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Seismic Treatment of Elastic Surface Waves in a Layered Solid Medium

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Abstract

Seismic activities in a homogeneous isotropic elastic layered solid medium are intense and the mathematical formulation very interesting. Changes to these mathematical formulations creates new opportunities and deeper understanding of the equations governing activities in such a medium as the solid earth. This paper attempts to calculate the effect of a displacement component of the earth's surface and therefrom establish a relationship between the seismic wave speeds and the material parameters.It is concluded that the effectiveness of the damping of elastic vibrations in elastic solid is a function of the strength of the solid material, that is, the more rigid a solid is, the greater is the damping of elastic vibrations passing through it.

Keywords: Seismic waves, seismic activity, p-wave, s-wave, Rayleigh waves, surface waves, elastic material, isotropic, damping, waveform, far field.

1.0 Introduction

The literature on seismic waves is very vast. Investigation on the propagation of elastic disturbances in layered media has received sustained patronage and interest over the past hundred years, mainly, due to its application in seismology, geophysical prospecting, and in many problems of acoustics and electromagnetism. The propagation of surface waves in elastic media is also of considerable importance in earthquake engineering and seismology due to the layers in the earth's crust.

Modern seismology has become a multifaceted discipline that focuses on issues of both scientific and societal concern. Investigation of earthquakes (causes, effects and control) as a physical process has yielded many important insights about the phenomenon. In the layered medium, the earth, the speed at which seismic waves travel through it is equally studied. This investigation has its emphasis in the wave seismic treatment of elastic surface waves in layered solid medium.

Techniques for characterizing seismic source, borrowed from earthquake seismology, can provide useful information for microseismic studies. Microseismic methods have emerged as an important tool for monitoring fluid process at the reservoir scale. Microseismic activity in a subsurface reservoir may result from brittle deformation of reservoir rocks due to fluid injection. The ability to pinpoint the locations of microseismic events provides a basis for investigating the state of stress in the reservoir. The spatial dimensions and rupture characteristics of microseismic events are encoded in the spectra of radiated seismic waves.

The genesis of the study of propagation of seismic waves in elastic media can be traced to the early investigation of [1-6]. Over the intervening years, great advances have been made and the subject matter has grown in leaps and bounds to attract contributions from other international scholars which include [7, 8].

The effect of gravity on wave propagation in an elastic solid medium was first considered in [1] who treated gravity as a type of body force. Love [6] extended the work of Bromwich in investigating the influence of gravity on surface waves showed that the Rayleigh wave velocity may be affected significantly by the gravity field.

In order to study seismic activities, microseismic waveforms are investigated. Ramirez showed that storm microseisms were predominantly Rayleigh waves and noted that the direction of the source could be determined within a few degrees by the times of arrival at seismic stations. Other researchers [9, 10] studied microseismic activities and established that double frequency signals can be generated by the interaction of opposing waves and the resulting pressure excitation pulse propagates energy to the

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seafloor in deep water. Efficient coupling of microseism energy to propagating seismic modes requires that the water column pressure waves at the seafloor match phase speeds with the seismic waves [11].
Much work has been done to identify the source regions of microseisms which are essentially Rayleigh surface waves propagating in the direction parallel to the earth's surface having the related energy trapped near the surface. This is essentially the combined activities of the inhomogeneous pressure and the vertical shear elastic waves both of which had gone through the process of total internal reflections along the air/solid earth surface [12]. Bromirski et al [13] correlated microseism power fluctuations with wave buoy data. When applied to body waves, this identifies the region of microseism generator. Some studies, [14, 15] indicate that mid–ocean storms generate microseism P-waves, and the source regions vary seasonally with the predominant waves without incorporatingthe effect of damping in their investigations. Other studies [16, 17] reported that surface stress plays a vital role in the propagation of waves due to the fact that the surface of a body exhibits properties that are quite different than those associated with the interior of the medium.

2.0 Model Parameters

Let the x-axis and y-axis be perpendicular and along the shoreline respectively. The z-axis points vertically downwards with $z=0$ as the earth's surface, $t>0$ is the time with $t=0$ giving the onset of the geophysical activities.
The behaviour of an isotropic solid is completely specified, if $\hat{\mu}$ and $\wp$ are given where $\hat{\mu}$ and $\wp$ are the Lame's constants. In particular, $\hat{\mu}$ defines the strength of the layered elastic solid, hence, the most important parameter in our present investigation. γ_s is the density of solid which in the case of horizontally stratified elastic half – space, will be a function of z only. Finally, the displacement components of the elastic half – space in response to the seismic events are $\bar{u},\bar{v},\bar{w}$ in the x, y and z directions respectively.

