Transactions of the Nigerian Association of Mathematical Physics
***Volume 2, (November, 2016*), pp 71 – 78**
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Thermal Elastic Stresses in an Infinite Thin Plate with a Circular Cylindrical Pore

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Abstract

The equations of thermo-elasticity are solved involving non-dimensional parameters and the results are obtained in integral forms. It is observed that the stresses are exponential in character and vanishes quickly with the cylindrical pore.

1.0 Introduction

The problem of a cylindrical hole embedded in an infinite medium is of significance in obtaining the results approximately valid for a thin hollow cylinder of finite height. The problem of the hollow cylinder in classical elasticity was first solved by Lame [1]. He obtained the results for the stress system submitted to uniform pressure on the inner and the outer surfaces.

In thermo-elasticity, the problem of circular hole within a cylinder was also solved in the steady state by with the surfaces free from external load. Liu and Chang [2] solved the dynamic problem of an infinitely long hollow cylinder subjected to an internal

axisymmetric blast and a sudden change in temperature, using the method of treating the time-dependent boundary conditions developed by Mindlin and Goodman [3]. Several other attempts had been made by different authors.

The general solution of steady-state two-dimensional non-axisymmetric mechanical and thermal stresses and mechanical displacements of a hollow thick cylinder made of fluid-

saturated functionally graded porous material was solved byM. Jabbari et al [4]. Jabbari et al [5] studied a general solution for mechanical and thermal stresses in a functionally graded hollow cylinder due to non-axisymmetric steady-state load Theyapplied separation of variables and Complex Fourier Series to solve the heat conduction and Navier equation. Poultangari et al.[6] presented Functionally graded hollow spheres under non-axisymmetric thermo-mechanical loads. Shariyat et al.[7] presented nonlinear transient thermal stress and elastic wave propagation analyses of thick temperature-dependent FGM cylinders, using a second-order point-collocation method. Lü Chen and Lim [8] presented elastic mechanical behavior of nano-scaled FGM films incorporating surface energies.. Lutz M.P., Zimmerman R.W.,[9] studied thermal stresses and effective thermal expansion coefficient of functionally graded sphere.

In this paper, a similar problem of a cylindrical pore in a thin plate of infinite radius is solved. The dynamic theory of thermo-elasticity is employed in which the heat conduction equation is taken independent of elastic field. The plate is assumed to be unstressed and initially to be at zero temperature. Then suddenly a uniform and constant temperature is applied on the inner surface of the hole. The heat thus flow through the plate parallel to the planes. The problem then reduces to one of a one-dimensional type and we are to find the stresses thus developed in the system. The method of solution is based on a simple theory of operators whereby much difficulties are avoided. The thermo-elastic equations are solved in terms of non-dimensional variables and the expressions for the stresses and displacements are in integral forms. It is possible to carry out the numerical computation of this of the results but this would be the subject of another paper.

2.0 Mathematical Formulations

The equation of motion in cylindrical polar coordinate system(r, θ , **z**), when the displacement vector (u_r, u_θ, u_z) is uniaxial is given by Nowaki [10]

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=m\frac{\partial T}{\partial r} \tag{1}$$

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Where,

$m = \dfrac{3\lambda + 2\mu}{\lambda + 2\mu}\alpha_T$, $\dfrac{1}{c^2} = \dfrac{\rho}{\lambda + 2\mu}$, λ *and* μ are the Lame constants and α_T is the coefficeint of thermal expansion.

The heat conduction equation when the temperature field is given as T = T (r, t) is,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{1}{k}\frac{\partial T}{\partial t} = 0 \tag{2}$$

Where k is the coefficient of thermal diffusity.

