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Use of Linear Programming in Allocating Buses to Routes

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Abstract

The objective of the study was to allocate the available buses of the transport authority to the authority's service intra and inter states routes in a manner that will yield optimum profit, taking all the constraints into consideration. The problem was modeled using Linear Programming (LP) and the TORA software yielded a maximum objective value of N897,214 per day after 20 iterations, which is a better result compared to the existingintuitive schedule by the authority with N766,046 per day. The difference of N131,168 (additional income) is a significant change that a transport company cannot ignore.

Keywords: NSTA, TORA, Linear Programming, Transport Service, Bus(es), Route(s), Objective Function

1.0 Introduction

Niger State Transport Authority (NSTA) is a state-own transport service operator in Minner, Niger state, Nigeria with the following seven commuting services:

i. Inter State services
ii. Intra State services
iii. City service within Minna municipal, Bida and Kontagora towns
iv. Towing, Haulage and Hire services to all parts of Nigeria
v. Ferry services from Rofia across to Zamare in Kebbi State and from Shiroro across to Lakpama, in Niger State
vi. Passenger Boat service from kofa to kabo in Suleja area council
vii. Civil servants and students Bus services within Minna municipal[1].

The first two services – inter and intra state services under consideration have the following routes:

Inter States Routes	**Intra State Routes**
MINNA-ILORIN	MINNA-BIDA
MINNA-LAGOS	MINNA-MOKWA
MINNA-KADUNA	MINNA-NEW BUSA
MINNA-KANO	MINNA-KTAGORA
MINNA-MAKURDI	MINNA-KAGARA
MINNA-JOS	MINNA-WUSHISHI
MINNA-BAUCHI	MINNA-SULEJA
MINNA-SOKOTO	MINNA-SERKI PAWA
MINNA-ZAMFARA	MINNA-LAPAI
MINNA-KATSINA	

The objective of this study is to applylinear programming model to optimally allocate the available buses of the Transport Authority to the service routes. This objective is predicated on the following background:

Niger State transport Authority (NSTA) operates ten intra and ten inter States routes transport serviceas can be seen above. The authority has 80, fourteen sitter buses (Fifty Hiace and thirty King Long Buses); from which six of the Hiace buses are usually on stand-by for emergency hire purposes. Twenty-five of the buses are used for city service, Civil servants and

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students Bus services within Minna municipal, Bida and Kontogora towns of the state. Forty-Nine are used for commuting the intra and inter states routes. According to findings, daily, the buses incur costs in four ways: fuel, percentage parking levy, routine service and maintenance (repair). Buses going to New-Busa, Mokwa, Ilorin and Lagos incur thrice the cost of repair (maintenance) due to bad road. The authority might want to turn from intuition and consider mathematically, an optimized way of allocating buses to routes in order to obtain daily optimal gain though there are seasonal variations in terms of passenger patronage during Christmas and Salla periods.

2.0 Brief History of NSTA

According to [1], Niger State Transport Authority was established on the 11th August, 1988 under edict No. 11 of 1988 enacted by the State Government. A task force was immediately constituted, headed by a military administrator to oversee the affairs of the organization. Since inception, as argued in[1], the authority has become a household name and has lived up to expectation been the only viable transport venture in the state and one of the best in the country vis-à-vis its contemporaries.

NSTA has the following organizational chart

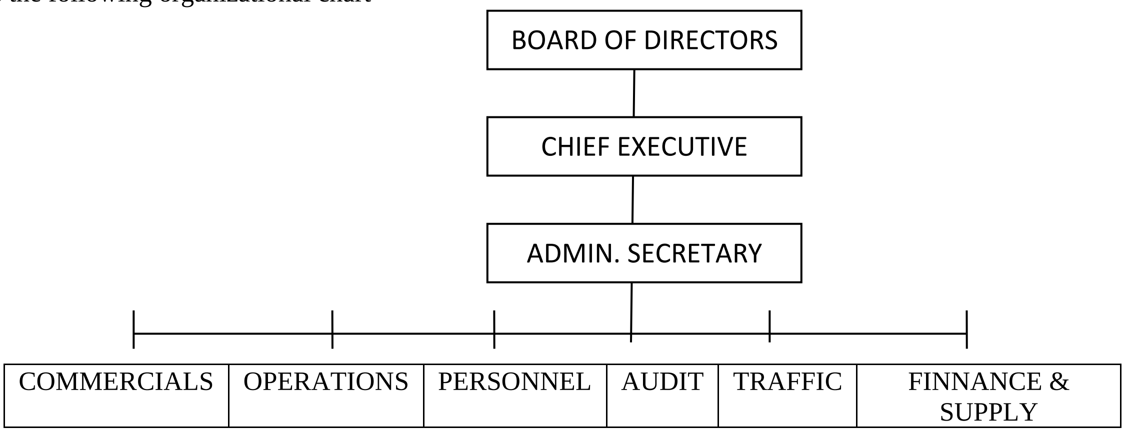


Figure 1: Organizational Chart [1].