3.0 Formulation of the Governing Equations

When a deformable body undergoes a change in configuration due to the application of a system of forces, the body is said to be strained. Within the body, any point P with space–fixed rectangular coordinates (x,y,z) is then displaced to a new position, the components of displacement being, respectively, $\bar{u},\bar{v},\bar{w}$. If P' is a neighbouring point $(x+dx, y+dy, z+dz)$, its displacement component can be given by a Taylor expansion in the form

$$\begin{aligned}\bar{v}(x+dx,y+dy,z+dz) = {} & \bar{v}(x_0+dx_0,y_0+dy_0,z_0+dz_0)\\ & +\left\{\bar{v}_{,x}(x+dx)+\bar{v}_{,y}(y+dy)+\bar{v}_{,z}(z+dz)\right\}\\ & +\frac{1}{2!}\left\{\bar{v}_{,xx}(x+dx)^2+2\bar{v}_{,xy}(x+dx)(y+dy)\right.\\ & +2\bar{v}_{,xz}(x+dx)(z+dz)+2\bar{v}_{,yz}(y+dy)(z+dz)\\ & \left.+\bar{v}_{,yy}(y+dy)^2+\bar{v}_{,zz}(z+dz)^2\right\}+\ldots \qquad (3.1b)\end{aligned}$$

$$\begin{aligned}\bar{w}(x+dx,y+dy,z+dz) = {} & \bar{w}(x_0+dx_0,y_0+dy_0,z_0+dz_0)\\ & +\left\{\bar{w}_{,x}(x+dx)+\bar{w}_{,y}(y+dy)+\bar{w}_{,z}(z+dz)\right\}\\ & +\frac{1}{2!}\left\{\bar{w}_{,xx}(x+dx)^2+2\bar{w}_{,xy}(x+dx)(y+dy)\right.\\ & +2\bar{w}_{,xz}(x+dx)(z+dz)+2\bar{w}_{,yz}(y+dy)(z+dz)\\ & \left.+\bar{w}_{,yy}(y+dy)^2+\bar{w}_{,zz}(z+dz)^2\right\}+\ldots \qquad (3.1c)\end{aligned}$$

If these displacement components are small, we may neglect higher terms, and have

$$\bar{u}\left(x+dx, y+dy, z+dz\right)=\bar{u}+\frac{\partial \bar{u}}{\partial x}dx+\frac{\partial \bar{u}}{\partial y}dy+\frac{\partial \bar{u}}{\partial z}dz$$

$$\bar{v}\left(x+dx, y+dy, z+dz\right)=\bar{v}+\frac{\partial \bar{v}}{\partial x}dx+\frac{\partial \bar{v}}{\partial y}dy+\frac{\partial \bar{v}}{\partial z}dz \qquad (3.1d)$$

$$\bar{w}\left(x+dx, y+dy, z+dz\right)=\bar{w}+\frac{\partial \bar{w}}{\partial x}dx+\frac{\partial \bar{w}}{\partial y}dy+\frac{\partial \bar{w}}{\partial z}dz$$

so that introducing the expressions obtained by the cyclic change of letters $x, y, z, \bar{u}, \bar{v}, \bar{w}$ and the expression

$$\varphi_z=\frac{1}{2}\left(\frac{\partial \bar{v}}{\partial x}-\frac{\partial \bar{u}}{\partial y}\right),\ e_{xy}=\frac{1}{2}\left(\bar{v}_{,x}+\bar{u}_{,y}\right) \qquad (3.2)$$

we can re-write the displacement components equation (3.1) in the form

$$\bar{u}\left(x+dx, y+dy, z+dz\right)=\bar{u}+\left(\varphi_y dz-\varphi_z dy\right)+\left(e_{xx}dx+e_{xy}dy+e_{xz}dz\right)$$

$$\bar{v}\left(x+dx, y+dy, z+dz\right)=\bar{v}+\left(\varphi_z dx-\varphi_x dz\right)+\left(e_{yx}dx+e_{yy}dy+e_{yz}dz\right) \qquad (3.3)$$

$$\bar{w}\left(x+dx, y+dy, z+dz\right)=\bar{w}+\left(\varphi_x dy-\varphi_y dx\right)+\left(e_{zx}dx+e_{zy}dy+e_{zz}dz\right)$$

where the first terms are the components of displacement of the point P. It has been shown that the terms in the first parentheses correspond to a pure rotation of a volume element and that the terms in the second parentheses are associated with deformation or strain of the element [18, 19]. Since e_{ij}, $i = x, y, z;\ j = x, y, z$ is symmetric, we have