The components of stress in this case will be

$$\sigma_{rr} = \lambda\left(\mu_{,r} + \frac{1}{r}u\right) + 2\mu_{,r} - (3\lambda + 2\mu)\alpha_T T \tag{3}$$

$$\sigma_{\theta\theta} = \lambda\left(\mu_{,r} + \frac{1}{r}u\right) + 2\mu\frac{u}{r} - (3\lambda + 2\mu)\alpha_1 T \tag{4}$$

$$\sigma_{zz} = \lambda\left(\mu_{,r} + \frac{1}{r}u\right) - (3\lambda + 2\mu)\alpha_1 T \tag{5}$$

$$\sigma_{r\theta} = \sigma_{rz} = \sigma_{\theta z}$$

The non-dimensional variables are introduced as follows:

$$U = \frac{u}{mT_0 b},\ \rho = \frac{r}{b},\ r = \frac{kt}{b^2},\ \theta = \frac{T}{T_0} \tag{6}$$

where

$b = \dfrac{k}{c}$, The equations (1) and (2) now gives

$$\frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho} - \frac{U}{\rho^2} - \frac{\partial^2 U}{\partial r^2} = \frac{\partial \theta}{\partial \rho} \tag{7}$$

and

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \theta}{\partial \rho} - \frac{\partial \theta}{\partial r} = 0 \tag{8}$$

Putting $u = \phi_{,\rho}$ in equation (2.7) and integrating once with respect to ρ, we get

$$\frac{\partial^2 \phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \phi}{\partial \rho} - \frac{\partial^2 \phi}{\partial r^2} = 0 \tag{9}$$

where ϕ is the non-dimensional displacement potential. The equations (3) – (4) also reduce to the forms

$$\sigma_{\rho\rho} = \frac{\sigma_{rr}}{mT_0(\lambda + 2\mu)} = \frac{\partial^2 \phi}{\partial \rho^2} + \frac{\upsilon}{1-\upsilon}.\frac{1}{\rho}\frac{\partial \phi}{\partial \rho} - \theta, \tag{10}$$

$$\sigma_{\phi\phi} = \frac{\sigma_{\theta\theta}}{mT_0(\lambda + 2\mu)} = \frac{\upsilon}{1-\upsilon}..\frac{\partial^2 \phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \phi}{\partial \rho} - \theta, \tag{11}$$

and

$$\sigma_{\xi\xi} = \frac{\sigma_{zz}}{mT_0(\lambda + 2\mu)} = \frac{\upsilon}{1-\upsilon}.\left[\frac{\partial^2 \phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \phi}{\partial \rho}\right] - \theta \tag{12}$$

where υ is the Poisson's ratio.

3.0 Method of Solution

Eliminating θ from equations (8) and (9), we get,

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial}{\partial r}\right)\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial r^2}\right)\phi = 0 \tag{13}$$

The solution of this equation can be taken in the form

$$\phi = \phi_1 + \phi_2 \tag{14}$$

where ϕ_1 is the solution of the equation

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial r^2}\right)\phi_1 = 0 \tag{15}$$

And ϕ_2 is that of

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial r^2}\right)\phi_2 = 0 \tag{16}$$

Substituting equation (14) in equation (9) and using (15), we get

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial r^2}\right)\phi_2 = \theta \tag{17}$$

Subtracting equation (r17) in equation (16), we have

$$\left(\frac{\partial^2}{\partial r^2} - \frac{\partial}{\partial r}\right)\phi_2 = -\theta \tag{18}$$

The solution of equation (18) is

$$\phi_2 = f_1(\rho) + e^{r} f_2(\rho) - \int e^{r}\left\{\int e^{-e\theta d r}\right\} d r \tag{19}$$

where $f_1(\rho)$ *and* $f_2(\rho)$ are the functions of ρ only and are such that equation (19) satisfies equation (16). Substituting equation (19) into equation (16), we get

$$f_1'(\rho) + \frac{1}{\rho} f_1(\rho) = \chi(\rho) \tag{20}$$

and

$$f_2''(\rho) + \frac{1}{\rho} f_2(\rho) - f_2(\rho) = 0 \tag{21}$$

where $\chi(\rho)$ is a known function of ρ only and is given as

$$\chi(\rho) = \left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial}{\partial r}\right)\int e^{r}\left\{\int e^{-e\theta d r}\right\} dr \tag{22}$$

