in (11) we have (25).

Using (38) and (41) in (35) we have

$$r(A_R + B_R x) = m_R x(t) + \frac{\beta^2 B_R^2 \big(1 - x(t)\big)}{4} - \delta B_R x(t). \quad (42)$$

Equating the coefficients of x we have

$$B_R = \frac{-4(r+\delta) \pm \sqrt{[4(r+\delta)]^2 - 4\beta^2(-4m_R)}}{2\beta^2} . \quad (43)$$

From (38) we have that B_R must not be negative. As such we have (29).

Equating constants we (32).

Using (39) and (41) in (36) we have

$$r(A_D + B_D x) = m_D x(t) + \frac{\beta^2 B_D B_R \big(1 - x(t)\big)}{2} - B_D \delta x(t). \quad (44)$$

Equating the coefficients of $x$and constant in (44) we have (30) and (33) respectively.

Using (40) and (41) in (37) we have

$$r(A_M + B_M x) = m_M x(t) + \frac{\beta^2 B_M B_R \big(1 - x(t)\big)}{2} - B_M \delta x(t). \quad (45)$$

Equating the coefficients of $x$and constant in (45) we have (31) and (34) respectively.■

The retail advertising strategy given in (25) is in consonance with our earlier assertion that a_R depends on the rate of increase of the retailer's payoff B_R, the advertising effectiveness and the unaware proportion of the market. Further we note that this B_R is very important to the retailer as can be seen in (26). Similarly B_D and B_M are very important to the distributor and manufacturer as can be seen from (27) and (28) respectively. It is therefore necessary for the players to be acquainted with how these rates can be increased.

Now, from (29) we observe that the only direct instrument within the retailer's reach through which he can increase B_R is his the margin m_R Thus the retailer can increase his margin if he intends to increase his payoff. This also applies to the distributor and manufacturer.

6.0 Stackelberg Equilibrium for Subsidised Retail Advertising

Theorem 5.1 Suppose that the distributor and manufacturer support retail advertising. Then the retailer's advertising effort is given by

$$a_R\big(x(t)\big|m_R, m_D, m_M, S_D(t), S_M(t)\big) = \frac{\beta(B_D + B_M)\sqrt{1 - x(t)}}{2} ; \quad (46)$$

the players' payoffs are given by

$V_R(x) = A_R + B_R x$, (47)

$V_D(x) = A_D + B_D x$, (48)

$V_M(x) = A_M + B_M x$,(49)

where

$$B_R = \frac{4m_R}{4(r+\delta) + \beta^2(B_D + B_M)} , \quad (50)$$

$$B_D = \frac{8m_D - \beta^2 B_R B_M}{8(r+\delta) + 2\beta^2(B_D + B_M) + \beta^2 B_R} , \quad (53)$$

$$B_M = \frac{8m_M - \beta^2 B_R B_D}{8(r+\delta) + 2\beta^2(B_D + B_M) + \beta^2 B_R} , \quad (52)$$

$$A_R = \frac{\beta^2 B_R (B_D + B_M)}{4r} , \quad (53)$$

$$A_D = \frac{\beta^2 (B_D + B_M)(2B_D + B_R)}{8r} , \quad (54)$$

$$A_M = \frac{\beta^2 (B_D + B_M)(2B_M + B_R)}{8r} . \quad (55)$$

Proof: From (19) and (22) we have that

$$-S_D(t)-S_M(t)=-\frac{\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}-\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}}{1-\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}}-\frac{\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}-\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}}{1-\frac{2V_{Dx}-V_{Rx}}{2V_{Dx}+V_{Rx}}\frac{2V_{Mx}-V_{Rx}}{2V_{Mx}+V_{Rx}}},$$

$$\Rightarrow \quad 1-S_D(t)-S_M(t)=\frac{V_{Rx}}{V_{Dx}+V_{Mx}}. \tag{56}$$

Now, let

$V_R(x)=A_R+B_Rx$ (57)

$V_D(x)=A_D+B_Dx$(58)

$V_M(x)=A_M+B_Mx$ (59)

$$\Rightarrow \quad V_{Rx}=B_R, \qquad V_{Dx}=B_D, \qquad V_{Mx}=B_M. \tag{60}$$

Using (60) in (56) and substituting the result into (7) we have (46).