$$e_{ij}=\begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix} \qquad (3.4a)$$

which represents the symmetric strain tensor at p. The three components

$$\begin{aligned} e_{ii}=e_{jj}&=e_{xx}=\bar{u}_{,x} &&, i=j=x \\ &=e_{yy}=\bar{v}_{,y} &&, i=j=y \\ &=e_{zz}=\bar{w}_{,z} &&, i=j=z \end{aligned} \qquad (3.4b)$$

represents simple extensions parallel to the x, y, z axes and the other three expressions e_{xy}, e_{yz}, e_{zx} are the shear components of strain which has been shown to be equal to half the angular changes in the xy, yz, zx planes respectively of an originally orthogonal volume element.

However, in the generalized form of Hooke's law, it is assumed that each of the six components of stress is a linear function of all the components of strain, and in the general case, 36 elastic constants appear in the stress-strain relations. As we shall see later, the number of elastic constants degenerates to two due to the symmetry associated with an isotropic body [20].

4.0 Derivation of the Governing Equations

To analyze waves in elastic materials, we must derive the equations governing the motions of such materials. The equations of motion are obtained by adding all the forces and the inertia terms

$-\gamma_s \dfrac{\partial^2 (.)}{\partial t^2}$ for each component.

With these specifications, the dynamic equations of motion for a three dimensional isotropic elastic solid medium are:

$$\gamma_s \bar{u}_{,tt} = (\wp + \hat{\mu})\Delta_{,x} + \hat{\mu}\nabla^2 \bar{u} \qquad (3.5a)$$

$$\gamma_s \bar{v}_{,tt} = (\wp + \hat{\mu})\Delta_{,y} + \hat{\mu}\nabla^2 \bar{v} \qquad (3.5b)$$

$$\gamma_s \bar{w}_{,tt} = (\wp + \hat{\mu})\Delta_{,z} + \hat{\mu}\nabla^2 \bar{w} \qquad (3.5c)$$

Where

$$\Delta = \bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z} \qquad (3.6)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (3.7)$$

In the expression, γ_s is the density of the medium.

Differentiate both sides of equation (3.5a), (3.5b), (3.5c) with respect to x, y, z respectively and get

$$\gamma_s \bar{u}_{,ttx} = (\wp + \hat{\mu})\left(\frac{\partial^3 \bar{u}}{\partial x^3} + \frac{\partial^3 \bar{v}}{\partial x^2 \partial y} + \frac{\partial^3 \bar{w}}{\partial x^2 \partial z}\right) + \hat{\mu}\left(\frac{\partial^3 \bar{u}}{\partial x^3} + \frac{\partial^3 \bar{u}}{\partial x \partial y^2} + \frac{\partial^3 \bar{u}}{\partial x \partial z^2}\right) \qquad (3.8a)$$

$$\gamma_s \bar{v}_{,tty} = (\wp + \hat{\mu})\left(\frac{\partial^3 \bar{u}}{\partial x \partial y^2} + \frac{\partial^3 \bar{v}}{\partial y^3} + \frac{\partial^3 \bar{w}}{\partial y^2 \partial z}\right) + \hat{\mu}\left(\frac{\partial^3 \bar{v}}{\partial y \partial x^2} + \frac{\partial^3 \bar{v}}{\partial y^3} + \frac{\partial^3 \bar{v}}{\partial y \partial z^2}\right) \qquad (3.8b)$$

$$\gamma_s \bar{w}_{,ttz} = (\wp + \hat{\mu})\left(\frac{\partial^3 \bar{u}}{\partial x \partial z^2} + \frac{\partial^3 \bar{v}}{\partial y \partial z^2} + \frac{\partial^3 \bar{w}}{\partial z^3}\right) + \hat{\mu}\left(\frac{\partial^3 \bar{w}}{\partial z \partial x^2} + \frac{\partial^3 \bar{w}}{\partial z \partial y^2} + \frac{\partial^3 \bar{w}}{\partial z^3}\right) \qquad (3.8c)$$