The staff strength of NSTA currently stands at 206 under the Chief Executive/General Manager and Admin secretary.

According to [1], Passengers are to adhere to the following guidelines:

i. The authority does not accept liability for loss of goods. Passengers are therefore advised to take good care of their goods/properties while waiting to board our vehicles and while on transit
ii. Preaching is strictly prohibited in our vehicles
iii. The habit of smoking in our vehicles is strictly prohibited
iv. You can only enter our vehicle when you have paid correct money and obtained a ticket for the journey
v. Tickets should be in passengers' possession until the end of the journey
vi. Ticket can only be issued to prospective commuters when they maintain a single queue
vii. Heavy loads, bags and boxes are paid for and tickets obtained before they are loaded in our vehicles
viii. Commuters are expected to present their tickets on demand to our traffic inspectors during the course of the journey when asked to do so
ix. Intentional damage to our vehicle seats, glasses etc. will not be accepted
x. Female passengers are seated at the back of the vehicle while male are seated at the front
xi. Hawking of goods inside our vehicles is strictly prohibited
xii. Passengers are to help us so that we can serve them better

3.0 Brief Literature Review on Linear Programming

Linear Programming (LP) is an optimal decision making tool in which theobjective is a linear function and the constraints on the decision problem arelinear equalities and/or inequalities. It is the most commonly applied formof constrained optimization. The four main elements of any constrained optimization are decision variables, objective function, constraints and variablebounds. In LP, all the mathematical expressions for the objective functionand constraints are linear. One might imagine that the restriction to linearmodels severely limits the ability to model real-world problems; but this isnot so. An amazing range of problems can bemodeled using LP from airlinescheduling to least cost petroleum processing and distribution [2, 3].

The popular Simplex method of solving LP problems obtains the optimum solution by moving along edges of the solution space from one extremepoint to another.

Linear programming problems havethe property that the constraints and the objective function are all linearfunctions of the input variables. The existence of a polynomial time algorithmfor solving linear programs and the multitude of optimization problems thatthey can encode makes them particularly useful in practice.

Generally, linear programming problem can be stated as follows:

Maximize the objective function

$Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots\ldots\ldots + c_nx_n$ (1)

Subject to the constraints

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{1n}x_{n1} \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{2n}x_{n2} \\ a_{31}x_1 + a_{32}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{3n}x_{n3} \\ . \quad . \quad . \\ . \quad . \quad . \\ . \quad . \quad . \\ a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{mn}x_{nm} \\ \\ x_1 \geq 0;\ x_2 \geq 0\ \ldots\ldots\ x_n \geq 0 \end{array}\right\} \quad (2)$$

We can write the problem in abbreviated form called the standard formor canonical form as follows:

Minimize C^Tx (3)

$$\left.\begin{array}{ll} \text{subjectto} & Ax \leq b \\ \text{and} & x \geq b \end{array}\right\} \quad (4)$$

Here,'x' is a vector of real-valued variables (sometimes assumed to be non-negative), 'C' and 'b' are vectors of real constants, and 'A' is a matrix of realconstants.

3.1 Definition of LP Terms

1. Objective Function: The linear compilation of variables that mustbe minimized or maximized is called the objective function. It is a partof linear programming model that express what needs to be optimizeddepending on the objectives of the problem.
2. Decision Variables: They represent activity which is in competitionwith other variables for limited resources.
3. Constraints: These are different kinds of limitation to which the objective function is subjected. It is expressed in the form of linear inequalities or equalities depending on the sign used.
4. Slack Variables: They are the variables which are added to an inequality constraint to transform it to an equality constraints depending onthe sign used.Slack variables cannot take on negative values.
5. Surplus Variables: They are variables which can be added to linear constraints. Consider inequality constraints of the sign "≥" i.e.

$\sum_{j=1}^{n} a_{ij\ x_j} = b_j$; I = 1,2,…,m (5)

Solution: - Any set $x_j$which satisfies the constraints on the linear programming problem.

6. Basic Solution: A basic solution for a set of m linear equation in nvariable is the solution in which (n-m) variables are set equal to zero.
7. Feasible Solution: A feasible solution to the linear programmingproblem is a vector.
8. Basic Feasible Solution: The basic feasible solution for a set of mlinear equation in n variable is the solution in which the value of allvariables are non-negativei.e. a basic solution which satisfies the non-negativity condition
9. Optimal Solution: An optimal solution is a feasible solution whichoptimizes the objective function or any point in the feasible regionwhich gives the optimum value.
10. Optimal Value: This is the minimum or maximum value of an objective function to be calculated in a linear programming problem.
11. Feasible Region: The set of all possible solution.
12. Feasible Solution: Any point within and on the boundary of thefeasible region are called feasible solution.
13. Non Degenerate Basic Feasible Solution: This is a basic feasible solution which has exactly m positive x_ii.e. all basic variables arepositive.
14. An Augmented solution: Is a solution for a problem that was originally in inequality form that has been augmented by the corresponding values of the slack or surplus variables to change the problem into equality form[2, 4].