Thus by solving equations (20) and (21)

$$f_1(\rho) = A\log\rho + B + F(\rho) \tag{23}$$

and

$$f_2(\rho) = C I_{(f}\rho) + D K_0(\rho) \tag{24}$$

where

$$F(\rho) = \int\frac{1}{\rho}\left\{\int \rho\,\chi(\rho)\, d\rho\right\} d\rho \tag{25}$$

The general solution of equation (15) can be taken in the form

$$\phi_1 = \int_{\delta}^{\infty} e^{-r\alpha}\left[N(\alpha)\, I_0(\alpha\rho) + M(\alpha)\, K_0(\alpha\rho)\right]d\alpha, \qquad \delta > 0 \tag{26}$$

where δ is small. Since the solution does not exist at $\alpha = 0$, the value at $\alpha = 0$ is excluded from the general solution (26). Thus the complete solution of equation (13) is

$$\phi = \int_{\delta}^{\infty} e^{-\alpha\tau}\left[N(\alpha)\, I_0(\alpha\rho) + M(\alpha) K_0(\alpha\rho)\right]d\alpha + A\log\rho + B + F(\rho) - \int e^{\tau}\left\{\int e^{-\tau}\theta\, d\tau\right\}\partial r \ldots (27)$$

where the term $e^{r} f_2(\rho)$ is merged in the value of $\emptyset_1$. The constants A and B and the function $N(\alpha)$ and $M(\alpha)$ can be determined by the boundary conditions of the problem.

4.0 Boundary Conditions

The surface of the hole is assumed initially at zero temperature and then suddenly heated and maintained at constant temperature T_0 uniformly. The surface is, otherwise, assumed to be free from external traction. The boundary conditions are therefore,

$$T = 0,\ t \le 0,\ \ and\ r \ge R \tag{28}$$

$$T = T_0\ ,\ r = R,\ \ and\ \ t > 0, \tag{29}$$

$$T = 0\ as\ r \to \infty,\ \ for\ \ all\ \ t > 0 \tag{30}$$

$$\sigma_{rr} = 0\ at\ r \to R,\ \ for\ \ all\ \ t > 0 \tag{31}$$

and all stresses should be zero at infinity.
In the form of non-dimensional variables, given as in equation (6), these boundary conditions reduce to :

$$\theta = 0,\quad \tau \le 0 \tag{32}$$

$$\theta = 1,\quad \rho = \alpha\left(= \frac{E}{b}\right),\ \ \tau > 0 \tag{33}$$

$$\theta = 0\ as\quad \rho \to \infty,\ \ for\ \ all\quad \tau > 0 \tag{34}$$

and

$$\sigma_{\rho\rho} = 0\ \ at\ \rho = \alpha \tag{35}$$

where R is the radius of cavity and α is the non-dimensional radius.

5.0 Solution

For finite displacement and to satisfy the regularity conditions at infinity, we equate $N(\alpha)$ to zeroin the equation (27). Hence $\emptyset$ becomes

$$\phi = \int_{\delta}^{\infty} e^{-\alpha\tau}\left[M(\alpha)K_0(\alpha\rho)\right]d\alpha + A\log\rho + B + F(\rho) - \int e^{\tau}\left\{\int e^{-\tau}\theta\, d\tau\right\}\partial r. \tag{36}$$

The solution of the heat conduction equation (8) with the boundary conditions (32) to (35) is given as in (Carslaw and Jeagar, (1969)[11]

$$\theta = -1 + \frac{2}{\pi}\int_0^{\infty} e^{-x^2\tau}\ \frac{J_0(x,\rho)\,Y_0(xa) - J_0(xa)\,Y_0(x\rho)}{J_0^2(x,a) + Y_0^2(x,a)}\cdot\frac{dx}{x} \tag{37}$$