Using (56) in (11), and then Using (57) and (60) in the resulting expression we have

$$r(A_R+B_Rx)=m_Rx(t)+\frac{\beta^2B_R(B_D+B_M)\big(1-x(t)\big)}{4}-B_R\delta x(t). \tag{61}$$

Equating the coefficients of x and constants in (61) we have (50) and (53) respectively.

Using (8) and (9) in (13), and thereafter using (58) and (60) in the resulting expression we have

$$r(A_D+B_Dx)=m_Dx(t)-\frac{\beta^2(2B_D-B_R)(B_D+B_M)\big(1-x(t)\big)}{8}$$
$$+\frac{\beta^2B_D(B_D+B_M)\big(1-x(t)\big)}{2}-B_D\delta x(t). \tag{62}$$

Equating the coefficients of $x$and constants in (62) we have (51) and (54) respectively.

Using (8) and (9) in (15), and thereafter using (59) and (60) in the resulting expression we have

$$r(A_M+B_Mx)=m_Mx(t)-\frac{\beta^2(2B_M-B_R)(B_D+B_M)\big(1-x(t)\big)}{8}$$
$$+\frac{\beta^2B_M(B_M+B_D)\big(1-x(t)\big)}{2}-B_M\delta x(t). \tag{63}$$

Equating the coefficients of $x$and constants in (63) we have (52) and (55) respectively. ∎

From (46) we observe that with subsidy the retail advertising effort depends on the growth rate of the payoffs of the distributor and manufacturer. Thus retail advertising increase with their increasing payoffs. As in the unsubsidised caseB_R, $B_D$and $B_M$are of much importance to the players as can be seen in (47), (48) and (49). Equations (50), (51) and (52) respectively show that their margins are possible tools through which increases in these parameters can be achieved. Further it is pertinent to note that the same parameters that affect the payoffs which were treated in the unsubsidised case also affect the payoff in the subsidised case.

As will be seen later the subsidised payoff is higher than the unsubsidised payoff. This is due to increased awareness resulting from the improvement (increase) in the advertising effort, which is obvious from the replacement of B_R [(associated with m_R) in the unsubsidised case] with B_D and B_M [(associated with m_D and m_M respectively) in the subsidised case]. For clarity recall that

$$a_{R(S_D,S_M=0)}=\frac{\beta B_R\sqrt{1-x(t)}}{2}$$

and

$$a_{R(S_D,S_M>0)}=\frac{\beta B_D\sqrt{1-x(t)}}{2}+\frac{\beta B_M\sqrt{1-x(t)}}{2},$$

where the subscripts $(S_D,S_M=0)$ and $(S_D,S_M>0)$ represent unsubsidised and subsidised advertising efforts respectively.

Since the manufacturer is the Stackelberg leader, it is natural that his margin be larger than those of the other players. Thus by the nature of equation (51) and (52)

$B_M\geq B_D$(since $m_M\geq m_D$).

Therefore

$$\frac{\beta B_D\sqrt{1-x(t)}}{2}+\frac{\beta B_M\sqrt{1-x(t)}}{2}\geq\frac{\beta B_D\sqrt{1-x(t)}}{2}+\frac{\beta B_D\sqrt{1-x(t)}}{2}=\frac{2\beta B_D\sqrt{1-x(t)}}{2}$$

But for subsidy

$2B_D>B_R$ (that is $B_R<2B_D$)

$$\Rightarrow \quad \frac{\beta B_R\sqrt{1-x(t)}}{2}<\frac{\beta B_D\sqrt{1-x(t)}}{2}+\frac{\beta B_D\sqrt{1-x(t)}}{2}$$

$$\leq \frac{\beta B_D\sqrt{1-x(t)}}{2}+\frac{\beta B_M\sqrt{1-x(t)}}{2}$$
$$\Rightarrow \quad \frac{\beta B_R\sqrt{1-x(t)}}{2}<\frac{\beta(B_D+B_M)\sqrt{1-x(t)}}{2}$$
$$\Rightarrow \quad a_{R(S_D,S_M=0)} < a_{R(S_D,S_M>0)}.$$