Adding the three equations, we get

$$\gamma_s \left(\bar{u}_{,ttx} + \bar{v}_{,tty} + \bar{w}_{,ttz}\right) = (\wp + \hat{\mu})\left(\frac{\partial^3 \bar{u}}{\partial x^3} + \frac{\partial^3 \bar{v}}{\partial x^2 \partial y} + \frac{\partial^3 \bar{w}}{\partial x^2 \partial z} + \frac{\partial^3 \bar{u}}{\partial y^2 \partial x} + \frac{\partial^3 \bar{v}}{\partial y^3} + \frac{\partial^3 \bar{w}}{\partial y^2 \partial z} + \frac{\partial^3 \bar{u}}{\partial z^2 \partial x} + \frac{\partial^3 \bar{v}}{\partial z^2 \partial y} + \frac{\partial^3 \bar{w}}{\partial z^3}\right) + \hat{\mu}\left\{\frac{\partial^3 \bar{u}}{\partial x^3} + \frac{\partial^3 \bar{v}}{\partial x^2 \partial y} + \frac{\partial^3 \bar{w}}{\partial x^2 \partial z} + \frac{\partial^3 \bar{u}}{\partial y^2 \partial x} + \frac{\partial^3 \bar{v}}{\partial y^3} + \frac{\partial^3 \bar{w}}{\partial y^2 \partial z} + \frac{\partial^3 \bar{u}}{\partial z^2 \partial x} + \frac{\partial^3 \bar{v}}{\partial z^2 \partial y} + \frac{\partial^3 \bar{w}}{\partial z^3}\right\} \qquad (3.9a)$$

$$\gamma_s \frac{\partial^2}{\partial t^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right) = (\wp + \hat{\mu})\left\{\frac{\partial^2}{\partial x^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right) + \frac{\partial^2}{\partial y^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right) + \frac{\partial^2}{\partial z^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right)\right\} + \hat{\mu}\left\{\frac{\partial^2}{\partial x^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right) + \frac{\partial^2}{\partial y^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right) + \frac{\partial^2}{\partial z^2}\left(\bar{u}_{,x} + \bar{v}_{,y} + \bar{w}_{,z}\right)\right\} \qquad (3.9b)$$

$$\gamma_s \frac{\partial^2 \Delta}{\partial t^2} = (\wp + \hat{\mu})\left(\frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial^2 \Delta}{\partial y^2} + \frac{\partial^2 \Delta}{\partial z^2}\right) + \hat{\mu}\left(\frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial^2 \Delta}{\partial y^2} + \frac{\partial^2 \Delta}{\partial z^2}\right) \qquad (3.9c)$$

$$\frac{\partial^2 \Delta}{\partial t^2} = \left(\frac{\wp + 2\hat{\mu}}{\gamma_s}\right)\nabla^2 \Delta \qquad (3.9d)$$

Hence

$$\frac{\partial^2 \Delta}{\partial t^2} = \eta^2 \nabla^2 \Delta \qquad (3.10)$$

where

$$\eta^2 = \left(\frac{\wp + 2\hat{\mu}}{\gamma_s} \right) \tag{3.11}$$

And Δ defines the wave of compression which moves with the speed η.

Further, differentiate both sides of equations (3.5a) w.r.t y and (3.5b) w.r.t x to get respectively

$$\gamma_s \bar{u}_{,tty} = (\wp + \hat{\mu}) \Delta_{,xy} + \hat{\mu} \nabla^2 \bar{u}_{,y} \tag{3.12a}$$

$$\gamma_s \bar{v}_{,ttx} = (\wp + \hat{\mu}) \Delta_{,yx} + \hat{\mu} \nabla^2 \bar{v}_{,x} \tag{3.12b}$$

Then subtracting equation (3.12b) from (3.12a), we get

$$\gamma_s \left(\bar{u}_{,tty} - \bar{v}_{,ttx} \right) = (\wp + \hat{\mu}) \left(\Delta_{,xy} - \Delta_{,yx} \right) + \hat{\mu} \left(\nabla^2 \bar{u}_{,y} - \nabla^2 \bar{v}_{,x} \right) \tag{3.12c}$$

$$\gamma_s \frac{\partial^2}{\partial t^2} \left(\bar{u}_{,y} - \bar{v}_{,x} \right) = \hat{\mu} \nabla^2 \left(\bar{u}_{,y} - \bar{v}_{,x} \right)$$

Differentiate both sides of equations (3.5a) w.r.t z and (3.5c) w.r.t x to get respectively

$$\gamma_s \bar{u}_{,ttz} = (\wp + \hat{\mu}) \Delta_{,xz} + \hat{\mu} \nabla^2 \bar{u}_{,z} \tag{3.12d}$$

$$\gamma_s \bar{w}_{,ttx} = (\wp + \hat{\mu}) \Delta_{,zx} + \hat{\mu} \nabla^2 \bar{w}_{,x} \tag{3.12e}$$