3.2 Advantages of LP

1. Linear programming helps in dealing with the problem of allocationof limited resources among different competitive activities in the mostoptimal manner.
2. It is concerned with determining the optimal allocation of scarce resources to meet certain objectives.
3. It provides practical and better quality of decision that reflects very precisely the limitations of the system. I.e. various restrictions under whichthe system must operate for the solution to be optimal.

4. Linear programming is an adaptive and flexible mathematical technique and hence can be used in analyzing a variety of multi-dimensional problem quite successfully.
5. The minimization of a function is equal to the maximization of thenegative of that same function. i.e. $P_{min}= 0 = -P_{max}$
6. The techniques help to make the best possible use of available productive resources.
7. Linear programming is applicable to transportation problem, diet problem, product mix problems, investment planning problem, marketingand distribution management etc.
8. According to [5], linear programs are expressed in aninequality form, which allows for the inexact computation of the algorithms direction of improvement, resulting in a significant computational advantage.

3.3 Disadvantages of LP

1. Linear programming is applicable only to problem where the constraintsand the objective functions are linear.
2. Factors such as uncertainty, weather conditions etc. are not taken intoconsideration.
3. There is restriction to linear objective function.
4. Reducing problems to a set of linear equation is usually very difficult.

4.0 Raw Data Collected

The data below were collected in December, 2012 from NSTA headquarters which is located at Shango by Paiko Road, Opposite Trade Fair Complex, Minna. Data on bus routes were obtained from the Interview with the desk officer; while that of bus service and maintenance were obtained from the interview with the Assistant Workshop Manager.

Table1: Data on Bus Routes

S/N	ROUTE	FREQUENCY OF TRIPS PER WEEK	FUEL CONSUMPTION (LTRS)	PERCENTAGE PARKING LEVY	NO. OF BUSES PER ROUTE PER DAY	MAX NO. OF TRIPS PER BUS PER DAY	NO. OF HOURS PER HALF TRIP	TRANSPORT FARE PER PERSON
1	MINNA-ILORIN	DAILY	125	NIL	2	1	6	1,700
2	MINNA-LAGOS	,,	195	10	3	1	10	3,500
3	MINNA-KADUNA	,,	85	7	2	1	3	800
4	MINNA-KANO	TURN BY TURN	155	NIL	4	1	6	1,500
5	MINNA-MAKURDI	DAILY	145	,,	1	1	6	2,000
6	MINNA-JOS	,,	125	7	1	1	6	2,000
7	MINNA-BAUCHI	2 TIMES PER WEEK	155	7	1	1	7	2,200
8	MINNA-SOKOTO	TURN BY TURN	185	7	1	1	9	2,500
9	MINNA-ZAMFARA	ONCE A WEEK	125	NIL	1	1	6	1,800
10	MINNA-KATSINA	TURN BY TURN	155	,,	1	1	7	2,000
11	MINNA-BIDA	DAILY	30	,,	3	3	1	300
12	MINNA-MOKWA	,,	75	,,	2	1	3	900
13	MINNA-N/BUSA	3 TIMES A WEEK	85	,,	1	1	4	1,200
14	MINNA-KTAGORA	DAILY	60	,,	2	1	2	600
15	MINNA-KAGARA	,,	30	,,	1	2	1.5	300
16	MINNA-W/SHISHI	,,	25	,,	1	3	1	150
17	MINNA-SULEJA	,,	30	,,	3	2	1.5	350
18	MINNA-S/PAWA	,,	30	,,	1	2	1	300
19	MINNA-LAPAI	,,	25	,,	1	2	1	150
20	MINNA-AGAI	,,	30	,,	1	1	1.5	200
	TOTAL		**1870**	**38**	**33**	**28**	**83.5**	**24450**

Explaining Table 1

Route: This column shows all the ten (10) inter and ten (10) intra States routes that NSTA buses ply.

Frequency of Trips per Week: This is the number of times a bus ply a particular route in a week. Note that, a trip means to and fro the destination. For example, Minna-Ilorin trip means the bus has gone from Minna to Ilorin and back. A bus that has gone from Minna to Ilorin only has made half trip.

Fuel Consumption in Liters: This is the number of liters of fuel consumed by one bus per trip. The fuel is given to bus drivers for a whole trip fiscally.

Percentage Parking Levy: This is a levy paid by NSTA for parking space to authorities of garages in towns where NSTA does not have its own facility or staff. The levy is an agreed percentage of the bus capacity per night.

