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# On the Study of Saturation Terms on the Infected Individual in Susceptible-Exposed Infected-Recovered- Susceptible (SEIRS) Epidemic Model

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***Abstract***

***In this paper, the effect of saturation terms for the infected individual in susceptible–exposed infected-recovered-susceptible epidemic model was considered. The saturation terms as used in the model are that parameters that determine the sociological and physiological effect on an infected individual during the outbreak or spread of a particular disease. Hence, the more the orientation of a particular disease to infected individual, the more the tendency to control or get rid such a disease***

***Therefore, the result shows that as the saturation term for the infected individual increases, the susceptible and recovered individual increase drastically and at a point the disease dies out.***

**Keywords:** Variational iteration method, saturation terms, epidemic model, susceptible and recovered individual, system of differential equations

## 1.0 Introduction

Differential equations play major role in the modeling of infectious diseases including modeling of HIV [1-2]. Epidemiological models can be categorized based on the described diseases, population and environment, as linear, non-linear, autonomous or non-autonomous model. Kunniya and Nakata [3] studied the long-term behavior of non-autonomous SEIRS epidemic model where $m_1 = m_2 = 0$ . They obtained new sufficient conditions for the permanence (uniform persistence) and extinction of infectious population of the model. The work done by Kunniya and Nakata was extended in [4] to include non-linear incidence rate to investigate the effect of saturation terms.

Many Authors [1-10] have used different numerical methods to solve different types of differential equations in attempt to search for better, accurate, efficient and elegant method for the solution.

In this paper the variational iteration method proposed by He [5-6]. was employed to further study the effect of saturation terms for the infected individual on the susceptible individual in susceptible–exposed infected-recovered-susceptible epidemic. The idea of variational calculus was proposed by Inokuti et al. [7] and later modified by J.H He into a variational iteration method. Variational Iteration Method has been shown to solve a large class of linear and nonlinear problems with approximation converging to exact solution rapidly.

This method and its modification has been applied to different types of differential equations [8-10].

In this work, we present VIM for the modeling of the effect of saturation terms on the susceptible individual in (SEIRS) Epidemic Model.

## 2.0 Mathematical Model of SEIRS Epidemic

In this section, the system of differential equations that described the model is presented as follows:

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$$
\left.\begin{aligned}
\frac{dS}{dt} &= N - \frac{\beta SI}{1+m_1S+m_2I} - \mu S + \delta R \\
\frac{dE}{dt} &= \frac{\beta SI}{1+m_1S+m_2I} - (\mu+\xi)E \\
\frac{dI}{dt} &= \xi E - (\mu+\gamma)I \\
\frac{dR}{dt} &= \gamma I - (\mu+\partial)R
\end{aligned}\right] \qquad (1)
$$

Where:
S(t) = susceptible individual
E(t) = exposed individual
I(t) = infected individual
R(t) = recovered individual
N = birth rate
$\beta =$ Disease transmission coefficient
$\mu =$ Mortality or death rate
$\xi =$ Recovery rate
$\gamma =$ Rate of losing immunity
$m_1 =$ Saturation term for susceptible individual
$m_2 =$ Saturation term for infected individual
$\frac{1}{1+m_1S+m_2I} =$ Incidence rate inclusive the saturation terms $m_1$ and $m_2$

## 3.0 The Variational Iteration Method

The idea of Variational calculus can be traced to Inokuti et al [3] and later, He [4-5] modified it and presented a Variational Iteration Method that has been proved elegant in the solution of different types of differential equations.
According to the Variational Iteration Method, we consider the differential equation.

$$L(u)+N(u)=g(s). \qquad (2)$$

Where L is a linear operator, N is a non-linear operator, and $g(s)$ is an inhomogeneous term. A correction functional to (1) can be constructed as :

$$U_{n+1}(s)=U_n(s)+\int_0^s \lambda\left[LU_n(\tau)+N\breve{u}_n(\tau)-g(\tau)\right]d\tau \qquad (3)$$

Where $\lambda$ is a general Lagrange multiplier which can be identified optimally by variational calculus and $\tilde{u}_n(\tau)$ is known as the restricted variation i.e. $\delta\tilde{u}_n(\tau)=0$.