Then subtracting equation (3.12e) from (3.12d), we get

$$\gamma_s \frac{\partial^2}{\partial t^2} \left(\bar{u}_{,z} - \bar{w}_{,x} \right) = \hat{\mu} \nabla^2 \left(\bar{u}_{,z} - \bar{w}_{,x} \right) \tag{3.12f}$$

Differentiate both sides of equations (3.5b) w.r.t z and (3.5c) w.r.t y to get respectively

$$\gamma_s \bar{v}_{,ttz} = (\wp + \hat{\mu}) \Delta_{,yz} + \hat{\mu} \nabla^2 \bar{v}_{,z} \tag{3.12g}$$

and

$$\gamma_s \bar{w}_{,tty} = (\wp + \hat{\mu}) \Delta_{,zy} + \hat{\mu} \nabla^2 \bar{w}_{,y} \tag{3.12h}$$

Then subtracting equation (3.12h) from (3.12g), we get

$$\gamma_s \frac{\partial^2}{\partial t^2} \left(\bar{v}_{,z} - \bar{w}_{,y} \right) = \hat{\mu} \nabla^2 \left(\bar{v}_{,z} - \bar{w}_{,y} \right) \tag{3.12i}$$

where $\Delta_{,i} - \Delta_{,j} = 0 \; ; i = x, y, z; \; j = x, y, z$

We now write equation (3.12c) in the form

$$\frac{\partial^2 \lambda_z}{\partial t^2} = \xi^2 \nabla^2 \lambda_z \tag{3.13}$$

where

$$\xi^2 = \frac{\hat{\mu}}{\gamma_s}, \lambda_z = \left(\bar{u}_{,y} - \bar{v}_{,x} \right) \tag{3.14}$$

In the same way, we obtain

$$\frac{\partial^2 \lambda_y}{\partial t^2} = \xi^2 \nabla^2 \lambda_y \tag{3.15}$$

and

$$\frac{\partial^2 \lambda_x}{\partial t^2} = \xi^2 \nabla^2 \lambda_x \tag{3.16}$$

The vector

$$\bar{\omega} = \left(\bar{\omega}_x, \bar{\omega}_y, \bar{\omega}_z\right) \tag{3.17}$$

gives the wave of rotation in the elastic solid. This wave moves with speed ξ. For the propagation of a Surface Rayleigh wave along the horizontal x-axis, we assume that the motion is uniform with respect to the y-axis. We introduce two displacement potentials $\phi_1(x,z,t)$ and $\phi_2(x,z,t)$ for the displacement of the elastic solid, which are related to the displacement components as follows;

$$\bar{u} = \frac{\partial \phi_1}{\partial x} - \frac{\partial \phi_2}{\partial z}, \ \bar{w} = \frac{\partial \phi_1}{\partial z} + \frac{\partial \phi_2}{\partial x} \tag{3.18}$$

The displacement potential $\phi_1(x,z,t)$ and $\phi_2(x,z,t)$ in the above equation are two distinct potentials, whose Laplacian specify compression and shear given by

$$\nabla_2^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{3.19}$$

Then,

$$\Delta = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z} = \nabla_2^2 \phi_1 \tag{3.20a}$$

$$\lambda_y = \bar{u}_z - \bar{w}_x = -\nabla_2^2 \phi_2 \tag{3.20b}$$

The potential $\phi_1(x,z,t)$ describes a wave of compressional motion which in the plane – wave case is longitudinal, while $\phi_2(x,z,t)$ describes a wave of shear motion which is transverse in the plane – wave. Thus, introducing equations (3.20a) and (3.20b) into equation (3.10) and (3.15) respectively, we obtain the wave equations in the form

$$\frac{\partial^2 \phi_1}{\partial t^2} = \eta^2 \nabla^2 \phi_1 \tag{3.21}$$

$$\frac{\partial^2 \phi_2}{\partial t^2} = \xi^2 \nabla^2 \phi_2 \tag{3.22}$$

4.1 Solving the Model Equations

In this consideration, we introduce a damping term into the governing equations to represent the effect of material inelasticity which we assumed to be small since the oscillations take place near the surface of the earth and the variations in the elastic parameters are slight.