Number of Buses per Route per Day: This is the number of buses that NSTA currently allocates to ply a particular route per day

Maximum Number of Trips per Bus per Day: This is the maximum number of times that a bus can possibly make a 'to-and-fro' journey in a day. For example, one bus can make two or more trips from Minna to Bida in a day. Note that, a bus that unavoidably makes only half trip from Minna to a destination, say Lagos, in a day is completed by another bus from that same destination (Lagos) back to Minna

Number of Hours per Half Trip: This is the number of hours a bus spends on the road under normal conditions from Minna.

Transport Fare per person: This is the cost charged per passenger from Minna to a destination. Usually, it is the same cost charged per passenger from that same destination back to Minna.

Table 2: Data on Bus Service

S/N	Required Service Items	Amount (N)
1	Oil filter	500
2	5-litre Engine oil	2800
3	Oil treatment	400
	Total	3700

Note: Buses in NSTA are serviced twice a month, which is after 15 days.

Table 3: Data on Bus Repair/Maintenance

S/N	Repair/Maintenance Items	Cost (N)	Duration it lasts
1	Tyre	25000	3 months
2	Front bearing	5000	6 months
3	Break disk	12000	2 years
4	Break pad	1000	2 weeks
5	Break lining	2400	1 month
6	Car battery	11000	2 years
7	Shocks filling	1000	4 months
8	Sparking plugs	4000	2 years
9	Fuel pump	5000	2 years
10	Release bearing	2000	2 years
	Total	**68400**	

Note: Buses going to New-Busa, Mokwa, Ilorin and Lagos incur thrice the cost of repair/maintenance due to bad road**.**

5.0 Data Analysis

Table 4: Daily cost of Bus Service

S/N	Required Service Items	Amount Per 15 Days (N)	Amount Per month (N)	Daily Service Cost Per Bus (N)
1	Oil filter	500	1000	1000÷30=33.3
2	5-litre Engine oil	2800	5600	5600÷30=186.7
3	Oil treatment	400	800	800÷30=26.7
	Total	3700	7400	247

Note: Buses in NSTA are serviced twice a month, which is after 15 days. This means that a bus in NSTA consumes N7400 in a month (Amount per 15 Days × 2). Dividing service amount per month by 30 days gives us N247 as the cost of servicing a bus in a day.

Table 5: Daily Cost of Bus Repair/Maintenance

S/N	Repair/Maintenance Items	Cost (N)	Duration it lasts	Duration in days	Cost per Day (N)
1	Tyre	25000	3 months	3×30 = 90	278
2	Front bearing	5000	6 months	6×30 = 180	28
3	Break disk	12000	2 years	2×12×30 = 720	17
4	Break pad	1000	2 weeks	2×7 = 14	71
5	Break lining	2400	1 month	1×30 = 30	15
6	Car battery	11000	2 years	2×12×30 = 720	80
7	Shocks filling	1000	4 months	4×30 = 120	08
8	Sparking plugs	4000	2 years	2×12×30 = 720	06
9	Fuel pump	5000	2 years	2×12×30 = 720	07
10	Release bearing	2000	2 years	2×12×30 =720	03
	Total	**68400**			**513**

Note: The cost of maintaining a bus in a day is calculated to be N513 (Cost ÷ Duration in Days). Since buses going to New-Busa, Mokwa, Ilorin and Lagos incur thrice the cost due to bad road, their repair/maintenance costs will be 513×3 = N1539 per bus per day.