## 4.0 Variational Iteration Method for the Solution of the Effect of Saturation terms on the SEIRS Epidemic Model:

In this section, the VIM will be used to study the effect of saturation terms in the susceptible individual in SEIRS epidemic model.To investigate the effect of $m_2$ , we proceed as follows:
Applying equation (3) in (1) we obtained the following system of correctional functional:

$$S_{n+1}(t)=S_n(t)+\int_0^t \lambda_1(\tau)\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2 I_n(\tau)}-\mu\bar{S}_n(\tau)+\partial\bar{R}_n(\tau)\right]d\tau$$

$$E_{n+1}(t)=E_n(t)+\int_0^t \lambda_2(\tau)\left[\frac{dE_n(\tau)}{dt}-\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$

(4)

$$I_{n+1}(t)=I_n(t)+\int_0^t \lambda_3(\tau)\left[\frac{dI_n(\tau)}{dt}-\xi\bar{E}_n(\tau)+(\mu+\gamma)\bar{I}_n(\tau)\right]d\tau$$

$$R_{n+1}(t)=R_n(t)+\int_0^t \lambda_4(\tau)\left[\frac{dR_n(\tau)}{dt}-\gamma\bar{I}_n(\tau)+(\mu+\partial)\bar{R}_n(\tau)\right]d\tau$$

Where $\lambda_1, \lambda_2, \lambda_3, and\ \lambda_4$ are general Lagrange Multiplier, $\tilde{S}_{n,}\tilde{E}_{n,}\tilde{I}_{n,}\ and\ \ \tilde{R}_n$ denote restricted variation i.e. $\delta\tilde{S}_n=\delta\tilde{E}_n=\delta\tilde{I}_n=\delta\tilde{R}_n=0$.
The stationary values that corresponds to the correctional functionals are:

$$\partial S_{n+1}(t)=\partial S_n(t)+\partial\int_0^t \lambda_1(\tau)\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}-\mu\bar{S}_n(\tau)+\partial\bar{R}_n(\tau)\right]d\tau$$

$$\partial E_{n+1}(t)=\partial E_n(t)+\partial\int_0^t \lambda_2(\tau)\left[\frac{dE_n(\tau)}{dt}-\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$

(5)

$$\partial I_{n+1}(t)=\partial I_n(t)+\partial\int_0^t \lambda_3(\tau)\left[\frac{dI_n(\tau)}{dt}-\xi\bar{E}_n(\tau)+(\mu+\gamma)\bar{I}_n(\tau)\right]d\tau$$

$$\partial R_{n+1}(t)=\partial R_n(t)+\partial\int_0^t \lambda_4(\tau)\left[\frac{dR_n(\tau)}{dt}-\gamma\bar{I}_n(\tau)+(\mu+\partial)\bar{R}_n(\tau)\right]d\tau$$

Therefore:

$$\lambda_1'(x,\tau)=0$$

$$1+\lambda_1(x,\tau)\Big|_{\xi=t}=0$$

$$\lambda_2'(x,\tau)=0$$

(6)

$$1+\lambda_2(x,\tau)\Big|_{\xi=t}=0$$

$$\lambda_3'(x,\tau)=0$$

$$1+\lambda_3(x,\tau)\Big|_{\xi=t}=0$$

$$\lambda_4^{'}(x,\tau)=0$$

$$1+\lambda_4^{'}(x,\tau)=0$$

Solving quations (6) give $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -1$.
With $\lambda_i, i = 1 \ldots 4$ , we obtained the following iterative scheme:

$$S_{n+1}(t)=S_n(t)-\int_0^t\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta S_n(\tau)\bar{I}_n(\tau)}{1+m_1S_n(\tau)+m_2I_n(\tau)}-\mu S_n(\tau)+\partial R_n(\tau)\right]d\tau$$

$$E_{n+1}(t)=E_n(t)-\int_0^t\left[\frac{dE_n(\tau)}{dt}-\frac{\beta S_n(\tau)I_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)E_n(\tau)\right]d\tau$$

$$I_{n+1}(t)=I_n(t)-\int_0^t\left[\frac{dI_n(\tau)}{dt}-\xi E_n(\tau)+(\mu+\gamma)I_n(\tau)\right]d\tau \tag{7}$$

$$R_{n+1}(t)=R_n(t)-\int_0^t\left[\frac{dR_n(\tau)}{dt}-\gamma I_n(\tau)+(\mu+\partial)R_n(\tau)\right]d\tau$$

Using the following initial and computational values;

$S_\circ(t)=25, E_\circ(t)=23, I_\circ(t)=16, R_\circ(t)=12, \beta=0.19, \mu=0.3, N=50,$

$\gamma=0.1, \partial=0.05, \xi=0.25.$

The following results can be readily obtained by Maple 18:

$$S_0(t)=25$$
$$E_0(t)=23$$
$$I_0(t)=16 \tag{8}$$
$$R_0(t)=12$$

$$S_1(t)=25+474.2300000t-18.m_1\,t-6.m_1m_2t$$
$$E_1(t)=23-29.32000000t-18.m_1\,t-6.m_1m_2\,t$$
$$I_1(t)=16+1.600000000t \tag{9}$$
$$R_1(t)=12-0.4500000000t$$

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## 5.0 Conclusion

The graph shown in Figures I-V, show the results when n=4. In Figure I, the graph reveals that the susceptible individuals are not either increasing or decreasing since the saturation terms have no effect while in Figure II the result reveals the asymptotic stability of the disease free equilibrium since the exposed and infected approaches zero. Figures III , IV and V show perfect asymptotic stability of the disease free equilibrium because the exposed and infected individuals die out rapidly as $m_2$ increases to a reasonable value.

The saturation terms as used in the model are that parameters that determine the sociological and physiological effect on an infected individual during the outbreak or spread of a particular disease. Hence, the more the orientation of a particular disease to infected individual, the more the tendency to control or get rid such a disease.

The simulation result reveals the stable and unstable nature of disease free equilibrium i.e. at stable and unstable nature. These results show the asymptotic stability nature of disease free equilibrium. Hence, the saturated term for infected individual plays a vital role in disease eradication.