Substituting the value of θ from equation (37 into equation (36) and simplifying, we get

$$\phi = \int_{\delta}^{\infty} e^{-\alpha\tau}\left[M(\alpha)K_0(\alpha\rho)\right]d\alpha + A\log\rho + B + F(\rho)$$
$$-\frac{2}{\pi}\int_{\delta^2}^{\infty}\frac{e^{-x^2\tau}}{x^3(x^2+1}\cdot\frac{J_0(x,\rho)\,Y_0(xa) - J_0(xa)\,Y_0(x\rho)}{J_0^2(x,a) + Y_0^2(x,a)}.dx \tag{38}$$

The integral from 0 to δ^2 can be neglected as the integrand → 0 as $x \to 0$. Substituting the value of θin $\chi(\rho)$ and then in F (ρ), we have

$$F(\rho) = \frac{1}{4}\rho^2 \tag{39}$$

Replacing $x^2 by\alpha$ and substituting the value of the function $F(\rho)$ from equation (39) in equation (38), we get

$$\phi = \int_{\delta}^{\infty} e^{-\alpha\tau}\left[M(\alpha)K_0(\alpha\rho)\right]d\alpha + A\log\rho + B + F(\rho)$$
$$-\frac{2}{\pi}\int_{\delta^2}^{\infty}\frac{e^{-x^2\tau}}{x^3(x^2+1}\cdot\frac{J_0(x,\rho)\,Y_0(xa) - J_0(xa)\,Y_0(x\rho)}{J_0^2(x,a) + Y_0^2(x,a)}.dx \tag{40}$$

Differentiating equation (40) with respect toρ, we have

$$\frac{\partial\phi}{\partial\rho}=\int_{\delta}^{\infty}e^{-\alpha\tau}\left[\alpha M(\alpha)K_0'(\alpha\rho)\right]d\alpha+\frac{A}{\rho}+\frac{1}{2}\rho$$

$$-\frac{2}{\pi}\int_{\delta^2}^{\infty}\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}.\sqrt{\alpha}\;\frac{J_0'(\rho\sqrt{\alpha})\,Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})\,Y_0'(\rho\sqrt{\alpha})}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})}.d\alpha \qquad (41a)$$

and

$$\frac{\partial^2\phi}{\partial\rho^2}=\int_{\delta}^{\infty}e^{-\alpha\tau}\left[\alpha\;M(\alpha)K_0'''(\alpha\rho)\right]d\alpha-\frac{A}{\rho^2}+\frac{1}{}$$

$$-\frac{2}{\pi}\int_{\delta^2}^{\infty}\frac{e^{-\alpha\tau}}{2\alpha(\alpha+1)}.\;\frac{J_0''(\rho\sqrt{\alpha})\,Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})\,Y_0''(\rho\sqrt{\alpha})}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})}.d\alpha \qquad (41b)$$

Now substituting from equations (41a) through (41b) into equation (10), we have

$$\sigma_{\rho\rho}=\int_{\delta}^{\infty}\alpha M\,e^{-\alpha\tau}\left\{\alpha K_0''(\alpha\rho)+\frac{\upsilon}{1-\upsilon}.\frac{1}{\rho}K_0'(\alpha\rho)\right\}dx\;-\frac{1}{2}\left(1+\frac{2A}{\rho^2}\right).\frac{1-2\upsilon}{1-\upsilon}$$

$$-\frac{2}{\pi}\int_{\delta}^{\infty}\frac{e^{-\alpha\tau}}{2\alpha(\alpha+1)}.\left[\begin{array}{l}\left\{\dfrac{J_0''(\rho\sqrt{\alpha})\,Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})\,Y_0''(\rho\sqrt{\alpha})}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})}\right\}\\+\dfrac{\upsilon}{1-\upsilon}.\dfrac{1}{\rho\sqrt{\alpha}}\\\times\left\{J_0'(\rho\sqrt{\alpha})\,Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})\,Y_0'(a\sqrt{\alpha})\right\}\\+(\alpha+1)\left\{J_0(x,\rho)\,Y_0(xa)-J_0(xa)\,Y_0(a\sqrt{\alpha})\right\}\end{array}\right]$$

$$\times\frac{dx}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})} \qquad (42)$$