Table 6: Daily Contribution per Bus

S/N	ROUTE	FREQUENCY OF TRIPS PER WEEK	FUEL CONSUMPTION PER BUS (LTRS)	FUEL CONSUMPTION PER BUS (N)	PERCENTAGE PARKING LEVY PER BUS (%)	MONEYTARY PARKING LEVY (N)	NUMBER OF BUSES PER ROUTE PER DAY	MAX NO OF TRIPS PER BUS PER DAY	EQUIVELENT NO OF BUSES PER ROUTE PER DAY	NO OF HOURS PER HALF TRIP	TRANSPORT FARE PER PERSON (N)	DAILY COST OF A BUS SERVICE (N)	DAILY COST OF A BUS REPAIR/ MAINTENANCE (N)	RETURN PER BUS PER TRIP (N)	DAILY TOTAL EXPENDITURE PER BUS (N)	DAILY CONTRIBUTION PER BUS (N)
1	MINNA-ILORIN	DAILY	125	15,000	NIL	0	2	1	2	6	1,700	247	1,539	47600	16786	30814
2	MINNA-LAGOS	,,	195	23,400	10	4,900	3	1	3	10	3,500	247	1,539	98000	30086	67914
3	MINNA-KADUNA	,,	85	10,200	7	780	4	1	2	3	800	247	513	22400	11740	10660
4	MINNA-KANO	TURN BY TURN	155	18,600	NIL	0	4	1	4	6	1,500	247	513	42000	19360	22640
5	MINNA-MAKURDI	DAILY	145	17,400	,,	0	1	1	1	6	2,000	247	513	56000	18160	37840
6	MINNA-JOS	,,	125	15,000	7	1960	1	1	1	6	2,000	247	513	56000	17720	38280
7	MINNA-BAUCHI	2 TIMES PER WEEK	155	18,600	7	2156	1	1	1	7	2,200	247	513	61600	21516	40084
8	MINNA-SOKOTO	TURN BY TURN	185	22,200	7	2450	1	1	1	9	2,500	247	513	70000	25410	44590
9	MINNA-ZAMFARA	ONCE A WEEK	125	15,000	NIL	0	1	1	1	6	1,800	247	513	50400	15760	34640
10	MINNA-KATSINA	TURN BY TURN	155	18,600	,,	0	1	1	1	7	2,000	247	513	56000	19360	36640
11	MINNA-BIDA	DAILY	30	3,600	,,	0	3	3	9	1	300	247	513	8400	4360	4040
12	MINNA-MOKWA	,,	75	9,000	,,	0	2	1	2	3	900	247	1,539	25200	10786	14414
13	MINNA-N/BUSA	3 TIMES A WEEK	85	10,200	,,	0	1	1	1	4	1,200	247	1,539	33600	11986	21614
14	MINNA-KTAGORA	DAILY	60	7,200	,,	0	2	1	2	2	600	247	513	16800	7960	8840
15	MINNA-KAGARA	,,	30	3,600	,,	0	1	2	2	1.5	300	247	513	8400	4360	4040
16	MINNA-W/SHISHI	,,	25	3,000	,,	0	1	3	3	1	150	247	513	4200	3760	440
17	MINNA-SULEJA	,,	30	3,600	,,	0	3	2	6	1.5	350	247	513	9800	4360	5440
18	MINNA-S/PAWA	,,	30	3,600	,,	0	1	2	2	1	300	247	513	8400	4360	4040
19	MINNA-LAPAI	,,	25	3,000	,,	0	1	2	2	1	150	247	513	4200	3760	440
20	MINNA-AGAI	,,	30	3,600	,,	0	1	1	1	1.5	200	247	513	5600	4360	1240
	TOTAL		**1870**	**224400**	**38**	**12246**	**35**	**28**	**47**	**83.5**	**24450**	**4940**	**14364**	**684600**	**255950**	**428650**

Note

i. 'Daily cost of service per bus' and 'daily cost of repair/maintenance per bus' are obtained in tables 4 and 5 respectively.

ii. 'Equivalent number of buses per route' is obtained by multiplying 'Number of buses per route' by 'number of trips per day'. Remember, a trip means to and fro minna.

iii. 'Return per bus per trip' is obtained by multiplying 'transport fare' per passenger by 2 (since a trip means to and fro) and 14 (number of passengers in a bus)

iv. 'Total expenditure per bus' is obtained by adding 'fuel consumption', Monetary parking levy', 'daily service per bus' and 'daily repair/maintenance per bus'.

v. 'Daily contribution per bus' is obtained by subtracting 'total expenditure' from 'return per bus'

Table 7: Daily Contribution per Route

S/N	ROUTE	MONEYTARY PARKING LEVY PER ROUTE	EQUIVELENT NO OF BUSES PER ROUTE PER DAY	FUEL CONSUMPTION PER ROUTE (N)	TRANSPORT FARE PER PERSON	DAILY COST OF BUS SERVICE PER ROUTE	DAILY COST OF BUS REPAIR/ MAINTENANCE PER ROUTE (N)	RETURN PER ROUTE (N)	DAILY TOTAL EXPENDITURE PER ROUTE (N)	DAILY CONTRIBUTION PER ROUTE (N)
1	MINNA-ILORIN	0	2	30000	1,700	494	3078	95200	33572	61628
2	MINNA-LAGOS	14700	3	70200	3,500	741	4617	294000	90258	203742
3	MINNA-KADUNA	1560	2	20400	800	494	1026	44800	23480	21320
4	MINNA-KANO	0	4	74400	1,500	988	2052	168000	77440	90560
5	MINNA-MAKURDI	0	1	17400	2,000	247	513	56000	18160	37840
6	MINNA-JOS	1960	1	15000	2,000	247	513	56000	17720	38280
7	MINNA-BAUCHI	2156	1	18600	2,200	247	513	61600	21516	40084
8	MINNA-SOKOTO	2450	1	22200	2,500	247	513	70000	25410	44590
9	MINNA-ZAMFARA	0	1	15000	1,800	247	513	50400	15760	34640
10	MINNA-KATSINA	0	1	18600	2,000	247	513	56000	19360	36640
11	MINNA-BIDA	0	9	32400	300	2223	4617	75600	39240	36360
12	MINNA-MOKWA	0	2	18000	900	494	3078	50400	21572	28828
13	MINNA-N/BUSA	0	1	10200	1,200	247	1539	33600	11986	21614
14	MINNA-KTAGORA	0	2	14400	600	494	1026	33600	15920	17680
15	MINNA-KAGARA	0	2	7200	300	494	1026	16800	8720	8080
16	MINNA-W/SHISHI	0	3	9000	150	741	1539	12600	11280	1320
17	MINNA-SULEJA	0	6	21600	350	1482	3078	58800	26160	32640
18	MINNA-S/PAWA	0	2	7200	300	494	1026	16800	8720	8080
19	MINNA-LAPAI	0	2	6000	150	494	1026	8400	7520	880
20	MINNA-AGAI	0	1	3600	200	247	513	5600	4360	1240
	TOTAL	**22826**	**47**	**431400**	**24450**	**232180**	**675108**	**1264200**	**498154**	**766046**