7.0 Subsidised and Unsubsidised Market (Awareness) Share

Using equation (7) in equation (1) we have

$$x'(t)=\beta\frac{B_R\beta\sqrt{1-x(t)}}{2\left(1-S_D(t)-S_M(t)\right)}\sqrt{1-x(t)}-\delta x(t)$$
$$=\frac{\beta^2 B_R}{2\left(1-S_D(t)-S_M(t)\right)}-\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}x(t) \qquad (64)$$

Using the integrating factor

$$I.F.=e^{\int\left(\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}\right)dt}=e^{\left(\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}\right)t} \qquad (65)$$

Multiplying (64) by (65) leads to

$$e^{\left(\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}\right)t}x'(t)+e^{\left(\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}\right)t}\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}x(t)$$
$$=e^{\left(\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}\right)t}\frac{\beta^2 B_R}{2\left(1-S_D(t)-S_M(t)\right)}$$

Integrating we have

$$x(t)=\frac{\beta^2 B_R}{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}+\frac{\left[\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta\right]x_0-\beta^2 B_R}{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}$$
$$\times\exp\left(-\frac{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}{2\left(1-S_D(t)-S_M(t)\right)}\right) \qquad (66)$$

Thus as $t\longrightarrow\infty$, (66) becomes

$$x(t)=\frac{\beta^2 B_R}{\beta^2 B_R+2\left(1-S_D(t)-S_M(t)\right)\delta}$$

8.0 Numerical Discussion

We recall that by definition $\beta\in[0,1]$. As such we let $\beta=0.6$. Also the decay rate $\delta<\beta$. Therefore, we let $\delta=0.2$. Since the players are foresighted, and the game is played on an infinite horizon, the discount rate must be very low. Thus we let $r=0.05$. The hierarchical decision process in which the manufacturer has the first mover's advantage followed by the distributor and then the retailer requires that $M_M\geq M_D\geq M_R$. As such we let $M_R=4, M_D=5$ and $M_M=6$

8.1 The Effect of Subsidy on Awareness Share

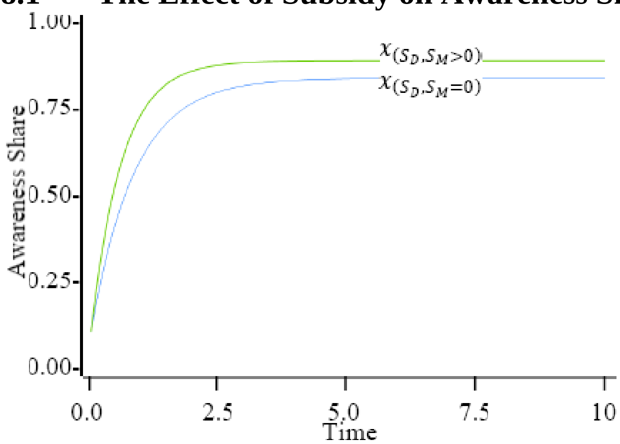


Figure 1: Subsidised and unsubsidised awareness share

From Figure 1 we observe that the awareness share increases with the provision of subsidies. It is therefore advisable that retail advertising effort should be subsidised. Now, the difference in awareness increased with subsidy until it stabilized in the long-run. This increasing difference leads to increasing payoffs showing that the retail effort should be subsidised. This is further explained in subsection 8.2.

8.2 The Relationship between the Awareness Share and Participation (Subsidy)

Figure 2 shows the relationship between the awareness share and subsidy for different values of t. Observe that for each value of time, increase in subsidy leads to increase in awareness. As time progresses the awareness converges to $x_{(s_D,s_M>0)\infty}$ (A constant long-run value) as $t \longrightarrow \infty$

This is further explained by Figure 3 which shows the relationship between awareness, subsidy and time.