Using the boundary conditions in equation (34) in equation (42), we have

$$0=\int_{\delta}^{\infty}e^{-\alpha\tau}\left[\begin{array}{l}M\alpha\left\{\alpha K_0''(\alpha a)+\dfrac{\upsilon}{1-\upsilon}.\dfrac{1}{\alpha}K_0'(a\alpha)\right\}\\-\dfrac{2}{\pi}.\dfrac{1}{2\alpha(\alpha+1)}.\dfrac{2}{\pi a^2\alpha}.\dfrac{1-2\upsilon}{1-\upsilon}.\dfrac{1}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})}\end{array}\right]dx \qquad (43)$$

$$-\frac{1}{2}.\left(1+\frac{2A}{\rho^2}\right).\frac{1-2\upsilon}{1-\upsilon}$$

Thus we have,

$$A=\frac{1}{2}a^2 \qquad (44)$$

and

$$M(\alpha) = \frac{1}{\alpha^2(\alpha+1)} \cdot \frac{2}{\pi a^2 \alpha^2} \cdot \frac{1-2\upsilon}{1-\upsilon} \cdot \frac{1}{J_0^2(a\sqrt{\alpha}) + Y_0^2(\alpha\sqrt{\alpha})} \times \frac{1}{\alpha K_0''(a\alpha) + \frac{\upsilon}{1-\upsilon} \cdot \frac{1}{\alpha} K_0'(a\alpha)} \qquad (45)$$

Hence we have,

$$\sigma_{\rho\rho} = \frac{2}{\pi}\int_\delta^\infty Z\, d\alpha - \frac{1}{2}\left(1 - \frac{a^2}{\rho^2}\right) \cdot \frac{1-2\upsilon}{1-\upsilon} \qquad (46)$$

Where

$$Z = \frac{e^{-\alpha\tau}}{2a^2(\alpha+1)} \left[\begin{array}{l} \frac{2}{\pi a^2} \cdot \frac{1-2\upsilon}{1-\upsilon} \cdot \frac{\alpha k_0(\alpha\rho) + \frac{1}{\rho} \cdot \frac{1-2\upsilon}{1-\upsilon} \cdot k_1(\alpha\rho)\,.}{\alpha k_0(a\alpha) + \frac{1}{a} \cdot \frac{1-2\upsilon}{1-\upsilon} \cdot k_1(a\alpha)} \\ + \frac{1-2\upsilon}{1-\upsilon} \frac{\sqrt{\alpha}}{\rho} \left\{ J_0(a\sqrt{\alpha})\, Y_1(\rho\sqrt{\alpha}) - J_1((\rho\sqrt{\alpha}))\, Y_0(a\sqrt{\alpha}) \right\} \\ -\alpha^2 \left\{ J_0(a\sqrt{\alpha})\, Y_1(\rho\sqrt{\alpha}) - J_1((\rho\sqrt{\alpha}))\, Y_0(a\sqrt{\alpha}) \right\} \end{array} \right] \times \frac{dx}{J_0^2(a\sqrt{\alpha}) + Y_0^2(\alpha\sqrt{\alpha})} \qquad (47)$$