Note

i. 'Monetary parking levy per route' is obtained by multiplying 'Monetary parking levy per bus' by 'Equivalent number of buses per route'

ii. 'Fuel consumption per route' is obtained by multiplying 'Fuel consumption per bus' by 'Equivalent number of buses per route'

iii. 'Daily cost of service per route' is obtained by multiplying 'Daily cost of service per bus' by 'Equivalent number of buses per route'

iv. 'Daily cost of repair/maintenance per route; is obtained by multiplying 'Daily cost of repair/maintenance per bus' and 'Equivalent number of buses per route'

v. 'Return per route' is obtained by multiplying 'Return per bus' and 'Equivalent number of buses per route'

vi. 'Total expenditure per route' is obtained by adding 'fuel consumption per route', Monetary parking levy per route', 'Daily cost of service per route' and 'Daily cost of repair/maintenance per route'.

vii. 'Daily contribution per route' is obtained by subtracting 'total expenditure per route' from 'Return per route'

6.0 NSTA Problem Formulation

The Problem Formulation is based on the information under the Problem Situation and table 6. The problem is formulated under the assumption that all the 49 buses that commute intra and inter states routes are working daily.

Based on the interview with NSTA, availability of passengers and the number of other transport services plying the same routes, among other factors, determine the number of buses that NSTA can assign to these routes. The following shows the possible number of buses that can be scheduled to routes:

Table 8: Maximum Number of Buses that can be Assigned to Routes

Inter States Routes	**Possible No of Buses**
MINNA-ILORIN	2-3
MINNA-LAGOS	3-4
MINNA-KADUNA	2-4
MINNA-KANO	3-5
MINNA-MAKURDI	1
MINNA-JOS	1
MINNA-BAUCHI	1
MINNA-SOKOTO	1
MINNA-ZAMFARA	1
MINNA-KATSINA	1
Intra State Routes	**Possible No of Buses**
MINNA-BIDA	9
MINNA-MOKWA	1-2
MINNA-NEW BUSA	1-2
MINNA-KTAGORA	2-3
MINNA-KAGARA	2-4
MINNA-WUSHISHI	1-2
MINNA-SULEJA	6
MINNA-SERKI PAWA	1-2
MINNA-LAPAI	0-1
MINNA-AGAI	0-1

It must be strongly noted that our allocation must not exceed 49 buses available for intra and inter states services. Below is our problem formulation:

$$\max z = 30814X1 + 67914X2 + 10660X3 + 22640X4 + 37840X5 + 38280X6 + 40084X7 + 44590X8 + 34640X9 + 36640X10 + 4040X11 + 14414X12 + 21614X13 + 8840X14 + 4040X15 + 440X16 + 5440X17 + 4040X18 + 440X19 + 1240X20 \quad (6)$$

subject to:

$$\left.\begin{array}{ll} X1 \le 3 \\ X2 \le 4 \\ X3 \le 4 \\ X4 \le 5 \\ X5 \le 1 \\ X6 \le 1 \\ X7 \le 1 \\ X8 \le 1 \\ X9 \le 1 \\ X10 \le 1 \\ X11 \le 9 \\ X12 \le 2 \\ X13 \le 1 \\ X14 \le 3 \\ X15 \le 3 \\ X16 \le 3 \\ X17 \le 6 \\ X18 \le 2 \\ X19 \le 2 \end{array}\right\} \quad (7)$$