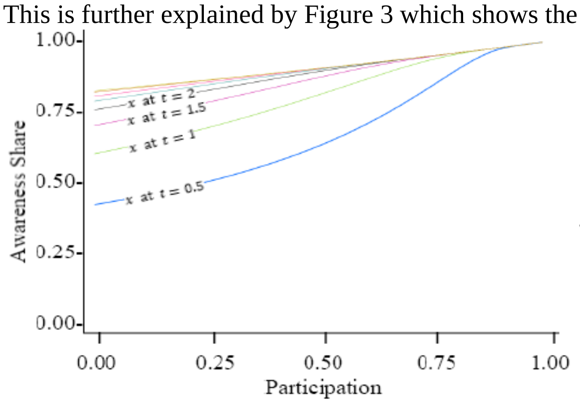


Figure 2: Relationship between the awareness share and participation forshare, using different times

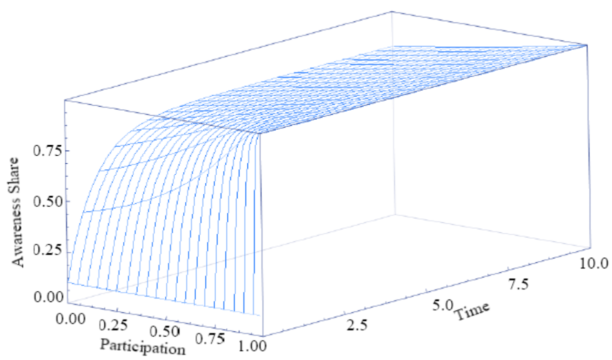


Figure 3: Relationship between the awareness participation and time using different times

8.3 Players' Payoffs

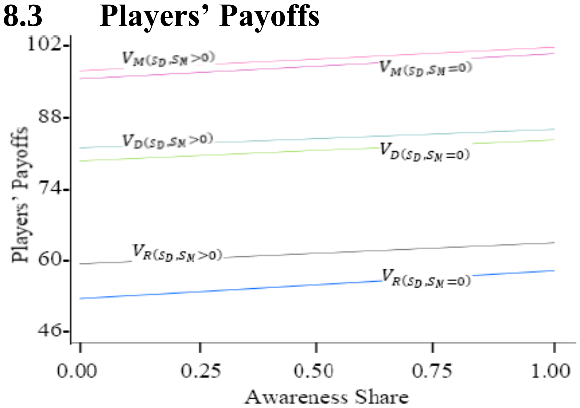


Figure 4: Manufacturer, distributor and channels' values (payoffs)

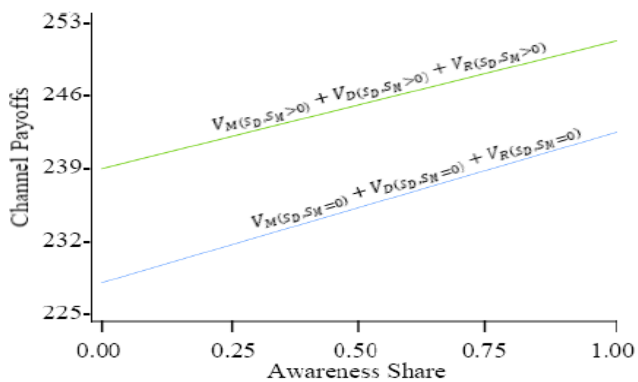


Figure 5: Subsidised and unsubsidised retailer's payoffs for subsidised and unsubsidised retail advertising.

The individual payoffs for subsidised retail advertising are larger than the unsubsidised payoffs. This is obvious in Figure 4. A reflection of this is clear in Figure 5, where the subsidised channel payoff is larger than the unsubsidised channel payoff.