Substituting the value of $M(\alpha)$ and A in equation (41 a), we obtain the displacement as:

$$U(\rho,T) = \frac{\partial\phi}{\partial\rho} = \frac{2}{\pi}\int_\delta^\infty \frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)} \left\{ \begin{array}{l} -\frac{2}{\pi a^2} \cdot \frac{1-2\upsilon}{1-\upsilon} \cdot \frac{K_1.(a\alpha)}{\alpha k_0(a\alpha) + \frac{1}{a}\frac{1-2\upsilon}{1-\upsilon} \cdot K_1(a\alpha)} \\ +\sqrt{a} \left\{ \begin{array}{l} J_1((\rho\sqrt{\alpha}))\, Y_0(a\sqrt{\alpha}) \\ -J_0(\rho\sqrt{\alpha})\, Y_1(a\sqrt{\alpha}) \end{array} \right\} \end{array} \right\} \times \frac{dx}{J_0^2(a\sqrt{\alpha}) + Y_0^2(\alpha\sqrt{\alpha})} - \frac{1}{2\upsilon}(a^2 - \rho^2). \qquad (48)$$

Substituting the values of equations (41a.b),(44) and (45) into equations (4) and (5), we have,

$$\sigma_{\phi\phi} = \frac{2}{\pi}\int_{\delta}^{\propto}\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}\left[\begin{array}{l} \frac{2}{\pi a^2}\cdot\frac{1-2\upsilon}{1-\upsilon}\cdot\frac{\frac{\upsilon}{1-\upsilon}\alpha K_0(\alpha\rho)-\frac{1}{\rho}\cdot\frac{1-2\upsilon}{1-\upsilon}.\,K_1(\alpha\rho)\,.}{\alpha K_0(a\alpha)+\frac{1}{a}\cdot\frac{1-2\upsilon}{1-\upsilon}.\,K_1(a\alpha)} \\ +\ \frac{1-2\upsilon}{1-\upsilon}\frac{\sqrt{\alpha}}{\rho}\left\{J_1(\rho\sqrt{\alpha})\,Y_0(\rho\sqrt{\alpha})-J_0((\rho\sqrt{\alpha}))\,Y_1(\rho\sqrt{\alpha})\right\} \\ -\alpha\left(\alpha+\frac{1-2\upsilon}{1-\upsilon}\right)\left\{J_0(\rho\sqrt{\alpha})\,Y_0(\alpha\sqrt{\alpha})-J_0((\alpha\sqrt{\alpha}))\,Y_0(\rho\sqrt{\alpha})\right\} \end{array}\right]$$

$$\times\ \frac{dx}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})}\ -\ \frac{1-2\upsilon}{2(1-\upsilon)}\left(1-\frac{a^2}{\rho^2}\right) \qquad (49)$$

and

$$\sigma_{\xi\xi}\ \frac{2\upsilon}{\pi(1-\upsilon)}\int_{\delta}^{\propto}\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}\left[\begin{array}{l} \frac{2}{\pi a^2}\cdot\frac{1-2\upsilon}{1-\upsilon}\cdot\frac{\alpha K_0(\alpha\rho)\ \ .}{\alpha K_0(a\alpha)+\frac{1}{a}\cdot\frac{1-2\upsilon}{1-\upsilon}.\,K_1(a\alpha)} \\ -\ \left\{J_0(\rho\sqrt{\alpha})\,Y_0(\alpha\sqrt{\alpha})-J_0((\alpha\sqrt{\alpha}))\,Y_0(\rho\sqrt{\alpha})\right\} \end{array}\right]$$

$$\times\ \frac{dx}{J_0^2(a\sqrt{\alpha})+Y_0^2(\alpha\sqrt{\alpha})} \qquad (50)$$

6.0 Conclusion

In this paper, the equations of thermo-elasticity are solved involving non-dimensional parameters and the results are obtained in integral forms. It is observed that the stresses are exponential in character and vanishes quickly with the cylindrical pore.The research work offers an in-depth explanation of characteristics of thermo-elastic materials and expressed them mathematically. This could offer great assistance to the industrialists and engineers to better their production and service delivery. It could also be a reference material to scholars of solid mechanics

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