We introduce $\phi_1(x,z,t), \phi_2(x,z,t)$ and a damping term 2δ into equations (3.5a) and (3.5b) and using the same notations therein to obtain:

$$Q(\alpha)e^{i\alpha(x-ct)} = \wp\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) + 2\hat{\mu}\left(\frac{\partial^2 \phi_1}{\partial z^2} - \frac{\partial^2 \phi_2}{\partial x \partial z}\right) + 2\delta \frac{\partial}{\partial t}\left(\frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial z}\right) \tag{4.1}$$

and

$$\hat{\mu}\left(2\frac{\partial^2\phi_1}{\partial x\partial z}-\frac{\partial^2\phi_2}{\partial x^2}+\frac{\partial^2\phi_2}{\partial z^2}\right)+2\delta\frac{\partial}{\partial t}\left(\frac{\partial\phi_1}{\partial z}-\frac{\partial\phi_2}{\partial x}\right)=0 \tag{4.2}$$

Consequently, the model equations adopted here will be the same as that of [14]. Thus the damping term in the equation of motion will be assumed to be proportional to the velocity of the material components.It will be further assumed that solid elastic media are welded together at the surface of contact, implying continuity of all forces and displacement components across the boundary. The system of equations generated above, form the boundary equations for our investigations. In equation (4.1), the term on the left hand side is the generating pressure field of the water waves; $Q(\alpha)$ being the amplitude spectrum of the bottom pressure. The wave number α and the phase speed C are such size as to match those of the seismic trapped modes below the seabed. Hence, α and C will refer to both the generating water waves and the seismic response of the elastic half – space. Thus, C is the phase velocity of the elastic surface wave.

The solution of the equations (4.1) and (4.2) are expressible in the form:

$$\phi_1(x,z,t)=Ae^{\left[i\alpha(rz+x-ct)\right]} \tag{4.3}$$

$$\phi_2(x,z,t)=Be^{\left[i\alpha(sz+x-ct)\right]} \tag{4.4}$$

where A and B do not depend on space and time but rather on frequency and wave number.

$$\frac{c^2}{\eta^2}-r^2=1 \tag{4.5a}$$

$$\frac{c^2}{\xi^2}-s^2=1 \tag{4.5b}$$

and $c<\xi<\eta$ being the condition for the surface Rayleigh wave. The effect of damping term introduced in equations (4.1) and (4.2) is to make the seismic waves inhomogeneous, thus α and C are complex with non-zero imaginary part. That is,

$$\alpha=\alpha_0+i\psi\alpha \tag{4.6a}$$

$$c=c_0+i\psi c \tag{4.6b}$$

but, $\alpha_0>>\psi\alpha$ and $c_0>>\psi c$.

On the earth's surface and in the far field, the waveforms are free, hence the equations (4.1) to (4.5a,b) gives

$$A\left[\xi^2\left(s^2-1\right)+2\delta c\right]+B\left[2\xi^2 s-2\delta sc\right]=0 \tag{4.7a}$$

$$A\left[2r\xi^2-2\delta sc\right]+B\left[\xi^2\left(1-s^2\right)-2\delta c\right]=0 \tag{4.7b}$$

Equations (4.7a) and (4.7b) are consistent if

$$f(c)=\left(2\xi^2-2\delta c\right)^2 rs+\left[\xi^2\left(1-s^2\right)-2\delta c\right]^2=0 \tag{4.8}$$

Eliminating r and S in equation (4.8) using (4.5), then

$$f(c)=\left(2\xi^2-2\delta c\right)^4\left[\left(\frac{\eta c}{\xi}\right)^2-1\right]-\left[\xi^2\left\{2-\left(\frac{c}{\xi}\right)^2\right\}-2\delta c\right]^4=0 \tag{4.9}$$

with ν as the medium Poisson's constant, we introduce the following notations:

$$\eta_1=\frac{\xi}{\eta}=\frac{1-2\nu}{2-2\nu},\ \nu=\frac{\wp}{2(\wp+\hat{\mu})},\ \alpha_1\frac{c}{\xi}\ \text{and}\ f(c)=f(\alpha_1)$$

Thus equation (4.9) gives

$$f(\alpha_1)=(2\xi-2\alpha_1\delta)^4\left(\eta_1^2\alpha_1^2-1\right)-\left\{\xi\left(2-\alpha_1^2\right)-2\delta\alpha_1\right\}^4=0 \tag{4.10}$$

$$=16(\xi-\alpha_1\delta)^4\left(\eta_1^2\alpha_1^2-1\right)-\left\{\xi\left(2-\alpha_1^2\right)-2\delta\alpha_1\right\}^4=0$$