$$X20 \le 1$$

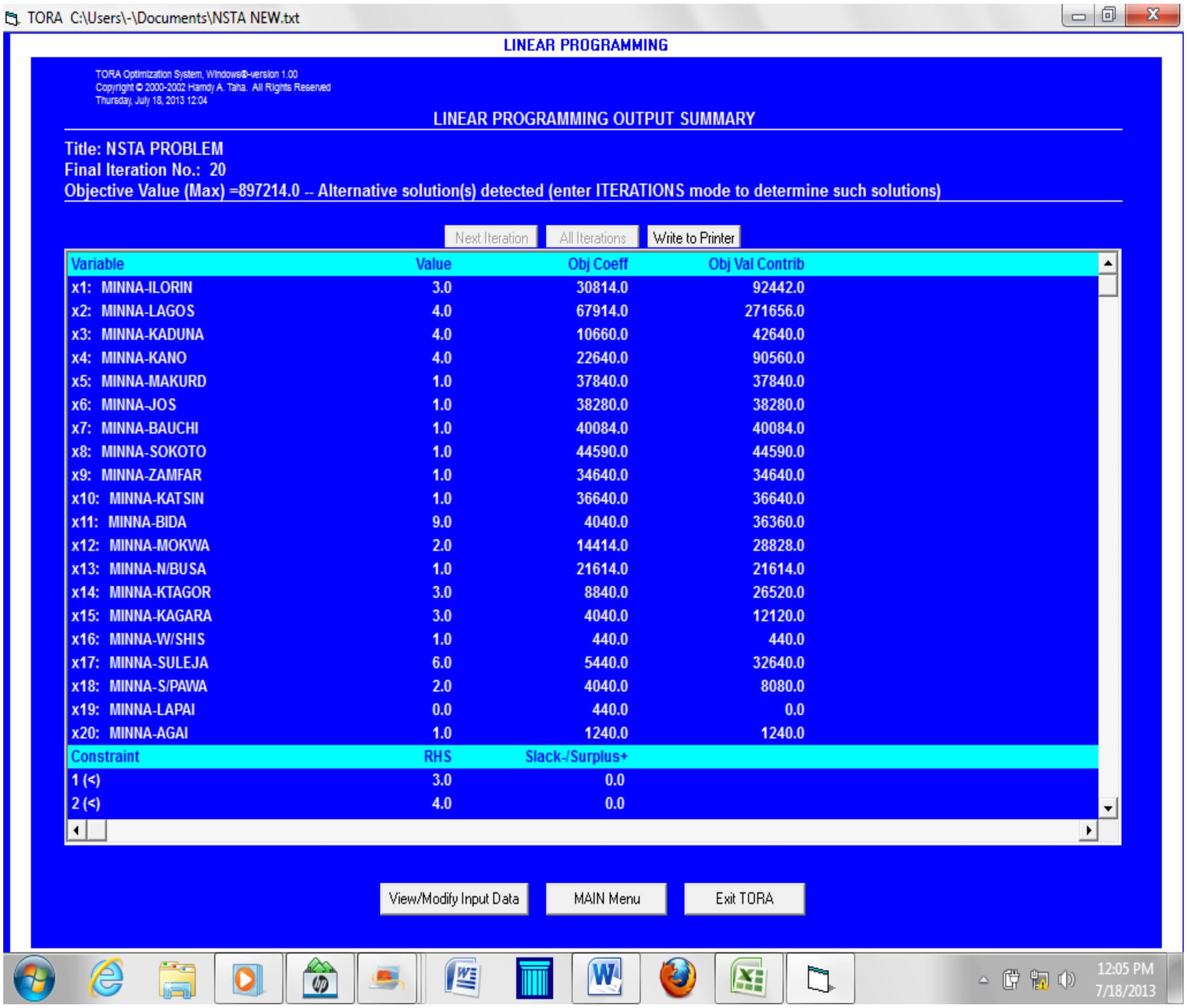
$$X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 \le 49$$

$$x1, x2, \ldots, x20 \ge 0 \quad (8)$$

Equation (6) is the objective function which seeks to maximize the daily contribution per bus for a given route. Equation (7) constitutes 21 constraints. Each route X1, x2,…, x20 forms a constraint with availability on the right hand side, which is the maximum available bus(es) per given route. The last (21^{st}) constraint conditions a maximum of 49 available buses to be allocated to the 20 routes.Equation (8) is a non-negativity condition since a negative bus will not make senseto us practically.