8.4 Effect of Advertising Effort on Payoffs

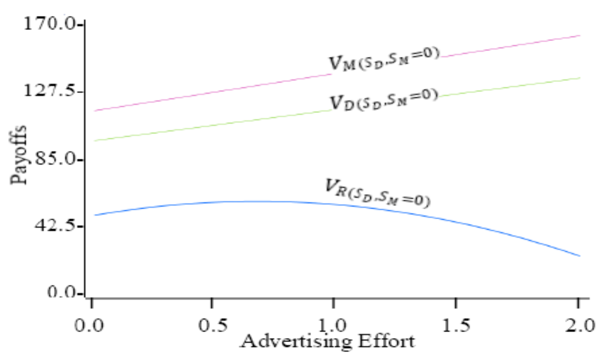


Figure 6: Effect of advertising effort on payoffs for unsubsidised retail advertising

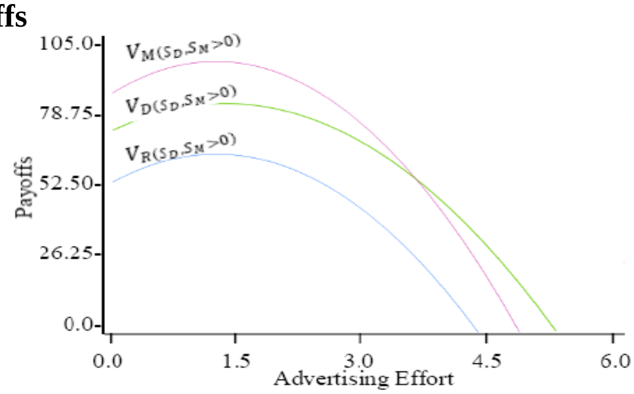


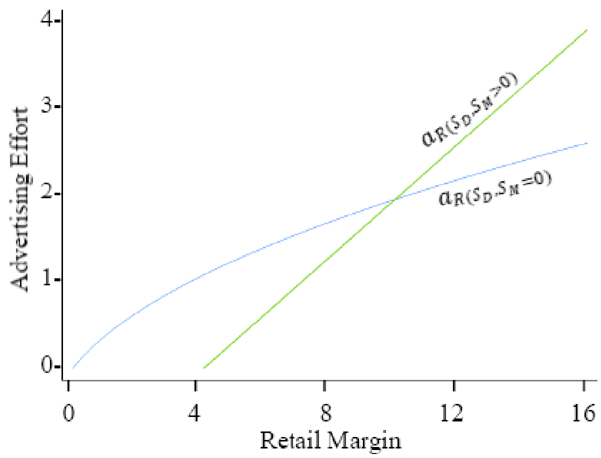
Figure 7: Effect of advertising effort on payoffs for subsidised retail advertising

With non-provision of subsidy, neither the manufacturer nor the distributor is directly or indirectly involved in advertising, and has no advertising expenditure. As such their values continue to increase with every effort by the retailer. On the other hand due to the retail advertising expenditure his value increases to a point and then starts exhibitingdiminishing returns. This is clear from Figure 6.

However looking at Figure 7, we observe that with the provision of subsidy by the manufacturer and distributor all the players' payoffs increase with retail advertising effort, but starts reducing after certain advertising level. This reduction in the manufacturer and distributor's payoffs (which is not the case with non-provision of subsidy) is as a result of their involvement in retail advertising. Thus diminishing return sets in after a certain level of advertising is attained. Clearly each player has a certain retail advertising level where his payoff is at a maximum.

Bearing in mind that in both cases a_R is the retailer's strategy, he can decide on the extent to which he wants to be involved in advertising. Such a level of advertising will determine the manufacturer and distributor's feasible maximum payoffs. Now, his maximum payoff is obtained at the turning points of $V_R$in Figure 6 and Figure 7. Observe that in both cases V_M and V_D are still increasing whereas V_R is already reducing. As such, by increasing his advertising effort above this level, he would be losing value(payoff), thus short changed.But, it would be irrational for the retailer to satisfy the other players to his detriment. Thus advertising is terminated at this point and the corresponding values become optimal for each player.