Rearranging equation (4.10) in powers of α_1, we get

$$f(\alpha_1) = \alpha_1^6\left(\xi^4 - \eta_1^2\delta^4\right) + 4\delta\xi\alpha_1^5\left(\xi^2 + 2\eta_1^2\delta^2\right) - \alpha_1^4\left[8\xi^4 - \delta^2\left\{6\xi^2\left(4\eta_1^2 - 1\right) - \delta^2\left(\eta_1^2 + 1\right)\right\}\right] + 4\delta\xi\alpha_1^3\left\{\delta^2\left(2\eta_1^2 + 1\right) + 2\xi^2\left(4\eta_1^2 - 3\right)\right\} + \alpha_1^2\left\{8\xi^4\left(3 - 2\eta_1^2\right) + 24\xi^2\eta_1^2\delta^2\right\} + 16\xi^2\delta\alpha_1\left(1 - 2\eta_1\right) - 16\xi^4\left(1 - \eta_1^2\right) = 0 \quad (4.11)$$

(4.11)

In an undamped elastic medium $(\delta = 0)$, equation (4.11) reduces to the usual equation for the non – dispersive Rayleigh waves in elastic solid. In this case, the equation reduces to a cubic equation in α_1^2 which has been thoroughly analysed, [5], to obtain the propagation properties of the surface waves for a range of values of ν. Equation (4.11) therefore is an example of the case of material dispersion in which α_1 and δ are coupled. Consequently, attenuation term induces material dispersion into an otherwise non – dispersive Rayleigh surface waves in the elastic material [14].

4.2 Conclusion

Equation (4.11) is a sixth order equation and so has six roots that are complex conjugate in the α_1- plane. It cannot be reduced to a cubic equation because it contains terms involving odd powers of α_1. However, quantitative analysis,[14], suggests that

$$f(0) = -16\xi^4\left(1 - \eta_1^2\right) < 0 \quad (4.12)$$

since $0 < \eta_1 < 1$ for surface waves.

$$f(1) > 0 \quad (4.13)$$

because $\delta^3\left\{\eta_1^2\left(2\delta - 4\xi\right) + \left(\delta - 4\xi\right)\right\} - 16\xi^2\delta^2\left(1 - \eta_1^2\right) + \xi^3\left(4\delta - \xi\right) < 0$ for each term in the bracket is negative since $\eta_1 < 1$ and $\delta < \xi/4$ because, δ is small. Thus, there is a root of (4.11) in $\alpha_1 \in [0,1]$.

For $f(-1)$, we have

$$\xi^4 + 4\delta\xi^3 + 6\xi^2\delta^2\left(8\eta_1^2 - 1\right) - 4\xi\delta^3\left(4\eta_1^2 + 1\right) - \delta^4\left(2\eta^2 + 1\right) > \frac{5}{8}\xi^4 + 59\frac{7}{8}\xi^2\eta_1^2\delta^2 + 3\frac{31}{32}\xi^3\delta > 0 \quad (4.14)$$

by introducing the realistic values of ξ and η_1.

Therefore, there is at least a root of equation (4.11) between $\alpha_1 = 0$ and $\alpha_1 = -1$. In brief, there are roots of equation (4.11) in the circle of unit radius $|\alpha_1| < 1$ and none on the circumference $|\alpha_1| = 1$. To do this, sequence $\{f_n(\alpha_1)\}, n = 1,2,3,4,5$ of Sturm's, [21], are computed from the equation (4.11).

We now let $f_0(\alpha_1) = f(\alpha_1)$ as in equation (4.11);

$f_n(\alpha_1)$ be taken as the first derivative of $\{f_n(\alpha_1)\}, n = 1,2,3,4,5$. Further, let $c(0)$ be the number assigned to the changes of sign in these sequence when $\alpha_1 = 0$. Attach an identical meaning to $c(1)$ when $\alpha_1 = 1$. In this consideration, it is deduced that the difference, $c(0) - c(1)$ in $0 < \alpha_1 < 1$ depends on the assigned values of δ. From symmetry, identical conclusion applies for the difference $c(0) - c(-1)$ in $-1 < \alpha_1 < 0$.

Consequently, if $0 \le \delta \le \frac{\eta_1^2 \xi}{40}$, then $c(0) - c(1) = 1$ and there is only one real root in each of the interval, $-1 < \alpha_1 < 0$ and $0 < \alpha_1 < 1$. In this case, propagating elastic waves are undamped. With regards to the Sturm's sequence, all the leading coefficients are positive. However, for higher values of δ in the range $\frac{\eta_1^2 \xi}{40} < \delta < \frac{\xi}{4}$, the leading coefficients for $n = 2$ and $n = 3$ in the Sturm's sequence are negative and $c(0) - c(1) = 2$. Thus, in $|\alpha_1| < 1$, there are four complex conjugate roots, one in each of the four quadrants of the α_1- plane. Consequently, this analysis convincingly proves that seismic waves in an elastic solid are effectively damped if the attenuation coefficient δ inherent in the solid exceeds the value $\frac{\eta_1^2 \xi}{40}$. Thus, it is concluded that, the effectiveness of the damping of elastic vibrations in elastic solid is a function of the strength of the solid material. Put differently, the more rigid a solid is, the greater is the damping of elastic vibrations passing through it. In practice, the upper limit of $\frac{\xi}{4}$ is never attained. In particular, the complex root in the first quadrant of α_1- plane for which $\mathrm{Re}(\alpha_1) > 0$, $\mathrm{Im}(\alpha_1) > 0$ corresponds to the observed damped seismic vibration.