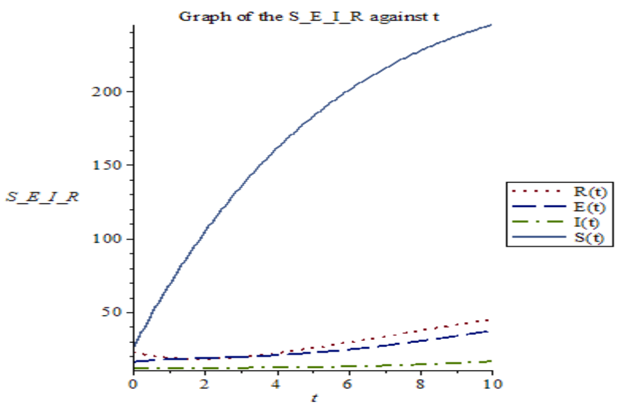


**Fig. 1:** Graph of simulated result when $m_1 = m_2 = 0$

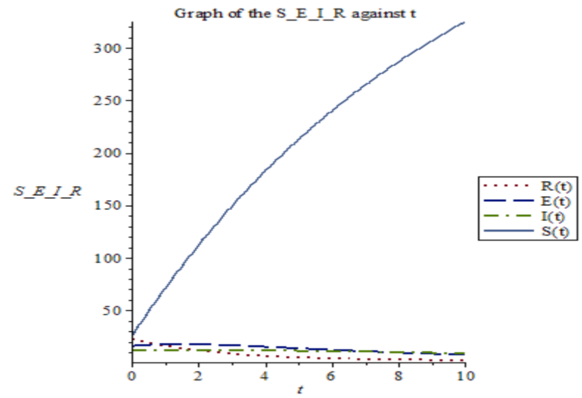


**Fig. 2:** Graph of simulated result when $m_2 = 0.2$

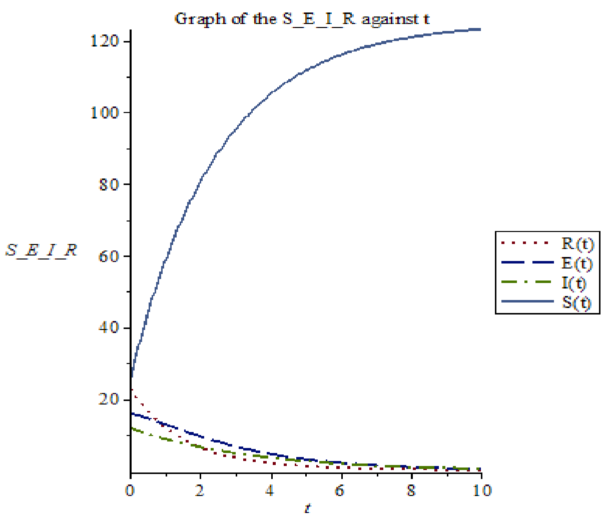


**Fig. 3:** Graph of simulated result when $m_2 = 0.4$

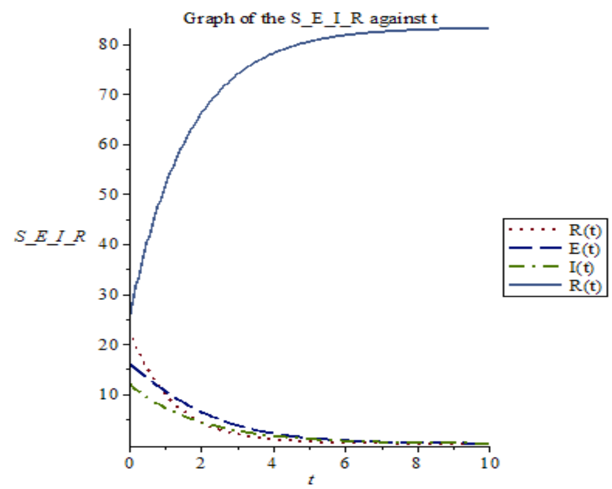


**Fig. 4:** Graph of simulated result when $m_2 = 0.6$

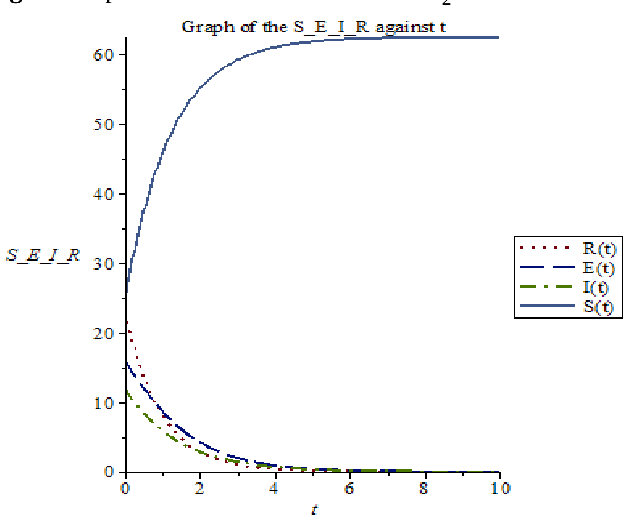


**Fig. 5:** Graph of simulated result when $m_2 = 0.8$

## 6.0 References


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