8.5 Advertising Effort Reactions to Change in Margins



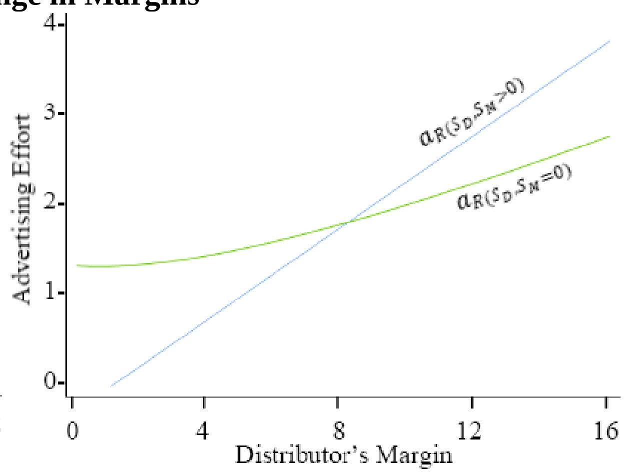


Figure 8: Relationship between advertising advertising effort distributor's margins

Figure 9: Relationship between effort retailer's margins

From Figure 8we observe that the advertising effort increases with the retail margin both for subsidised and unsubsidised cases. Further not until $m_R = 1.3017$ the rate of increase in retail effort is lower for subsidised.

A similar scenario plays out inFigure 9 and Figure 10 where the retailer's advertising effort also increases with the players' margins. However, in both cases the subsidised is higher than the unsubsidised until $m_D = 6.4583$ and $m_M = 8.0694$.

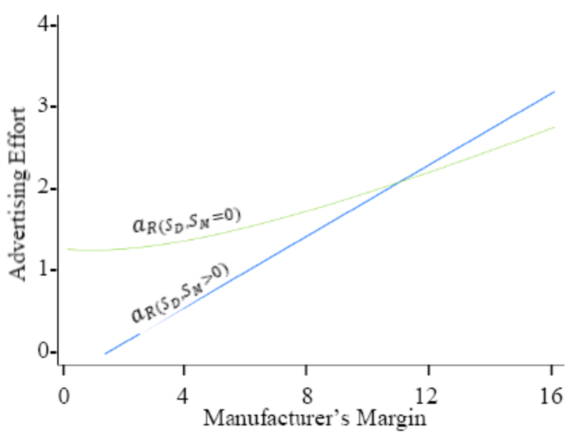


Figure 10: Relationship between Advertising Effort Manufacturer's Margins

8.6 The Effect of Margins on Participation (Subsidy)

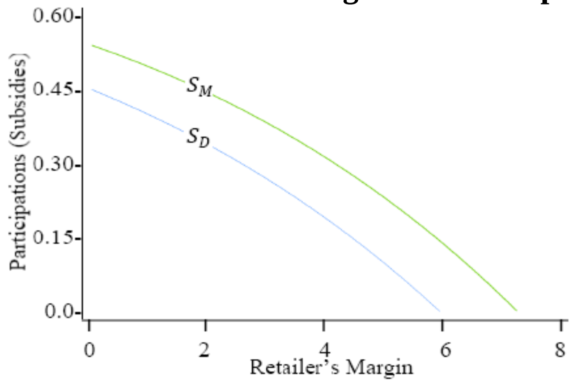
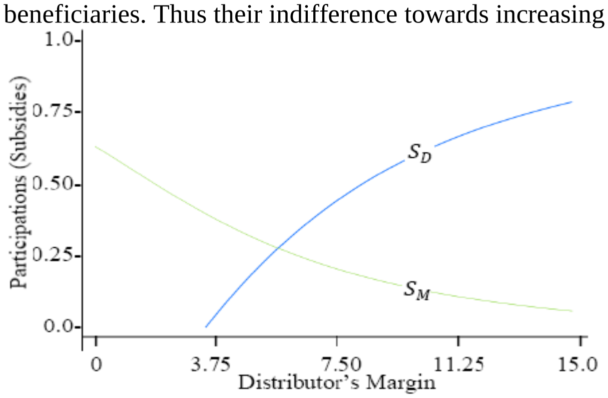


Figure 11: Effect of Retailer's Margins on Participation (Subsidy)

Figure 11 shows that the subsidies from the manufacturer and distributor reduce with increase in the retailer's margin. Ideally this is not out of place because with increasing retail margin they (the manufacturer and the distributor) are not the direct beneficiaries. Thus their indifference towards increasing their support with increasing retail margin is not out of place.