We now apply this result to the microseismic signals recorded on land below which is made of fairly hand rock. With this earth's structure, the phase speed, c_0 of the seismic signal ranges from 1.1kmsec^{-1} to 1.8kmsec^{-1}. Using the value $c_0 = 0.8\xi$, the corresponding value of δ is between 0.021kmsec^{-1} and 0.04kmsec^{-1}. This range of values of δ is between $\frac{\eta_1^2 \xi}{40}$ and $\frac{\xi}{4}$ suggesting strongly that the microseismic signals propagating from the source to the recording station in the far field are damped appreciably.

5.0 References

[1.] Bromwich, T.J. (1898). On the influence of gravity on elastic waves, and in particular, on the vibrations of an elastic globe. Proc. London Math. Soc., 30. Pp. 98-120.

[2.] Stoneley, R. (1924).Proc. R. Soc. A 806, pp. 416-428.

[3.] Sezawa, K. (1927). Dispersion of elastic waves propagated on the surface of stratified bodies and on curved surface. Bull. Earthq. Res. Inst. Tokyo, 3. Pp. 1-18.

[4.] Ewing, W.M., Jardetzky, W.S., Press, F. (1957): Elastic waves in layered media, (380pp), McGraw-Hill Book Company, Inc., New York

[5.] Bullen K.E. and Bolt B.A; (1985): An Introduction to the theory of Seismology Cambridge University Press.

[6.] Love, A.E.H. (1965). Some Problems of Geodynamics. Cambridge University Press, London.

[7.] Haskell, N.A. (1953). The dispersion of surface waves in multilayered media. Bull. Seis. Soc.Amer. 43. Pp. 17-34.

[8.] Biot, M.A. (1965). Mechanics of incremental Deformations, J. Willy.

[9.] Ramirez, J.E. (1940); An experimental investigation of the nature and origin of microseisms. Bulletin of the Seismological Society of America 30: 35-84, 139-78

[10.] Longuet-Higgins M.S., (1950): A Theory of the Origin of Microseisms, Phil. Trans. R. Soc. A243, 1 - 35.

[11.] Hasselmann, K; (1963): A Statistical Analysis of the Generation of the Microseisms. Rev. Geophys; 1, 177-210.

[12.] Okeke E.O and Asor V.E. (1998): Further on the Damping of low Frequency Seismic Waves in a Semi-Infinite Elastic Medium, J. of Nigeria Math Phys. 2, 48 - 55.

[13.] Bromirski, P.D., Flick, R.E. and Graham, N.(1999). Ocean wave height determined from island seismometer data and Implications for investigating wave climate changes inthe NE pacific, J. Geophys. Res., 104(c9), 20, 753-766

[14.] Asor, V.E. (2000): Effect of Earth's layering on Micro Earth Tremors, PhD Thesis, University of Benin, unpublished.

[15.] Gerstoft, P., Bromirski, P. D. and Zhang, J.(2010). Pelagic and coastal source for P-wave microseisms: Generation under tropic cyclones. Geophysical Research Letters, vol. 37, L15301, doi: 10.1029/2010GL044288.

[16.] Gurtin, M.E. and Murdoch, A.I. (1976). Effect of surface stress on wave propagation in solids. J. Appl. Phys. 47. Pp. 4414-4429.

[17.] Chandrasekharaih, D.S. (1987). Effect of surface stresses and voids on Rayleigh waves in an elastic solid. Int. J. Eng. Sci. Great Britain 25. Pp. 205-211.
[18.] Kolsky, H. (1963): Stress waves in solids, Dover Publications, Inc., (213pp)
[19.] Okeke E.O. and Asor V.E. (2000): On the microseisms associated with coastal sea waves, Geophys. J. Int.141(3), 672-678.
[20.] Bedford, A. and Drumheller, D.S. (1994). Introduction to Elastic Wave Propagation. John Wiley & Sons Ltd Chichester, England.
[21.] Kurosh A; (1980): Higher Algebra, MIR Publishers (Moscow).