7.0 Discussion of Result

The formulated problem was solved using TORA software (an optimization software) which has been developed by HamdyTaha[6]. The 20th iteration reached the optimal solution and the objective value of 897, 214 is obtained as can be seen on TORA windowin figure 2:



TORA C:\Users\-\Documents\NSTA NEW.txt

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Thursday, July 18, 2013 12:04

LINEAR PROGRAMMING OUTPUT SUMMARY

Title: NSTA PROBLEM
Final Iteration No.: 20
Objective Value (Max) =897214.0 -- Alternative solution(s) detected (enter ITERATIONS mode to determine such solutions)

Next Iteration | All Iterations | Write to Printer

Variable	Value	Obj Coeff	Obj Val Contrib
x1: MINNA-ILORIN	3.0	30814.0	92442.0
x2: MINNA-LAGOS	4.0	67914.0	271656.0
x3: MINNA-KADUNA	4.0	10660.0	42640.0
x4: MINNA-KANO	4.0	22640.0	90560.0
x5: MINNA-MAKURD	1.0	37840.0	37840.0
x6: MINNA-JOS	1.0	38280.0	38280.0
x7: MINNA-BAUCHI	1.0	40084.0	40084.0
x8: MINNA-SOKOTO	1.0	44590.0	44590.0
x9: MINNA-ZAMFAR	1.0	34640.0	34640.0
x10: MINNA-KATSIN	1.0	36640.0	36640.0
x11: MINNA-BIDA	9.0	4040.0	36360.0
x12: MINNA-MOKWA	2.0	14414.0	28828.0
x13: MINNA-N/BUSA	1.0	21614.0	21614.0
x14: MINNA-KTAGOR	3.0	8840.0	26520.0
x15: MINNA-KAGARA	3.0	4040.0	12120.0
x16: MINNA-W/SHIS	1.0	440.0	440.0
x17: MINNA-SULEJA	6.0	5440.0	32640.0
x18: MINNA-S/PAWA	2.0	4040.0	8080.0
x19: MINNA-LAPAI	0.0	440.0	0.0
x20: MINNA-AGAI	1.0	1240.0	1240.0
Constraint	**RHS**	**Slack-/Surplus+**	
1 (<)	3.0	0.0	
2 (<)	4.0	0.0	

View/Modify Input Data | MAIN Menu | Exit TORA

12:05 PM 7/18/2013

Figure 2: TOR Result Window of Problem Formulation.

From figure 2, 'variable' is the name of the route. For example, x1 represents Minna - Ilorin route. 'Value' is the number of buses to be allowedto ply Minna-Ilorin. 'Objective coefficient' is the contribution made per bus. 'Objective value contribution' is the result of multiplying 'value' and the 'objective coefficient'. The summary of our recommendation which comes from the result from figure 2 is shown in table 9:

Table 9: Recommended Schedule

S/NO	ROUTE	NSTA CURRENT SCHEDULE	RECOMMENDED SCHEDULE
1	MINNA-ILORIN	2	3
2	MINNA-LAGOS	3	4
3	MINNA-KADUNA	2	4
4	MINNA-KANO	4	4
5	MINNA-MAKURDI	1	1
6	MINNA-JOS	1	1
7	MINNA-BAUCHI	1	1
8	MINNA-SOKOTO	1	1
9	MINNA-ZAMFARA	1	1
10	MINNA-KATSINA	1	1
11	MINNA-BIDA	9	9
12	MINNA-MOKWA	2	2
13	MINNA-N/BUSA	1	1
14	MINNA-KTAGORA	2	3
15	MINNA-KAGARA	2	3
16	MINNA-W/SHISHI	3	1
17	MINNA-SULEJA	6	6
18	MINNA-S/PAWA	2	2
19	MINNA-LAPAI	2	0
20	MINNA-AGAI	1	1

From S/No 1 in table9, the current (existing) NSTA bus schedule from Minna to Ilorin is 2 buses per day, while our model recommends 3 buses from Minna to Ilorin per day. This interpretation applies the same way to S/No 20.The existing intuitive NSTA schedule has a daily contribution of N766, 046 (see table7). Our recommended schedule will yield N897, 214 when implemented. This means that our model has an improvement by yielding additional N131, 168 to the company.

8.0 Conclusion

This research work has demonstrated the application of quantitative technique of linear programming to the allocation of buses to routes in a transport company. The case study of NSTA Minna provides a particular illustration which is a model of how other transport companies can seek to improve their own bus allocations in order to maximize profit. This is a good departure from the traditional method of intuition where guess and memory predominate.

9.0 References

[1] NSTA (n.d), Niger State Transport Authority at a Glance, Yan Ju Prints, Minna, Nigeria

[2] Chinnech J.W. (2001), Practical Optimization; A Gentle Introduction. Retrieved from http://www.sce.carleton.ca/faculty/Chinneck/po.html

[3] Ramsey D. (2012), Operations Research I. Retrieved from http://www3.ul.ie/ramsey/Lectures/OperationsResearch1=or1slides3:pdf.

[4] Hillier F.S and Lieberman G.J (1986), Introduction to operations research. Holden-day, Inc. USA

[5] Adler I., Karmarkar N, Mauricio G.C. and Veiga G.(1995), An Implementation of Karmarkar's Algorithm for Linear Programming. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi =10:1:1:48:8163rep = rep1type = pdf

[6] Taha H.A.(2002), TORA Optimization System. Prentice-hall, Inc. London