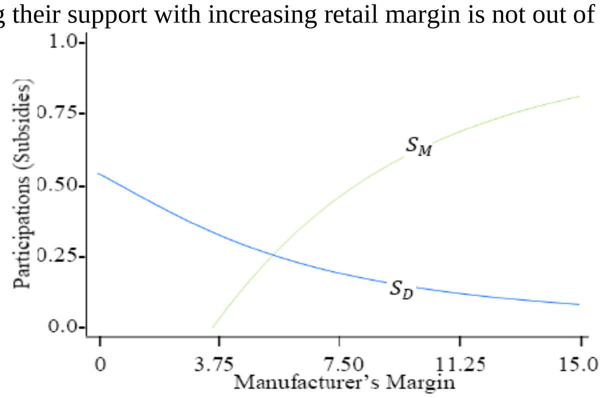


Figure 12: Effect of retailer's margins on on Participation (Subsidy)

Figure 13: Effect of Retailer's Margins participation (Subsidy)

In Figure 12 we observe that the distributor's subsidy to the retailer increases with his (distributor's) margin while the manufacturer's subsidy reduces with the distributor's margin. A similar trend plays out Figure 13where increase in manufacturer's margin results in increase in subsidy from the manufacturer, but decrease in subsidy from the distributor.

9.0 Concluding Remarks

In this work we set out to develop cooperative advertising models in a manufacturer-distributor-retailer setting using differential game theory. We used the models to consider the relationship between subsidy and awareness share (showing how subsidies affect the awareness share). We further considered its effect on the players' margins and payoffs.

This work has extended the cooperative advertising literature by categorically incorporating the distributor into the literature as a link between the classical (traditional) cooperative advertising supply chain members (the manufacturer and retailer). It considered a situation where only the retailer is involved in advertising while the distributor and manufacturer participate indirectly in advertising by subsidising retail advertising.

Two types of equilibria were identified. The first is on a situation where retail advertising is not subsidised, while the other deals with a situation where subsidy is provided by both the manufacturer and the distributor.

We obtained a number of results. We saw that the retail advertising effort depends on a number of factors, particularly the awareness share and subsidies which must not be total. We also observed that both the distributor and the manufacturer have great influence on each other's participation in retail advertising, and these two players should only subsidise retail advertising if the rate of increase of their individual payoffs are twice greater than that of the retailer.

Expectedly, both the distributor and manufacturer reduce their subsidies with retail margin, while each player's subsidy increases with his margin but decreases with the other's margin. Further, we saw that the awareness share increases with subsidy. This also leads to larger payoff for the players. Also, the advertising effort increases with all the players' margins.

Considering both channels, we observe that with subsidy the retail advertising effort is higher, leading to larger payoffs for the subsidised channel. Thus the subsidised channel performs better than the unsubsidised channel.

This work has some limitations. We assumed a trilateral monopolistic situation. A possible extension can involve a number of competing manufacturers, distributors and retailers. In this work the manufacturer is the Stackelberg leader. An extension can consider a situation where the retailer or the distributor can dictate terms to the manufacturer. Further, it is possible to consider a situation where a product may be new in the target market, or has better competing substitute(s). This may require an aggressive advertising approach. This can call for all the players'involvement in advertising, while the distributor and manufacturer also participate in retail advertising. An extension can consider integrated channel structure where the manufacturer, the distributor and the retailer play a Nash game. Using this extension, the performance of the channel structures can be compared based on the advertising efforts, the awareness shares and the payoffs.

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